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Effects of Strong Magnetic Fields on the Equation of State of Cold Non-Accreting Neutron-Star Crusts

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Abstract. Using the latest experimental atomic mass data complemented with a microscopic atomic mass model, we have determined the equilibrium structure of the outer crust of cold non-accreting neutron stars endowed with strong magnetic fields. The equation of state is found to be markedly affected by the Landau quantization of electron motion. In particular, a strongly quantizing magnetic field not only changes the crust composition but also makes the crust more incompressible.

1 Introduction

Neutron stars are among the most strongly magnetized objects in the Universe. Radio pulsars are endowed with typical magnetic fields of order 10^{12} G. Magnetic fields up to 2×10^{15} G have been estimated in soft-gamma repeaters and anomalous X-ray pulsars [1]. Currently about 20 such objects have been detected [2]. Even stronger fields might exist in the interior of these stars. According to the virial theorem, the upper limit on the neutron star magnetic fields is about 10^{18} G [3]. Such fields are believed to be already present in newly-born neutron stars (see *e.g.*Ref. [4] and references therein) and could therefore alter their chemical evolution. In this paper, we study the impact of strong magnetic fields on the equilibrium composition of the outer crust of cold non-accreting neutron stars along the lines of Ref. [3] by using recent experimental atomic mass data complemented with a theoretical mass table based on the Hartree-Fock-Bogoliubov method [5]. In Section 2, we present the microscopic model used to describe the outer crust of a strongly magnetized neutron star. Results are discussed in Section 3.

2 Microscopic Model of Magnetized Neutron Star Crusts

In the model we adopt here [3], the neutron-star crust is assumed to be made of cold catalyzed matter, *i.e.* matter in its ground state at zero temperature and in a uniform magnetic field. The atoms are supposed to be fully ionised and arranged in a body centered cubic lattice. We have determined the equilibrium composition of each layer of the outer crust at a given pressure P by minimising the Gibbs free energy per nucleon

$$g = \frac{\mathcal{E} + P}{\bar{n}} \tag{1}$$

where $\mathcal{E}$ is the average energy density and $\bar{n}$ the average nucleon number density. Assuming that each layer of the outer crust contains only one nuclear species with proton number Z and atomic number A, the energy density can be expressed as

$$\mathcal{E} = n_N M'(Z, A) + \mathcal{E}_e + \mathcal{E}_L \tag{2}$$

where $n_N = \bar{n}/A$ is the number density of nuclei, $M'(Z, A)$ their mass (including the rest mass of nucleons and Z electrons), $\mathcal{E}_e$ the energy density of electrons (without rest mass energy) and $\mathcal{E}_L$ the lattice energy density. The nuclear mass $M'(Z, A)$ can be obtained from the atomic mass $M(Z, A)$ after substracting out the binding energy of the atomic electrons

$$M'(A, Z) = M(A, Z) + 1.44381 \times 10^{-5}\, Z^{2.39} + 1.55468 \times 10^{-12}\, Z^{5.35} \tag{3}$$

where both masses are expressed in units of MeV. As in Ref. [3], we will ignore the effects of the magnetic field on nuclear masses. This approximation is reasonable unless the magnetic field exceeds $\sim 10^{17}$ G [16]. In Ref. [3], the authors used the experimental atomic masses from Ref. [7] supplemented with by the mass model of Ref. [8]. In this paper, we have made use of the most recent experimental atomic mass data from a preliminary unpublished version of an updated Atomic Mass Evaluation (AME) [9]. For the masses that have not yet been measured, we have employed the microscopic nuclear mass model HFB-21 of Ref. [10] based on the Hartree-Fock-Bogoliubov method with a generalized Skyrme effective nucleon-nucleon interaction [11] supplemented with a microscopic contact pairing interaction [12]. The parameters of the Skyrme interaction were fitted to the 2149 measured masses of nuclei with N and $Z \geq 8$ given in the 2003 AME [13] with a rms deviation as low as 0.58 MeV. The parameters were simultaneously constrained to reproduce the zero-temperature equation of state of homogeneous neutron matter, as determined by many-body calculations with realistic two- and three-nucleon forces [14], from very low densities up to the maximum density found in stable neutron stars. With this constraint, the model is very well-suited for describing the highly-neutron rich nuclei found in the outer crust of a neutron star, as shown in Figure 1. This model can also be reliably extrapolated to the deeper regions of a neutron star

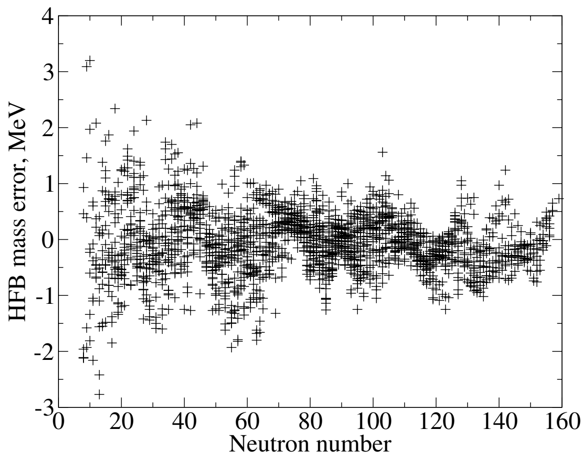

Figure 1. Differences between experimental and theoretical nuclear masses as a function of neutron number. Experimental masses are taken from the 2003 AME [13], whereas theoretical masses have been obtained from the model HFB-21 of Ref. [10].

where nuclei dissolve into an homogeneous mixture of nucleons and leptons. We have ensured that no spurious instabilities occur in nuclear matter for all densities prevailing in the interior of neutron stars [15].

The electrons are treated as a relativistic Fermi gas. In the presence of a strong magnetic field, the electron motion perpendicular to the field is quantized into Landau levels. Neglecting the small anomalous magnetic moment of electrons, the energies of these Landau levels (which were actually first found by Rabi as early as 1928 [17]) are given by

$$e_\nu = \sqrt{c^2 p_z^2 + m_e^2 c^4 (1 + 2\nu B_\star)} \tag{4}$$

where ν is any non-negative integer, p_z is the component of the momentum along the field, and $B_\star = B/B_c$ is the strength of the magnetic field B measured in units of the critical field

$$B_c = \frac{m_e^2 c^3}{e\hbar} \simeq 4.4 \times 10^{13}\,\mathrm{G}\,. \tag{5}$$

In the following we will assume that only the lowest Landau level $\nu = 0$ is filled. This approximation is actually exact provided the electron number density n_e satisfies the inequality

$$n_e < \frac{1}{\sqrt{2}\pi^2 a_m^3} \tag{6}$$

where $a_m = \sqrt{\hbar c/eB}$. Since the average nucleon density is given by $\bar{n} = (A/Z)n_e$, Eq. (3) can be equivalently expressed as $\bar{n} < \bar{n}_B$ with

$$\bar{n}_B \simeq 1.24 \times 10^{-9} \frac{A}{Z} B_\star^{3/2}\,\mathrm{fm}^{-3}\,. \tag{7}$$

Introducing the dimensionless parameter

$$x = \frac{\pi^2 \lambda_e^3 n_e}{B_\star} \tag{8}$$

where λ_e is the electron Compton wavelength, the electron energy density is given by (see *e.g.*Ref. [3] and references therein)

$$\mathcal{E}_e = \frac{B_\star m_e c^2}{(2\pi)^2 \lambda_e^3} \left[x\sqrt{1+x^2} + \log\{x + \sqrt{1+x^2}\} \right] . \tag{9}$$

According to the Bohr-van Leeuwen theorem [18], the lattice energy density is not affected by the magnetic field. For point-like ions in a body centered cubic lattice, the lattice energy density is approximately given by [19]

$$\mathcal{E}_L = -1.44423 Z^{2/3} e^2 n_e^{4/3} . \tag{10}$$

The pressure P can be decomposed into an electronic part P_e and a lattice part P_L,

$$P = P_e + P_L , \tag{11}$$

where

$$P_e = \frac{B_\star m_e c^2}{(2\pi)^2 \lambda_e^3} \left[x\sqrt{1+x^2} - \log\{x + \sqrt{1+x^2}\} \right] , \tag{12}$$

and

$$P_L = \frac{1}{3}\mathcal{E}_L \tag{13}$$

respectively.

3 Equilibrium composition and equation of state

We have calculated the composition of the outer crust of a magnetized neutron star as follows. For any given pressure P and for all proton numbers $1 \leq Z \leq 110$ and neutron numbers $1 \leq N \leq 250$ (more than 8000 nuclei), we have calculated the Gibbs free energy per nucleon g from Eq. (1) by first extracting the electron density n_e from Eqs. (11),(12) and (13) and second by using Eqs. (9) and (10). The equilibrium composition is that which yields the lowest value of g. We have repeated the calculations by increasing the pressure from $P = 10^{-11}$ MeV fm^{-3} to $P = P_{\rm drip}$ for which g equals $m_n c^2$ where m_n is the neutron mass. At higher pressures, neutrons start to drip out of nuclei thus delimiting the boundary between the outer crust and the inner crust of a neutron star. For $B_\star < 1.3 \times 10^3$, we have found that electrons start to fill the Landau level $\nu = 1$ at a pressure $P_B < P_{\rm drip}$. For this reason, we have chosen $B_\star = 1500$ so that only the lowest Landau level is occupied in any region of the outer crust. Results are summarized in Table 1. For comparison, we also show the composition of the outer crust in the absence of magnetic field using the well-known expressions of the energy and pressure of a relativistic Fermi gas instead of Eqs. (9) and (10) respectively. Note that our results are slightly different from those given in Table III of Ref. [6] because of our neglect of electron exchange as well as other small corrections to g that were included in Ref. [6]. Our results for $B_\star = 0$ also differ from those obtained previously in Ref. [3] because of the use of more recent experimental and theoretical atomic mass data. In particular, the elements ^{124}Ru and ^{118}Kr that were found in Ref. [3] are now absent, whereas ^{79}Cu, ^{80}Ni, ^{124}Sr and ^{121}Y are present. Using the latest experimental mass tables the outer crust is found to contain 12 nuclides with experimentally measured masses *vs.*only 6 in the calculations of Ref. [3]. In a strong magnetic field, the maximum density at which each nuclide is present in the crust increases. We have also found that the equilibrium composition is significantly changed. In particular, the nickel isotopes ^{66}Ni, ^{78}Ni and ^{80}Ni disappear whereas the elements ^{88}Sr, ^{128}Pd and ^{126}Ru are now present.

Table 1. Composition of the outer crust of cold non-accreting neutron stars

	Maximum density $\bar{n}$, fm^{-3}	
Nuclide	$B_\star = 0$	$B_\star = 1500$
^{56}Fe	4.93×10^{-9}	3.84×10^{-6}
^{62}Ni	1.59×10^{-7}	1.69×10^{-5}
^{58}Fe	1.65×10^{-7}	–
^{64}Ni	7.99×10^{-7}	1.84×10^{-5}
^{66}Ni	9.22×10^{-7}	–
^{88}Sr	–	2.43×10^{-5}
^{86}Kr	1.86×10^{-6}	4.02×10^{-5}
^{84}Se	6.79×10^{-6}	5.95×10^{-5}
^{82}Ge	1.67×10^{-5}	7.96×10^{-5}
^{80}Zn	3.18×10^{-5}	9.95×10^{-5}
^{79}Cu	4.35×10^{-5}	1.06×10^{-4}
^{78}Ni	5.42×10^{-5}	–
^{80}Ni	7.99×10^{-5}	–
^{128}Pd	–	1.23×10^{-4}
^{126}Ru	–	1.37×10^{-4}
^{124}Mo	1.23×10^{-4}	1.63×10^{-4}
^{122}Zr	1.48×10^{-4}	1.76×10^{-4}
^{121}Y	1.74×10^{-4}	1.88×10^{-4}
^{120}Sr	1.95×10^{-4}	1.97×10^{-4}
^{122}Sr	2.39×10^{-4}	2.13×10^{-4}
^{124}Sr	2.56×10^{-4}	2.16×10^{-4}

As can be seen in Figure 2, the quantization of electron motion due to the strong magnetic field leads to a drop of pressure at densities $\bar{n} \ll \bar{n}_B$. This figure also shows that the surface layers of a strongly magnetized neutron star are almost incompressible over a wide range of pressures. In the upper layers of

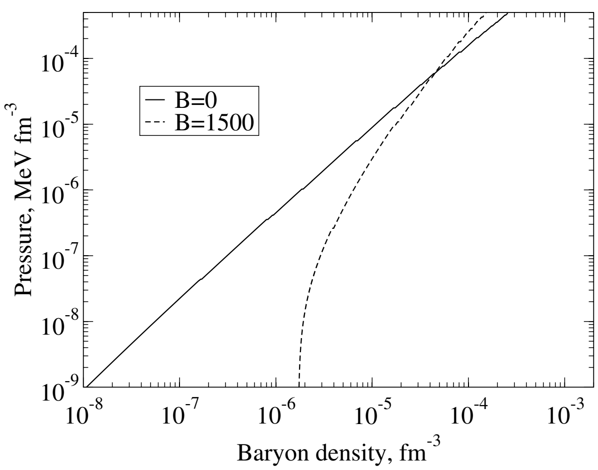

Figure 2. Equation of state of the outer crust of cold non-accreting neutron stars in the absence of a magnetic field (continuous line) and in a uniform magnetic field with $B_\star = 1500$ (dashed line).

the crust $x \ll 1$ and the electron pressure is approximately given by

$$P_e \simeq \frac{1}{3} m_e c^2 n_e^3 \left[\frac{2\pi^2 \lambda_e^3}{B_\star}\right]^2 . \tag{14}$$

Substituting Eq. (14) into Eq.(11) with $P = 0$ using Eq. (13) yields the average density at the surface of a magnetized neutron star

$$\bar{n}_s \simeq \frac{A_s}{Z_s}\left[\frac{1.44423 Z_s^{2/3} e^2}{m_e c^2}\left(\frac{B_\star}{2\pi^2\lambda_e^3}\right)\right]^{3/5} , \tag{15}$$

with Z_s and A_s the proton number and the charge number of the equilibrium nuclide at the surface. Assuming that the surface of a neutron star is made of iron with $Z_s = 26$ and $A_s = 56$ leads to

$$\bar{n}_s \simeq 2.501 \times 10^{-10} B_\star^{6/5}\,\mathrm{fm}^{-3} . \tag{16}$$

This simple formula shows that the stronger the magnetic field is, the higher is the surface density. It should be stressed however that Eq. (16) provides only a rough estimate of the surface density. The present model is not suited for describing the surface layers of a neutron star because of the nonuniformity of the electron gas [20].

4 Conclusion

Using the most recent experimental atomic mass data complemented with the latest Hartree-Fock-Bogoliubov mass model, we have computed the equilibrium composition and the equation of state of the outer crust of cold non-accreting neutron stars in the presence of strongly quantizing magnetic fields $B \gg m_e^2 c^3/(e\hbar)$. The magnetic field changes the composition and increases the maximum density at which each nuclide appears. This leads to a dramatic drop of pressure in the surface layers of the star. Moreover the magnetic field makes the crustal matter much more incompressible.

Acknowledgements

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