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Low lying $K = 0^+$ Bands in Gadolinium Nuclei

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Abstract. In this paper we investigate the nature of the low-lying excited 0^+ states in ^{156,158,160}Gd nuclei, in the framework of the pseudo-SU(3) model. We study the energy levels, their wave function contents, and the corresponding $B(E2)$ transitions related to the excited 0^+ states. The theory uses a realistic Hamiltonian with single-particle energies and monopole pairing interactions in addition to the usual quadrupole-quadrupole term, with the pairing and quadrupole-quadrupole strengths taken from systematics. The calculations are carried out with a set of basis states with pseudo-spin zero proton and neutron configurations. The calculated $B(E2)$ values suggest that the first excited 0^+ state might have a mixed nature of collective vibration, possible with quasiparticle excitations that are not considered in this calculation.

1 Introduction

Heavy deformed nuclei in the lanthanide region have a large number of $J^\pi = 0^+$ states at relatively low excitation energies (between 1-3 MeV). In the experiment [1] based on the $^{160}Gd(p,t)^{158}Gd$ reaction, 13 low-lying excited 0^+ states were detected in ^{158}Gd below 3 MeV. The nature of these states is not quite understood. Calculations using the IBA model [2], the projected shell model [3] and the quasiparticle-phonon model [4] have answered partially this question and showed the complicated nature of these states.

In this paper, we study the energy levels, their wave function contents, and the corresponding $B(E2)$ transitions in ^{156,158,160}Gd nuclei, in the framework of the pseudo-SU(3) model.

2 Model Space and Interaction

The building blocks of the model are the pseudo-SU(3) proton and neutron states with pseudo spin zero. Many-particle states are built as pseudo-SU(3) coupled states with well defined particle number and total angular momentum [5,6]. Nucleons occupying the intruder orbits are considered implicitly through the use of effective charges. To determine occupancies of the normal- and unique-parity sectors, a deformed Nilsson single-particle scheme is employed, with levels

filled from the bottom up. Since in a quadrupole-quadrupole driven hamiltonian, the states corresponding to highest deformation are the most important, we extract from this scheme the proton and neutron SU(3) irreducible representations (irreps) corresponding to the highest C_2 values which, in turn, are coupled to final SU(3) irreps that have good total angular momentum.

For ^{156}Gd nucleus, from all of the allowed proton (32) and neutron (48) SU(3) irreps with pseudo-spin zero, five proton and five neutron irreps, corresponding to highest deformations, were used to obtained resultant coupled SU(3) irreps. And out of a total of 1774 such coupled irreps, the first thirty with the largest C_2 values were chosen to build the configuration space. These irreps are given in Table 1. Similar procedure was used to determine the irreps used in the calculation for ^{158,160}Gd nuclei.

Table 1. SU(3) irreps (obtained by coupling five proton and five neutron irreps with pseudo-spin zero) used to describe the low-energy spectra of ^{156}Gd.

(λ_π, μ_π)	(λ_ν, μ_ν)	(λ, μ)
(10,4)	(18,0)	(28,4), (26,5), (27,3),(24,6)
(10,4)	(15,3)	(25,7), (26,5), (27,3), (28,1), (23,8), (24,6)
(10,4)	(12,6)	(22,10),(23,8), (24,6), (20,11)
(10,4)	(13,4)	(23,8), (24,6)
(10,4)	(14,2)	(24,6)
(12,0)	(18,0)	(30,0), (28,1)
(12,0)	(15,3)	(27,3)
(12,0)	(12,6)	(24,6)
(8,5)	(18,0)	(26,5), (24,6)
(8,5)	(15,3)	(23,8), (24,6)
(8,5)	(12,6)	(20,11)
(5,8)	(18,0)	(23,8)
(5,8)	(15,3)	(20,11)
(9,3)	(18,0)	(27,3)
(9,3)	(15,3)	(24,6)

The Hamiltonian includes spherical Nilsson single-particle terms for the protons and neutrons, $H_p^{sp} + H_n^{sp}$, the quadrupole-quadrupole ($Q \cdot Q$) and pairing interactions, H_p^P and H_n^P, with interaction strengths taken from systematics, as well as four 'rotor-like' terms which preserve the SU(3) symmetry,

$$H = H_p^{sp} + H_n^{sp} + G_p H_p^P + G_n H_n^P + \chi QQ + aL^2 + bK_J^2 + a_{sym} C_2 + a_3 C_3. \quad (1)$$

The strength of the quadrupole-quadrupole ($Q \cdot Q$) and pairing interactions (H_σ^P) were fixed, respectively, at values typical of those used by other authors, namely, $\chi = 35A^{5/3}$ MeV, $G_\pi = 21/A$ MeV and $G_\nu = 19/A$ MeV. The spherical single-particle terms in this expression have the form:

$$H_\sigma^{sp} = \hbar\omega \sum_{i_\sigma} (-2\kappa_\sigma \mathbf{l}_{i_\sigma} \cdot \mathbf{s}_{i_\sigma} - \kappa_\sigma \mu_\sigma \mathbf{l}_{i_\sigma}^2), \quad (2)$$

where $\sigma = p$ for protons, and $\sigma = n$ for neutrons. The single-particle orbit-orbit (l^2) and spin-orbit ($l \cdot s$) interaction strengths were fixed by systematics [11].

The parameters (a, b, a_{sym}, a_3) of the 'rotor-like' part of the Hamiltonian are used for fitting the band-head energy of the first excited $K^\pi = 0^+$, $K^\pi = 2^+$, and $K^\pi = 1^+$ states, and the moment of inertia of the ground band. Parameter values, given in Table 2, are close to those used in the description of neighboring even-even and odd-A nuclei [7, 8]. The single-particle terms together with the proton and neutron pairing interaction mix the SU(3) basis states allowing for a realistic description of the inter-band $B(E2)$ transitions.

Table 2. Coefficients used in the SU(3) Hamiltonian for ^{156,158,160}Gd.

coefficients	^{156}Gd	^{158}Gd	^{160}Gd
$a_3 \times 10^{-4}$	0.94	1.57	1.93
a_{sym}	0.00045	0.0035	0.0035
a	0.003	0.00	0.001
b	0.155	0.18	0.153

3 Results and Analysis

The experimental energy values [9] of the excited $K^\pi = 0^+$, and $K^\pi = 2_1^+$ states in even-even nuclei display interesting behavior. The first one increases from ^{154}Gd to ^{160}Gd, while the latter increases from 0.99 MeV in ^{154}Gd to 1.19 MeV in ^{158}Gd, and then decreases back to 0.99 MeV in ^{160}Gd (see Table 3). Since the energy values of $K^\pi = 2^+$ states are an indication of the internal deformation of the nucleus, it is interesting to notice the way the energy values of the $K^\pi = 0^+$, and $K^\pi = 2_1^+$ states vary with the number of neutrons.

Table 3. Experimental energies for the excited $K^\pi = 0_2^+$ and $K^\pi = 2_1^+$ states in four Gadolinium nuclei.

	nucleus			
	^{154}Gd	^{156}Gd	^{158}Gd	^{160}Gd
Energy [MeV] of $K^\pi = 0_2^+$	0.68	1.05	1.19	1.33
Energy [MeV] of $K^\pi = 2_2^+$	0.99	1.15	1.19	0.99

The low energy spectra of ^{156,158,160}Gd nuclei is given in Figure 1. Figure 1 shows few more bands in each nucleus. Using the eigenvalues obtained from diagonalizing the Hamiltonian, the $B(E2)$ transitions were calculated. We used the $E2$ operator given by [10] :

$$Q_\mu = e_\pi Q_\pi + e_\nu Q_\nu \approx e_\pi \frac{\eta_\pi + 1}{\eta_\pi} \tilde{Q}_\pi + e_\nu \frac{\eta_\nu + 1}{\eta_\nu} \tilde{Q}_\nu. \qquad (3)$$

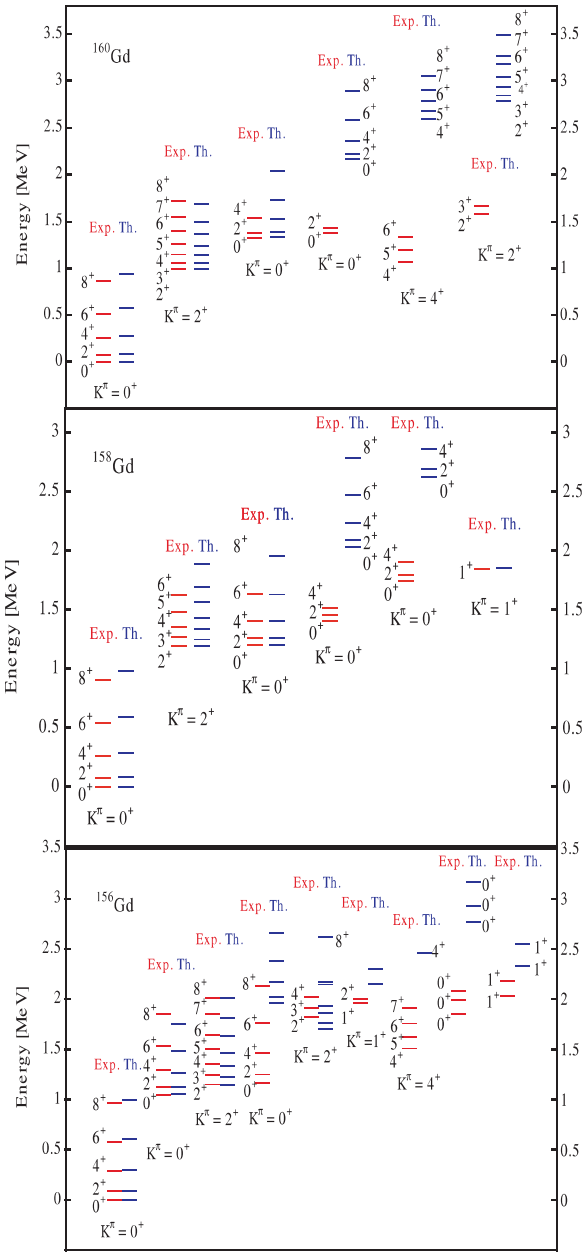

Figure 1. Experimental and calculated energy levels in ^{156,158,160}Gd nuclei.

with the effective charges e_π and e_ν. To account for the contribution to the quadrupole moments from nucleons in the unique parity orbitals, the values of the effective charges are larger than those used in standard calculations of $B(E2)$ strengths [11].

The lowest three energy bands are described well using only a few even-even irreps with pseudo-spin zero. The SU(3) content of calculated eigenvectors for states in the ground-state, $K = 2^+$, and $K = 0_2^+$ bands in ^{156}Gd are given in Table 4. The percentage distribution of each eigenvector across the (λ, μ) values are given in the last column. All the basis states that contribute more than 2% are identified. These percentages are similar for states within the same band. For the states in the ground state and $K = 2^+$ bands, these percentages are also similar.

Table 4. SU(3) content of eigenstates for the $K = 0_1^+$, $K = 2_\gamma^+$, $K = 0_2^+$, and $K = 0_3^+$ band-heads for ^{156}Gd, calculated with the set of basis states described in the text. All the basis states that contribute more than 2% are identified.

J_k	(λ, μ)	(λ_π, μ_π)	(λ_ν, μ_ν)	%
0_1	(28, 4)	(10, 4)	(18, 0)	49.4
	(30, 0)	(12, 0)	(18, 0)	8.3
	(22, 10)	(10, 4)	(12, 6)	3.5
	(24, 6)	(10, 4)	(14, 2)	34.5
2_γ	(28, 4)	(10, 4)	(18, 0)	45.6
	(22, 10)	(10, 4)	(12, 6)	6.3
	(24, 6)	(10, 4)	(14, 2)	42.2
0_2	(28, 4)	(10, 4)	(18, 0)	2.3
	(22, 10)	(10, 4)	(12, 6)	74.4
	(24, 6)	(10, 4)	(14, 2)	5.2
	(24, 6)	(12, 0)	(12, 6)	16.0
0_3	(24, 6)	(10, 4)	(15, 3)	2.5
	(22, 10)	(10, 4)	(12, 6)	4.9
	(24, 6)	(10, 4)	(12, 6)	74.4
	(24, 6)	(12, 0)	(12, 6)	14.2

The dominant $SU(3)$ irreps in the wave functions of $K = 0_1^+$ and $K = 2_1^+$ states are the same. The percentage distribution changes a lot in the second and third $K = 0^+$ states. The dominant $SU(3)$ irreps in the second and third $K = 0^+$ states are different than the ones in the ground state, and different from each other. The SU(3) content of calculated eigenvectors for states in the ground-state, $K = 2^+$, and $K = 0_2^+$ bands in ^{156,158,160}Gd are given in Table 5 for comparison. In all three nuclei, the dominant $SU(3)$ irreps are the same for the $K = 0_1^+$ and $K = 2_1^+$ states, and different for $K = 0_2^+$ state. The percentages of the dominants irreps are changing slightly from $K = 0_1^+$ to $K = 2^+$, but still the dominant irreps are the same ones. Even though, the energy value of

Table 5. $SU(3)$ content in [%] of wave functions of the collective ground state, $K^\pi = 2^+$ and $K^\pi = 0_2^+$ states in the three considered nuclei. All the basis states that contribute more than 2% are identified.

nucl	state	$(\lambda_\pi, \mu_\pi) \otimes (\lambda_\nu, \mu_\nu) \rightarrow (\lambda, \mu)$	$0_{g.s.}$	2_γ^+	0_2^+
^{156}Gd	a	$(10,4) \otimes (18,0) \rightarrow (28,4)$	49.4	45.6	2.3
	b	$(10,4) \otimes (12,6) \rightarrow (22,10)$	3.5	6.3	74.4
	c	$(10,4) \otimes (14,2) \rightarrow (24,6)$	2.9	2.8	5.2
	d	$(10,4) \otimes (14,2) \rightarrow (24,6)$	8.2	9.2	
	e	$(10,4) \otimes (14,2) \rightarrow (24,6)$	23.4	30.2	
	f	$(12,0) \otimes (18,0) \rightarrow (30,0)$	8.3		
	g	$(12,0) \otimes (12,6) \rightarrow (24,6)$			16.0
^{158}Gd	a	$(10,4) \otimes (18,4) \rightarrow (28,8)$	49.1	81.6	12.4
	b	$(10,4) \otimes (18,4) \rightarrow (30,4)$			65.5
	c	$(10,4) \otimes (20,0) \rightarrow (30,4)$	14.6	5.8	2.9
	d	$(12,0) \otimes (18,4) \rightarrow (30,4)$	22.0	12.2	2.6
	e	$(10,4) \otimes (18,4) \rightarrow (32,0)$	3.0		13.0
	f	$(12,0) \otimes (20,0) \rightarrow (32,0)$	9.4		3.7
^{160}Gd	a	$(10,4) \otimes (18,4) \rightarrow (28,8)$	62.7	85.5	11.1
	b	$(10,4) \otimes (18,4) \rightarrow (30,4)$			63.7
	c	$(10,4) \otimes (20,0) \rightarrow (30,4)$	11.0	4.4	5.6
	d	$(12,0) \otimes (18,4) \rightarrow (30,4)$	17.5	9.3	5.6
	e	$(10,4) \otimes (18,4) \rightarrow (32,0)$	2.4		7.9
	f	$(12,0) \otimes (20,0) \rightarrow (32,0)$	5.6		6.0

the $K = 2_1^+$ state is higher than the one of the $K = 0_2^+$ state in ^{156}Gd, and the energy values are reversed in ^{160}Gd, the distribution of the dominant $SU(3)$ irreps follow the same pattern. This pattern in the percentage distribution of the wave function holds in the case when the μ quantum number is non-zero. The situation would have been different if $\mu = 0$ in the dominant irrep [12].

The $B(E2)$ transition probabilities between states within the ground ($K^\pi = 0^+$), $K = 2^+$, and excited $K = 0_2^+$ bands were calculated and compared to the experimental data in Table 6. These transitions are very collective, as shown by the fact that their average value is about 200 W.u., and are well reproduced within quoted experimental uncertainties.

The inter-band $B(E2)$ transitions are of particular interest because their values are a factor of approximately 100 smaller than the collective intra-band transitions. As a consequence, they are sensitive to the parameters of the model. Their values are given in Table 7. As shown, good agreement – within a factor of two – was obtained for transitions between the states in the $K = 2^+$ band and the states in the ground state band.

As for the $B(E2)$ transitions between states in the ground state band and the states in the $K = 2^+$ band, the transitions between the states in the $K = 0_2^+$ band and the states in the ground state band are very sensitive to the model

Table 6. Experimental and calculated $B(E2)$ transition probabilities within the bands in ^{156}Gd. Experimental data are taken from [9].

$B(E2; J_i, K_i^\pi \to J_f, K_f^\pi)$[W.u.]											
J_i	J_f	Exp.	Th.	J_i	J_f	Exp.	Th.	J_i	J_f	Exp.	Th.
$K^\pi = 0^+_{g.s.} \to K^\pi = 0^+_{g.s.}$				$K^\pi = 0^+_2 \to K^\pi = 0^+_2$				$K^\pi = 2^+_1 \to K^\pi = 2^+_1$			
2	0	187 (5)	187	2	0		171	3	2		325
4	2	236 (5)	265	4	2		245	4	2		108
6	4	295 (8)	289	6	4		278	5	3		172
8	6	320 (17)	297	8	6		296	5	4		174
								6	4		203
								7	6		97
								8	6		235

parameters. The ability of the model to reproducing these values is a strong confirmation of the validity of the theory. These transitions are about one order of magnitude smaller then the former ones. The calculated values for these transition probabilities are about a quarter of their experimental counterparts.

One strength of our model is the correct reproduction of the $B(E2)$ transition between the band head of the γ-band, the $K = 2^+$ state, and the 0^+ of the ground state. Reproducing this transition means that the model reproduces the correct mixing in the wave-functions of these two states. In the same time, all the transition between the states in the $K^\pi = 2^+$, and $K^\pi = 0^+_1$ are well reproduced.

Table 7. Experimental and calculated inter-band $B(E2)$ transition probabilities in ^{156}Gd. Experimental data are taken from [9]

$K^\pi = 0^+_2 \to K^\pi = 0_{g.s.}$		
$J_i \to J_f$	Exp.[1]	Th.
0 → 2	4.18 → 10.97	0.50
2 → 0	0.55 → 1.13	0.29
2 → 2	3.06 → 6.28	0.30
2 → 4	3.58 → 7.35	0.88
4 → 2	0.94 → 2.11	0.88
4 → 4	1.97 → 4.45	0.54
4 → 6	1.58 → 3.57	1.60

The experimental and calculated $B(E2; 0^+_2 \to 2_{g.s})$ and $B(E2; 2^+_{K=2} \to 0_{g.s.})$ transitions are also given in Table 8. The calculated $B(E2)$ values for the first transition are smaller than the experimental ones for all three nuclei. The experimental $B(E2)$ value decreases by about a factor of five from ^{156}Gd to ^{158}Gd. This trend is reproduced by the calculations. However, the calculated $B(E2)$ values for the latter transitions are over estimated by about a factor of two

Table 8. Calculated and experimental values for two $B(E2)$ transitions in three Gadolinium nuclei are included. Experimental data for ^{156}Gd are extracted from [14]. Experimental data for ^{158,160}Gd extracted from [13]

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Exp. $B(E2; 0_2^+ \rightarrow 2_{g.s.})$[W.u]	$4.18 \rightarrow 10.97$	1.2	
Th. $B(E2; 0_2^+ \rightarrow 2_{g.s.})$[W.u]	0.5	0.08	0.07
Exp. $B(E2; 2_{K=2}^+ \rightarrow 0_{g.s.})$[W.u]	$2.73 \rightarrow 4.25$	3.4(3)	3.80(22)
Th. $B(E2; 2_{K=2}^+ \rightarrow 0_{g.s.})$[W.u]	6.4	6.8	8.1

for all three nuclei. Having reproduced the correct trend in the $B(E2)$ values for these transitions in all three nuclei, we conclude that the missing $B(E2)$ strength from the excited 0^+ state may come from other excitations and configurations, that are not considered in this model.

4 Conclusion

This work demonstrates the usefulness of the pseudo-SU(3) model as a shell model, one that can be used to describe deformed rare-earth and actinide isotopes by performing a symmetry dictated truncation of the Hilbert space. The study further shows that pseudo-spin zero neutron and proton configurations with a relatively few pseudo-SU(3) irreps with largest C_2 values suffice to obtain good agreement with known experimental results. The good agreement between calculated and experimental results was found not only for the low-lying states in the ground ($K = 0^+$) but for the gamma ($K = 2^+$) and the excited $K = 0_2^+$ bands as well. The theory also successfully described the intra-band $B(E2)$ transitions. Especially important is the description of the inter-band $B(E2)$ transitions, between the states in the $K = 0$, $K = 2$, and the second $K = 0$ bands, since in this case the transition probabilities are a factor of approximately 100 smaller than the intra-band transitions.

To better understand the mechanism behind the low-lying $K = 0^+$ bands, we need to further study in detail more $K = 0^+$ bands and in more neighboring nuclei. Since the $SU(3)$ symmetry provides a solid foundation for truncating the basis, we can use it to smartly enlarge the dimension of the space to be able to include more $K = 0^+$ states in the calculations. Work in this direction is underway.

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