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Collective Excitations of Deformed Nuclei and Their Coupling to Single Particle States

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Abstract. Traditionally the collective excitations of deformed even-even nuclei, that give rise to intrinsic band heads within the pairing gap, have been regarded as β ($\mathrm{K}^\pi = 0^+$), γ ($K^\pi = 2^+$) and octupole ($K^\pi = 0^-$ to 3^-) vibrations. However the properties of the lowest excited 0_2^+ states in deformed nuclei do not generally have the properies of a β vibration . The low-lying 0_2^+ states in transitional rare earth nuclei have been shown to be 2p-2h, or 4qp, neutron states involving the [505]$11/2^-$ Nilsson orbit extruded by the deformation to the Fermi surface from the filled $\mathrm{h}_{11/2}$ shell. This is demonstrated by the blocking of the the coupling of [505]$11/2^-$ neutrons in odd-A nuclei to their core 0_2^+ states in N=88 and N=90 nuclei.

This experimental observation leaves γ and octupole vibrations as the remaining collective states within the pairing gap. It demonstrates that nuclei, in general, are stiff to $\mathrm{K}^\pi = 0^+$ vibrations along the symmetry axis, even in transitional regions where the nuclear shape is changing rapidly. It also demonstrates the futility of expecting non-microscopic theories to be able to describe 0_2^+ states if the effects of Pauli blocking cannot be included in the models.

In this presentation I review the experimental data on $\mathrm{K}^\pi = 2^+$ "γ-bands" in deformed nuclei, built both on alignments in even-even nuclei and coupling to single particles in odd-A nuclei.

1 First Excited 0⁺ States in Deformed Nuclei

1.1 Tradition

The iconic and perennial tome of Bohr and Mottelson gives the quadrupole oscillations in the shape of a deformed nucleus in terms of volume conserving changes in the radius as [1]:

$\delta R \propto (3\cos^2\theta - 1)\cos\omega_\beta$t for β-vibrations along the symmetry z-axis and

$\delta R \propto \sin^2\theta\cos(2\varphi \pm \omega_\gamma$t) for γ-vibrations in the (x, y) directions, perpendicular to the z-symmetry axis.

In the rotation-vibration model [2,3] the energies of the states that arise from these collective vibrations are given, in an obvious notation, by

$$E_x(n_\beta n_\gamma IK) = \hbar\omega_\beta(n_\beta + 1/2) + \hbar\omega_\gamma(2n_\gamma + 1/2 \mid K \mid +1) + [I(I+1) - K^2]\hbar^2/2\mathcal{J} \quad (1)$$

This equation is derived by starting with the 5 dimensional Bohr Hamiltonian [4, 5] which includes the β and γ degrees of freedom. The Bohr Hamiltonian is constructed by quantizing the Classical energy [6] of an incompressible, irrotational liquid drop. This is done by replacing the appropriate Classical cannonical variables with differential operators. This leaves an undetermined mass parameter **B** which is then fitted to experimental data in the hope that this will account for the fact that the experimental moments-of-inertia correspond neither to irrotational flow nor to rigid-body behaviour. For a recent extensive and profound review of the construction of Bohr Hamiltonians, see Ref. [7].

If n_β = 1 and n_γ = 0 in (1), then a rotational band exists with its band-head at an excitation energy $E_x = \hbar\omega_\beta$, spin and parity $I_\pi = 0^+$ and spin projection on the symmetry axis $K_\pi = 0^+$. This band is referred to as the "β-vibrational band" and is identified with β deformation oscillations of the nuclear shape along the symmetry axis.

Early microscopic models of collective vibrations in deformed nuclei [8–12] assume the existence of a vibrational "phonon" or "boson". They then have to construct this entity out of some set of basis states. This they do by postulating an interaction, expanding their collective phonon in a truncation of this basis and then using the variation principle to minimise the phonon energy in terms of their interaction parameters. Needless to say an industry is created [13–17] discussing optimisation of bases, truncations and "fitting" the parameters! One of the many difficulties with this approach is that, without exception, it is assumed that the lowest excited 0_2^+ states in deformed nuclei are β-vibrations, *i.e.* one phonon states. It is the "mother's milk" of all text books, undergraduate nuclear physicis courses "101" and data compilations that such a band should be identified with the lowest excited 0_2^+ band that lies within the pairing gap of even-even deformed nuclei. Unfortunately comparison with experimental data [18] does not support this view.

1.2 Experimental Data for 0_2^+ States

Over the years there have been many experimental papers in which the authors mention that the 0_2^+ band in the nucleus they are studying, does not have the properties associated with a β-vibration; for examples see Refs. [19–26]. The very thorough review of the properties of 0_2^+ states by Paul Garrett [18] shows that the decay strengths from previously assigned "β-bands" do not have the properties expected of states associated with a vibration along the symmetry axis. We have recently pointed out [27] that the 0_2^+ states in the rare earth transitional nuclei with N = 88 and 90, that lie at low energies within the pairing gap, are actually 2-particle, 2-hole neutron states, or 4-quasiparticle states in the HFB nomenclature. Such states can exist in the pairing gap when there is a high-Ω Nilsson oblate orbital that has been extruded to the Fermi surface by the deformation. This orbital does not contribute to the normal pairing [28] as it is decoupled from the high density of low-Ω prolate orbitals that are driving the deformation.

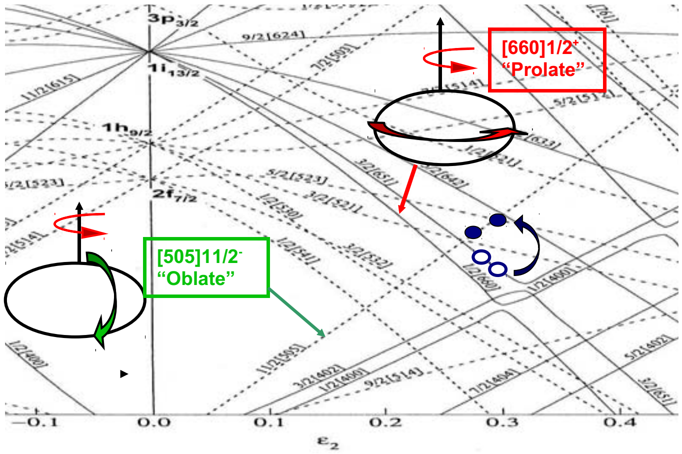

Figure 1. Nilsson diagram illustrating the configuration of the 0_2^+ states in $N = 90$ nuclei, *e.g.* ^{154}Gd$_{90}$. Two neutrons are taken out of a down-sloping prolate orbital and put in the up-sloping [505]11/2$^-$ oblate orbital from the h$_{11/2}$ shell. Pairing is configuration dependent, decoupling the high density of down-sloping prolate orbitals from the low density of up-sloping oblate orbitals.

This mechanism was first pointed out by Griffin, Jackson and Volkov [29] to explain the 0_2^+ states in actinide nuclei, observed by Maher [19], that did not have the properties of a β-vibration. The core idea is that the oblate-prolate pairing force G_{op} is significantly weaker than the oblate-oblate G_{oo} and prolate-prolate G_{pp} pairing forces. Central to this model is the paucity of oblate Nilsson levels near the Fermi surface. This decoupling of the polar and equatorial orbitals leads to the oblate pairing energy Δ_o , and hence the oblate quasi-particle energy, being reduced and permitting the existence of low-lying 0^+ states. Also the two neutron transfer cross-section to these states is no longer reduced by the normal pairing effects. Ragnarsson and Broglia [30] coined the term "Pairing Isomers" for such 0^+ levels. This very simple concept is illustrated in Figure 1 for N = 90 nuclei, for example ^{154}Gd$_{90}$. The relationship between the experimental excitation energies of 0_2^+ states in nuclei with even proton number Z and neutron numbers $N = 96 - 98$ and the excitation energies of the intruder [505]11/2$^-$ Nilsson states in the neighbouring odd neutron nuclei, is shown in Figure 2. This relationship between extruded orbitals and low-lying 0_2^+ states has been commented on in many previous publications [20–23].

The interpretation of the 0_2^+ states as "pairing isomers" [30] or as "second vacua" [27] means that 0_2^+ and 0_1^+ states have different charge distributions which accounts for the strong E0 transitions observed for $0_2^+ \rightarrow 0_1^+$ transitions in this region (see [31] and references therein).

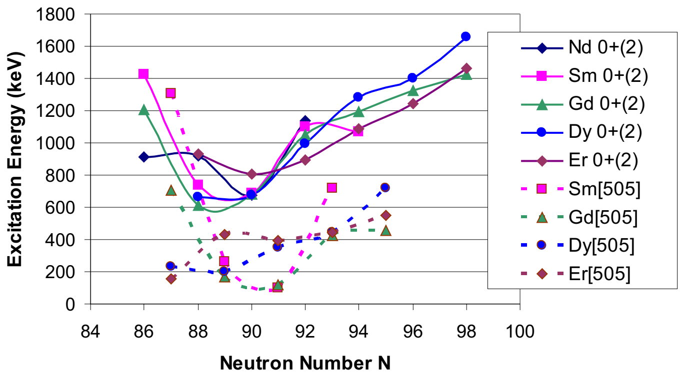

Figure 2. Systematics of the excitation energies of 0_2^+ states in even-even nuclei, continuous lines, and of the [505]11/2$^-$ neutron states in the neighbouring odd-N nuclei, dotted lines.

1.3 Coupling to 0_2^+ States

In many cases the single-particle orbitals dominating the configuration of a nuclear state can be found from its population in direct reactions. However the 0_2^+ states in N=90 nuclei are very weakly populated in single particle transfer [32–35] and electron scattering [36] experiments. They are also relatively weakly populated in (p,t) two neutron pick-up reactions [37, 38] but strongly in (t,p) two neutron stripping reactions [39, 40]. These latter data indicate that a considerable part of the 0_2^+ configuration consists of two quasi-neutrons in time reversed orbits. However, these L=0 two neutron transfer data give no information on which time reversed orbits are involved.

In looking for an unambiguous test of the microscopic structure of 0_2^+ states we realize that the single particle orbitals in odd nuclei, having an even-even $N = 90$ nucleus as a core, will couple to any collective excitations of that core. Thus classically the single neutron orbitals in ^{155}Gd should couple to any β-, γ- and octupole vibrations of its ^{154}Gd core. Should any of these collective modes have the major part of its wave function composed of two quasi-neutrons in a particular time reversed orbit, then the coupling of this particular quasi-neutron to that collective mode will be blocked in the odd neutron nucleus.

As an example, the coupling of the ground state [521]3/2$^-$ neutron in the $N = 91$ nucleus ^{154}Gd to the 0_2^+ core excitation at 681 keV in ^{154}Gd has been well established in transfer reactions [41, 42]. In Figure 3 we show the results of Schmidt *et al.* [43] for ^{155}Gd. They carried out an extensive investigation of the low-spin levels using the (n,γ), (d,p) and (d,t) reactions. They identified the coupling of the lowest $K_\pi = 3/2^\pm$ orbitals, the ground state [521]3/2$^-$ and [651]3/2$^+$ at 105 keV, to the 0_2^+ core excitation at 681 keV in ^{154}Gd, to have their band head energies at 592 keV and 815 keV respectively. These bands also couple to the γ-vibration of the ^{154}Gd core, producing states with $K_< =| K_{\text{band}} - 2 |$

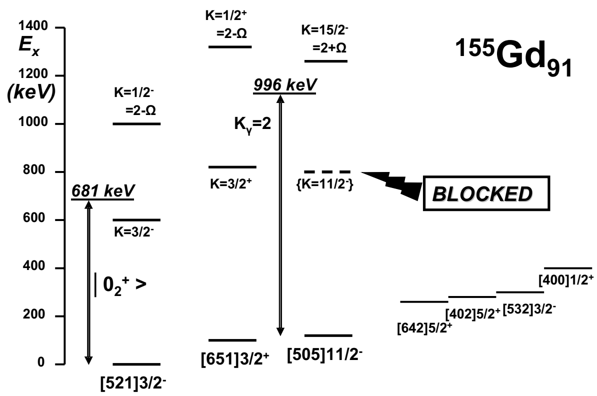

Figure 3. Schematic showing the rotational band heads arising from the coupling of the 0_2^+ state and the γ-vibration, at 681 keV and 996 keV respectively, in ^{154}Gd to the Nilsson single particle neutron orbits in ^{155}Gd with $K = \Omega$. The data for the $K \leq 5/2$ orbitals are taken from Ref. [43] and the data for the [505]11/2$^-$ orbital are from [44].

at 1003 keV ($K_\pi = 1/2^-$) and 1332 keV ($K_\pi = 1/2^+$) respectively. Only the lower $K_<$ band is seen when the bands couple to the γ-vibration because the $K_> = (K_{\text{band}} + 2)$ coupling has higher spin and could not be reached by the low angular momentum reactions used by Schmidt *et al.* [43].

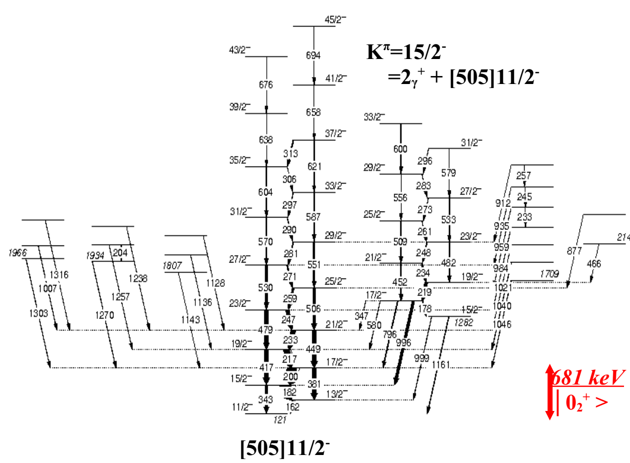

Figure 4. Partial decay scheme for ^{155}Gd showing the [505]11/2$^-$ band at 121 keV and the high-K levels that decay to it. The levels above 1282 keV are conjectured to be formed by the [505]11/2$^-$ neutron coupled to the $K^\pi = 2^+$ γ-vibration of the ^{154}Gd core. Levels due to the coupling of the 0_2^+ at 681 keV in ^{154}Gd to the [505]11/2$^-$ quasi-neutron, to produce a $K = 11/2^-$ band, are conspicuous by their absence.

We used the AFRODITE spectrometer [44] of iThemba LABS to measure $\gamma\gamma$ coincidences in the ^{154}Sm(α,3n)^{155}Gd reaction at a beam energy of 35 MeV. The decay scheme divides itself into two; one set of levels decaying to $K = 1/2$, 3/2 and 5/2 bands; the other set of levels decaying to the [505]11/2$^-$ band. This is because it would take at least a $\Delta K = 3$ transition to cross the gap between the $K \geq 11/2$ states and the $K \leq 5/2$ states. In Figure 4 we show the levels that decay to the [505]11/2$^-$ band. In this experiment we see no evidence whatsoever for γ decay to members of the [505]11/2$^-$ band, that could be associated with a $K^\pi = 11/2^-$ band formed by coupling 0_2^+ to the [505]11/2$^-$ neutron. This blocking of the coupling of the 0_2^+ core excitation to the oblate [505]11/2$^-$ neutron confirms the 4qp nature of the 0_2^+ states in $N = 88$ and 90 nuclei.

2 $K_\pi = 2^+$ γ-Vibrations in Deformed Nuclei

It is very clear that the γ degree of freedom, in describing the shapes of deformed nuclei, is indispensible. A nice illustration of this is the self-consistent relativistic mean field plus BCS calculations of the München group and colleagues [45, 46]. Figure 5 shows that for the deformed nuclei ^{148}Nd and ^{150}Nd, strong minima with oblate shapes seen in the calculations using only β deformation [45], turn out to be saddle points on a very γ-soft total energy surface when the γ degree of freedom is included in the calculations [46].

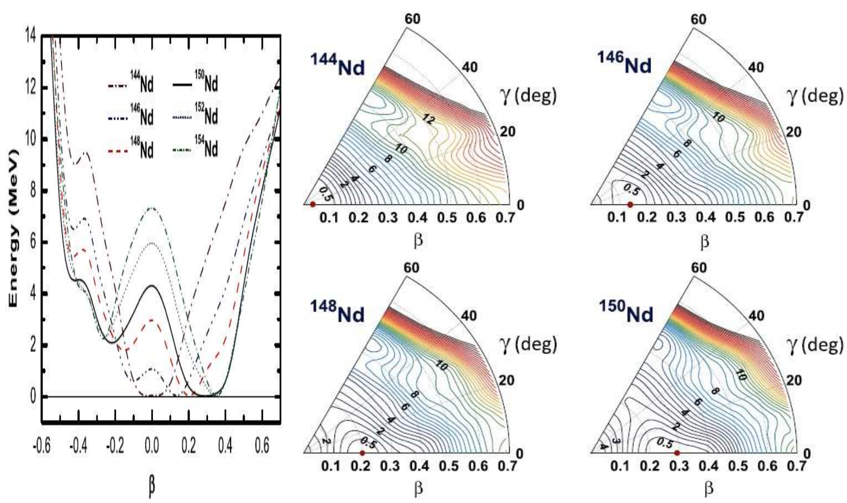

Figure 5. Self-consistent relativistic mean field plus BCS calculations for the even Nd isotopes. On the left are the ground state energies calculated [45] varying only the axial β degree of freedom. To the right are the total energy surfaces calculated [46] when the axial-symmetry breaking γ degree of freedom is included. These calculations show that the oblate minima on the left are really saddle points associated with the deeper prolate minima due to γ softness.

2.1 Transfer Reactions to γ-bands

Two particle transfers, *e.g.* by (p,t) and (t,p) reactions from the target ground 0^+ state of an even-even nucleus, will populate the 2^+_γ band head with an $L = 2$ transition but give no information on the single particle orbits involved, other than how much pairs of neutrons are involved in the wavefunction. Single particle direct reactions to a γ-band, *e.g.* (d,p), (p,d), (^{3}He,d), (d,^{3}He), give considerable information as the spin/parity of the odd target nucleus and the transferred nucleon must add vectorally to the final state spin. As pairing is an important feature of deformed nuclei, a first order description of their properties is given by introducing pairing interactions in the Nilsson basis. Hence single particle transfer gives information on the quasi-particle/hole structure of any states not in the ground state band of the final nucleus. States that are strongly populated will consist of the target odd quasi-particle coupled to some other quasi-particle. As K is a good quantum number for axially symmetric states, any p-h component of the γ-band configuration should be composed of quasi-particles in Nilsson orbits $[Nn_z\Lambda]\Omega$ of the same parity and where $\Delta K =\mid \Omega_{\text{target}} \pm \Omega_{\text{transfer}} \mid = 2$ [47, 48].

The (p,t) pick-up reactions usually populate the γ-band very weakly, for instance in the Gd isotopes [37, 38] and W isotopes [21]. The two neutron pick-up to the ^{168}Er γ-band [26] is stronger than most at about 15% of the intensity to the ground state. The γ-bands can also be clearly identified in the two neutron pick-up to ^{228,230}Th and ^{232}U [25]. The (t,p) reaction often has no sign at all of the γ-band in even-even nuclei, as in the famous paper by Casten *et al.* [49] and for the ^{152}Gd(t,p)^{154}Gd reaction [40]. No transfer to the γ-bands is also observed in the (t,p) reaction to the even Sm isotopes [39].

The (d,p) reaction has been used to populate states in the deformed nuclei ^{158}Gd, ^{164}Dy, ^{172}Yb and ^{173}Yb [47]. A straightforward calculation using Nilsson wavefunctions and the classical boson approach of [10] gives a good account of the relative strengths of the relative populations for the ground state and γ bands in all four nuclei. All the configurations involved have $\Delta K = 2$. Similarly the ^{151}Sm(d,p)^{152}Sm reaction [32] strongly populates the γ-band as the ground state of ^{151}Sm is [523]5/2$^-$ and the [521]1/2$^-$ orbital, giving $\Delta K = 2$, is available above the Fermi Surface. In contrast, the neutron pick-up reaction ^{151}Sm(p,d)^{150}Sm does not populate the γ-band [32] as there is no suitable $\Delta K = 2$ orbital to couple to below the Fermi Surface.

Proton stripping reactions to ^{154}Gd using the (^{3}He,d) and (α,t) reactions [35]populate the γ-band very strongly. The target nucleus ^{153}Eu has its odd proton in the [413]5/2$^+$ orbit and the $\Delta K = 2$ orbit [411]1/2$^+$ is just above the Fermi Surface. Again, in contrast, the (t,α) proton pick-up reaction to the nuclei ^{152}Sm [50], ^{164}Dy [51] and ^{174}Yb [52] do not populate the γ-band at all. Again this is because there are no suitable $\Delta K = 2$ orbitals below the Fermi Surface.

2.2 Systematic Properties of γ-Bands

A partial decay scheme showing positive parity levels observed [27] in the prolate deformed nucleus ^{154}Gd using (α,xn) reactions is shown in Figure 6. The levels to the right are a γ-band based on the ground state intrinsic state. The levels are divided into even 2^+, 4^+, 6^+ ... and odd 3^+, 5^+, 7^+ ... spin levels for clarity. The even spin levels decay to the levels in the ground state band (gsb) by not only $\Delta J = 2$ transitions, but also $\Delta J = 0$ and $\Delta J = -2$ transitions. As $\Delta K = 2$ in these out-of-band transitions, M1 components are K-forbidden in the $\Delta J = 0$ γ-rays. Similarly the $\Delta J = \pm 1$ transitions from the odd spin members to the gsb will be mostly E2 and contain very small M1 components at most [53, 54].

Generally in-band $\Delta J = 1$ transitions between the even and odd spin members of γ-bands are very weak. This means that $g_K \approx g_R$ for γ-bands.

Equation (1) may be used to calculate $\hbar\omega_\gamma$ for a series of nuclei from the excitation energies of their γ-bands and their moments-of-inertia. These are

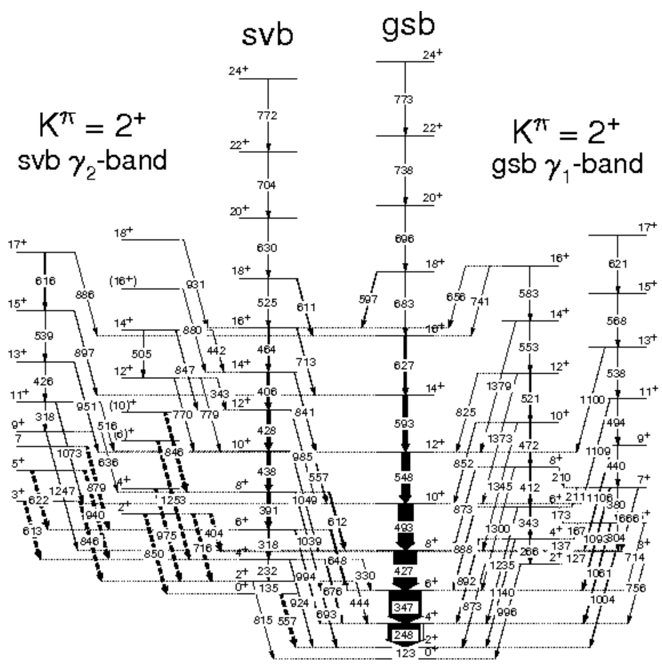

Figure 6. Partial decay scheme for ^{154}Gd showing positive parity bands in (α, xn) experiments [27]. The widths of the γ-rays are proportional to the intensities found in the $(\alpha, 4n)$ experiment. The γ-rays shown as dashed arrows were only observed in the $(\alpha, 2n)$ experiment. The ground state band 0_1^+ is labelled **gsb**, the second vacuum band 0_2^+ is labelled **svb**. Each has its own $K^\pi = 2^+$ γ-band.

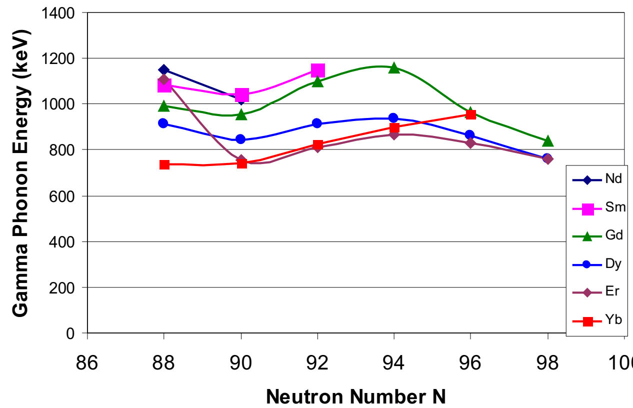

Figure 7. Gamma phonon energy $\hbar\omega_\gamma$ calculated using Eq. (1) for even-even nuclei with neutron number from $N = 88$ to 98 and proton number $Z = 60$ (Nd) to 70 (Yb). The nuclear deformation decreases with increasing Z. This phonon energy is relatively stable with (Z, N) compared to the excitation energies of corresponding 0_2^+ states shown in Figure 2.

shown in Figure 7 for deformed nuclei with $Z = 60$ (Nd) to 70 (Yb). It can be seen that values of $\hbar\omega_\gamma$ lie mainly between 750 and 1200 keV.

It is not very usual for γ-bands to be identified much above spin 12^+ as they are usually about 1.0 MeV above the yrast line. This makes it difficult to populate such states in fusion-evaporation (HI,xn) reactions as they are embedded in other structures which compete for intensity. The use of very heavy ion beams

Table 1. Some even-even nuclei in which the γ-band has been observed above 15^+

Nucleus	Beam		Highest Spin Reached			Ref.
	Species	E(MeV)	Yrast band	γ-even	γ-odd	
^{104}Mo	ff		20^+	18^+	17^+	[59]
^{154}Gd	α	45	24^+	16^+	17^+	[27]
^{156}Dy	^{12}C	65	32^+	28^+	27^+	[55]
^{156}Er	^{48}Ca	215	26^+	26^+	15^+	[60]
^{160}Er	^{48}Ca	215	50^+	-	43^+	[58]
^{162}Dy	^{118}Sn	780 Coulex	24^+	18^+	17^+	[61]
^{164}Dy	^{118}Sn	780 Coulex	22^+	18^+	11^+	[61]
^{164}Er	^{9}Be	59	24^+	14^+	19^+	[62]
^{164}Er	^{18}O	70	24^+	88^+	21^+	[22, 57]
^{170}Er	^{238}U	1358 Coulex	26^+	18^+	19^+	[63]
^{180}Hf	^{136}Xe	750 Coulex	18^+	16^+	13^+	[64]
^{238}U	^{209}Bi	1130&1330 Coulex	30^+	26^+	27^+	[65]

to Coulomb excite the most deformed nuclei has, in favourable cases, allowed γ-bands to be traced to much higher spins. In Table 1 we list some even-even nuclei in which the γ-band has been observed above 15^+, giving both the reaction used and the highest spin reached both for even and odd spins.

A notable feature of γ-bands is that they track the intrinsic configuration, usually the ground state, that they are based on. An example of this is shown in Figure 8 for the γ-band in ^{156}Dy [55]. Here the γ-band tracks the ground state configuration up to its highest spin of 28^+. The aligned band in ^{156}Dy, which causes a back-bend in the band based on the 0_2^+ state [56], shows no sign of any interaction with the γ-band. The γ-band has a small signature splitting at higher spins.

The question then arises, are there γ-vibrations based on the aligned configurations? Indeed such a crossing has been observed in ^{164}Er [22, 57]. The data shows that the ground state band is crossed by the usual aligned- or S-band. The γ-band tracks the ground state band at lower spins and then aligns to follow the aligned yrast states. The alignment in the γ-band comes at a slightly lower spin and hence a slightly lower rotational frequency than for the yrast states. This is because the band crossing is just from two different configurations; the γ-band built on the ground state 0_1^+ is crossed by a band which is the γ-band built on the aligned band $\{0_1^+ + (i_{13/2})^2\}$. An even more spectacular example of a γ-band built on sequential alignments has been found in ^{160}Er and is shown in Figure 9 from the very recently published work of Ollier *et al.* [58]. Only the odd spin members of the γ-band are seen, but they extend up to spin 43^+ and track the

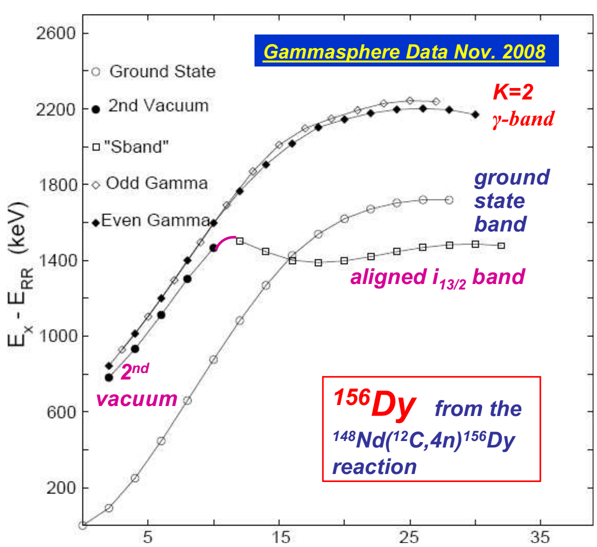

Figure 8. The excitation energies, minus a rotational energy, of positive parity bands in ^{156}Dy [55].

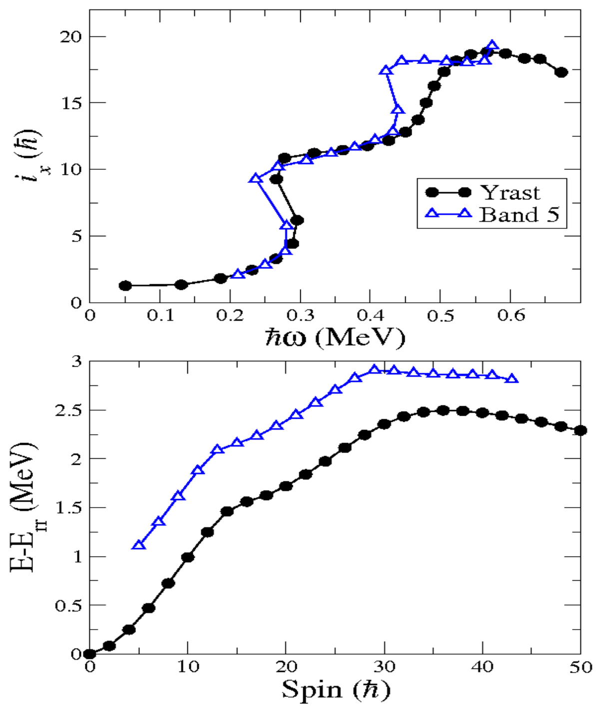

Figure 9. The position of the odd-spin γ-band (band 5) with respect to the yrast states in ^{156}Er [58]; (upper) the aligned angular momentum as a function of rotational frequency, (ower) excitation energy, minus a rigid rotor energy, against spin.

yrast states around both the $\nu(i_{13/2})^2$ neutron alignment and then around the $\pi(h_{11/2})^2$ proton alignment (second back-bend). Again the data indicates that the crossings of the γ-band come at slightly lower spins and frequencies than in the yrast states. The data in Figure 9 is quite remarkable. It shows that on every intrinsic or aligned configuration a γ-band is built with an added $K+2$ quantum number. Whatever the γ-bands are built with, it does not seem to be affected by the configurations causing the alignments.

2.3 Coupling of single particle states to γ-vibrations

One strong test of any theory of γ-vibrations is the experimental evidence of how single particle states couple to the core γ-vibration. Each single particle state with Nilsson quantum number Ω can couple to the core $K^\pi = 2^+$ either in a parallel mode to give $K_> = \Omega + 2$ or in an anti-parallel mode to give $K_< =| \Omega - 2 |$. There can be a splitting of the band heads of these two bands which will give information on the particle-vibration interaction. Clearly the band with $K_>$ will usually be nearer yrast and therefore easier to detect in (HI,xn) reactions.

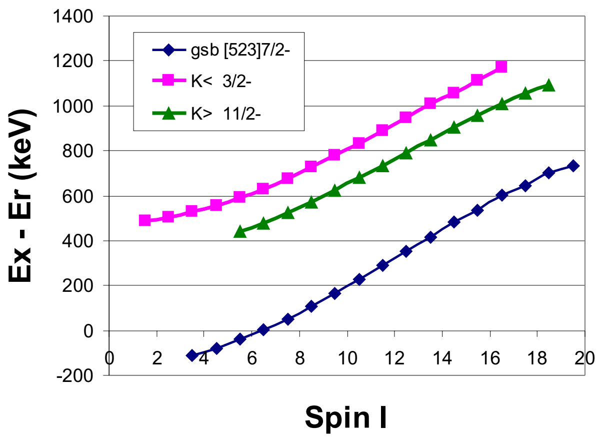

Figure 10. Excitation energies, minus a rigid rotor energy, for the [523]7/2$^-$ ground state band in $^{165}_{67}$Ho$_{98}$ and the $K_>$ and $K_<$ bands formed by coupling the [523]7/2$^-$ proton to the γ-vibration of the $^{166}_{68}$Er$_{98}$ core [66].

However the $K_<$ band can be found, when $K_<$ is small, in experiments such as (n,γ) and (n,n$^{'}\gamma$) experiments [43].

The most complete data sets on the coupling of the ground state nucleon in an odd nucleus to a core γ-vibration are Coulex experiments [66] on $^{165}_{67}$Ho$_{98}$ and $^{167}_{68}$Er$_{99}$, which share the core nucleus $^{166}_{68}$Er$_{98}$, and fission fragment spectroscopy [67] on the trio $^{103}_{41}$Nb$_{62}$, $^{104}_{42}$Mo$_{62}$ and $^{105}_{42}$Mo$_{63}$.

In Figure 10 we show the excitation energies of the ground state $K_>$ and $K_<$ bands in $^{165}_{67}$Ho$_{98}$. The energies of the $K_>$ and $K_<$ band heads are 689 and 515 keV respectively, giving a splitting of 174 keV. The core γ-bandhead in $^{166}_{68}$Er$_{98}$ is at 786 keV. It can be seen that the $K_>$ and $K_<$ bands again track the ground state band and that there is no significant signature splitting in any of the 3 bands.

3 Conclusions

The experimental evidence is quite clear; it is the underlying microscopic quantum states of the protons and neutrons that determine the physical properties of deformed nuclei. The lowest 0_2^+ states are two particle, two hole states and any β-vibrations lie near the top of the pairing gap in even-even nuclei or well above it where they will mix with other configurations. Deformed nuclei are stiff against β-vibrations, even in the region of transitional nuclei.

The K$_\pi$=2$^+$ bands that we associate with γ-vibrations are truly "collective" structures. They invariably track their intrinsic structures, both in even and in odd nuclei. It is not yet clear to me if the "γ-vibrational" bands are just a $K_\pi = 2^+$ projection of the zero point motion on the symmetry axis, or if they are more of a traditional Boson or Phonon? I am impressed by the relative successes of RPA [68–71] and TPSM [72–76] calculations and I am forced, by the experimental data, to regard non-microscopic models as no use at all!

Unlike the phantom β-vibrations, γ-vibrations are a REAL collective motion!

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