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Elastic Scattering of Protons at the Nucleus ^{6}He in the Glauber Multiple Scattering Theory

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Abstract. Calculation is submitted for the differential cross sections of elastic p^6He-scattering at energies of 70 and 700 MeV/nucleon within the framework of the Glauber theory of multiple diffraction scattering. We used the three-particle wave functions: α-*n*-*n* with realistic intercluster potentials. The sensitivity of elastic scattering to the proton-nuclear interaction and the structure of nuclei had been investigated. It is shown that the contribution of small components of the wave function as well as the multiplicity of the scattering operator Ω should be considered to describe a cross-section in broad angular range . A comparison with available experimental data was made.

1 Introduction

A series of calculations are presented for the differential cross section (DS) of the elastic scattering of protons on the isotope ^{6}He within the method of Glauber multiple diffraction scattering. We used the wave function (WF), obtained in the current three-particle nuclear models: α-n-n with realistic inter-cluster potentials and the expansion of the Glauber operator into a series of multiple scattering in a manner well adapted to the picture of weakly bound clusters in the halo nuclei. We also compare the different approaches to the assessment of DS in order to identify the validity of different models and the importance of higher-order terms in a number of multiple scattering.

It can now be considered established that the neutron-rich isotope ^{6}He consists of core and two valence neutrons. The microscopic multicluster model predicts thickness of skin (the neutron skin) to be 0.8 fm for ^{6}He, which (together with data on the mean square radius of nuclear matter listed in Table 1. and in comparison with other characteristics) makes us conclude that the nucleus ^{6}He does not show a clear halo structure, but rather a skin-core, in which the presence of excess neutrons does not lead to a marked increase in the radius, but only to an excessive concentration of neutrons in the surface region of the nucleus.

2 Formalism

Glauber multiple scattering operator in the general form is written as an alternating series of one-, two-, ..., A-tuple (where A is the number of nucleons in the target nucleus) collisions of the incident nucleus with the target:

$$\Omega = 1 - \prod_{j=1}^{A}(1 - \omega_j(\rho - \rho_j)) = \sum_{j=1} \omega_j - \sum_{j<\mu} \omega_j \omega_\mu + \sum_{j<\mu<\eta} \omega_j \omega_\mu \omega_\eta + \cdots (-1)^{A-1} \omega_1 \omega_2 \cdots \omega_A, \quad (1)$$

where ω_j – profile functions, depending on the elementary f_{xN} – amplitude:

$$\omega_j\left(\rho - \rho_j\right) = \frac{1}{2\pi ik} \int d^2q \exp\left[-iq\left(\rho - \rho_j\right)\right] f_{xN}(q), \quad (2)$$

where ρ, ρ_j – the impact parameter and the particle coordinates of nucleons, being the two-dimensional vectors in the Glauber theory, k, k' – momenta of the incident and the emitted hadron, q – the transferred momentum:

$$q = k - k' \quad , \quad k = k', \quad |q| = 2k\sin(\theta/2). \quad (3)$$

Proton-nucleon amplitude is parametrized in the following standard way:

$$f_{pN} = \frac{k\sigma_{pN}}{4\pi}\left(i + \gamma_{pN}\right) \exp\Big(-\frac{1}{2}\beta_{pN} q^2\Big). \quad (4)$$

The parameters of the amplitude are the input parameters of the theory, but they are determined from independent experiments. Summary of parameters used by us is shown in the table.

We use wave functions of ^{6}He nucleus, obtained in a cluster αnn-model. It assumes an inert α-partial core, but fully takes into account all the interactions between the two valence neutrons and a core, therefore, the WF is not factored in the one-particle density, and is written as:

$$\Psi^{JM_J} = \sum_{\mu,m,M_L,M_S} \left\langle \lambda\mu lm \right| LM_L\rangle \left\langle LM_L SM_S \right| JM_J\rangle \Phi_\alpha \Phi_{\alpha nn}, \quad (5)$$

Here $\Phi_\alpha, \Phi_{\alpha nn}$- wave functions of α-particle and relative αnn motion, the latter is obtained by solving the three-particle Schrodinger equation with the following potentials of intercluster interactions:

Model 1: NN-Reid, with the soft core, Nα – Sak-Bidenharn-Breit

Model 2: NN-Reid, with the soft core, Nα – split on parity.

Configuration of wave function is determined by the quantum numbers λlLS, where l – angular momentum of relative motion of α-particle and the center of mass of two neutrons , λ – angular momentum of relative motion of two neutrons, L and S – total orbital and spin momenta of the nucleus. Since the total spin of the two valence nucleons can be only 0 or 1, the condition $L = S$ limits the wave function of ground-state by S- and P-configurations: $\lambda = l = L$= 0, S=0 (S-wave, its weight is 95,7% in model 1 and 88% in model 2) and $\lambda = l = L$=1, S=1 (P-wave, its weight is 4.3% in model 1 and 12% in model 2).

According to the parametrization of wave function in the form of (5), in the operator Ω it is easier to consider the collision of a proton not with all nucleons, but with α- cluster and two valence neutrons. Formula (1) can be rewritten as follows:

$$\Omega_{^6\mathrm{He}} = \Omega_\alpha + \Omega_{n_1} + \Omega_{n_2} - \Omega_\alpha\Omega_{n_1} - \Omega_\alpha\Omega_{n_2} - \Omega_{n_1}\Omega_{n_2} + \Omega_\alpha\Omega_{n_1}\Omega_{n_2}. \quad (6)$$

Assuming α-particle as structureless, the operator of scattering on the α-particle can be written in the form of a profile function: $\Omega_\alpha = \omega_\alpha$, but with the elementary amplitude $f_{\alpha N}$ instead of f_{pN} in the formula (4), the $f_{\alpha N}$ amplitudes are listed in the table.

Scattering matrix element in the Glauber theory is written as follows:

$$M_{if}(q) = \sum_{M_J M'_J} \frac{ik}{2\pi} \int d^2\rho \prod_{\nu=1^6} dr_\nu \exp(iq\rho)\delta(R_A) \cdot \left\langle \Psi_f^{JM_J} \right| \Omega | \left. \Psi_i^{JM_J} \right\rangle, \quad (7)$$

where $\vec{R}_A$- the coordinate of the center of mass of the target nucleus. Substituting into (7) WF (5) and operator (6), taking the coordinates of the Jacobi, the matrix element can be calculated analytically. Technique for computing the matrix element with the three-particle wave functions in detail is in [17].

The differential cross section is the square modulus of the matrix element:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2J+1} \left| M_{if}(q) \right|^2 \quad . \quad (8)$$

3 Discussion of Results

In this paper, DS, calculated within the framework of diffraction theory are compared with experimental data obtained from plants in the Petersburg Nuclear Physics Institute (Russia) [1], and RIKEN (Japan) [13].

Figure 1 shows the DS of the model wave function: three-body computed in model 1 (solid curve), 2 (dashed) and shell (dotted line) at a) $E = 0.071$ GeV/nucleon, b) $E = 0.717$ GeV/nucleon. All the figures show that the cross sections calculated with the three-particle wave functions are very close to each other, indicating a low sensitivity of αnn wave function to the various inter-cluster interaction potentials. The calculation with the shell wave function (taken as a harmonic oscillator wave function: [2] 31S weighing 0.973 and 33P with a weight of -0.23 [18]) differs from the cluster one, most notably in the areas of cross-section minima (in dotted curve, minima is deeper). There are also

(a)

(b)

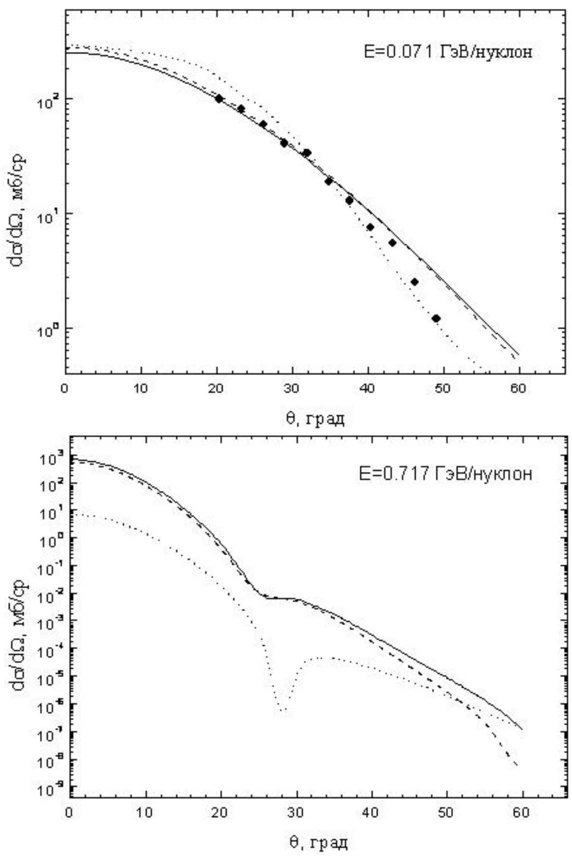
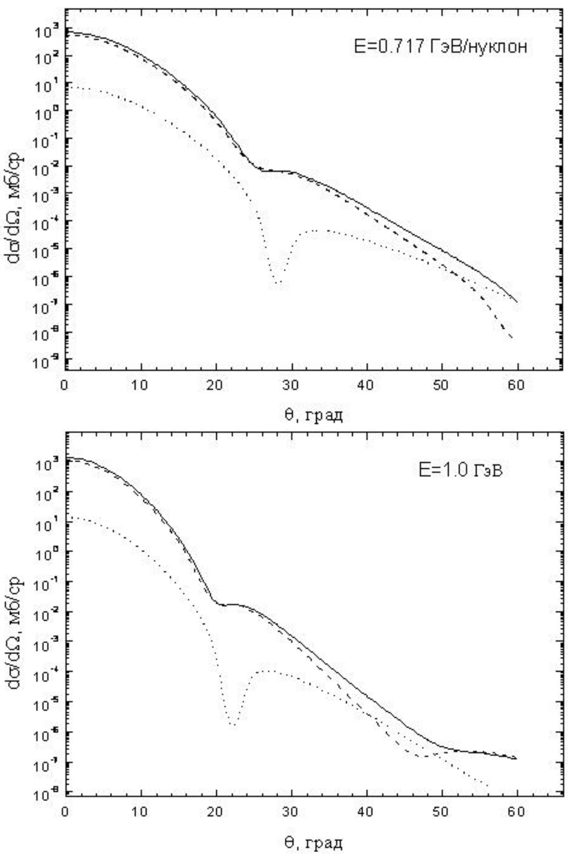


Figure 1. The calculation of DCS with different model wave functions: the solid and dotted curve – αnn wave function in models 1 and 2, dashed curve – oscillator wave function, with: (a) $E = 0.071$ GeV/nucleon, the experimental points from [13], (b) $E = 0.717$ GeV/nucleon, the experimental points from [1].

differences both at small and large scattering angles, and they are not very small, counting the logarithmic scale on the ordinate.

Comparison of theoretical and experimental results at 0.071 GeV/nucleon [13] shows that for small scattering angles ($\theta < 38°$) DS with three-particle wave function is in good agreement with experimental data, at large angles ($\theta > 38°$) calculated curve lies above the experimental points . The cross section calculated with the shell WF describes an experiment at small scattering angles worse than the three-body one, the coincidence of the same experiment and theory at large angles may be accidental. Differences in the description of DS with different model wave functions are associated with their behavior in the core and the periphery. In the case of small scattering angles the momentum transfer is



(a)

(b)

Figure 2. Dependence of DS on the contribution of the cross section of different components of the WF (model 1): dot-dashed curve – the contribution of S-waves, the point – the contribution of the P-wave, solid – total contribution, at: (a) $E = 0.717$ GeV/nucleon, (b) $E = 1.0$ GeV/nucleon.

small (at $\theta = 2°$, $q = 0.011$ GeV/c, see formula (3)) and the peripheral region of the nucleus (*i.e.*, the asymptotic of wave function)can be only probed. In the three-particle model, wave function is more extended than that of the shell one, which decreases rapidly and does not convey the real behavior of the nuclear wave function. At large scattering angles the transferred momentum increases (reaching a value $q = 0.217$ GeV/c at $\theta = 40°$), particles interact more in the core, where the subtle effects of particle correlations (which actually distinguish one model from another) should be more visible and we see different behavior of the angular distributions.

Comparison with experiment at $E = 0.717$ GeV/nucleon [1] shows that for small scattering angles the agreement is good, except for very small angles

($\theta < 2^\circ$), where the Coulomb interaction (which is not taken into account in our calculation) begins to dominate.

Thus, the sensitivity of the Glauber amplitude for the model wave functions had been demonstrated, although each of them leads to approximately the same root-mean-square radius equal to 2.3 fm.

Figure 2 shows the dependence of the DS on the contribution of different components of the three-particle wave functions into it (model 1): dot-dashed curve – the contribution of S-waves, the point – the contribution of the P-wave, solid – total contribution at a) $E = 0.717$ GeV/nucleon, b) $E = 1.0$ GeV/nucleon.

Figures 2a, b show that the cross section, calculated with the S-component, in accordance with its weight (95.7%), completely dominates at small scattering angles. The cross section for this component has a rather monotonous appearance with a soft minimum at $\theta \approx 25^\circ$ for 2a and two minima at $\theta \approx 20^\circ,\ 46^\circ$ for 2b.

The configuration of the P-state gives a small contribution to the cross section, noticeable only at large scattering angles $\theta > 50^\circ$ at 0.717 GeV/nucleon and $\theta > 35^\circ$ at 1.0 GeV/nucleon. In the section of the P-wave the sharp minimum can be seen at $\theta \approx 29^\circ$ (0.717 GeV/nucleon) and at $\theta \approx 22^\circ$ (1.0 GeV/nucleon), not giving, however, contribution to the total cross section, as absolute value of the cross section with the S-wave in this area is greater by 4 orders of magnitude. However, for large angles the contribution of the P-wave is comparable with the contribution of the S-wave and even exceeds it by filling out the second minimum, the existing section in the S-wave (Figure 2b), thereby increasing the total cross section.

We now consider the cross section dependence on the contributions of different multiplicities of scattering into the Glauber operator Ω. Substituting in (7) the operator (6), we obtain:

$$M_{if}(q) = M^1_{if}(q) - M^2_{if}(q) + M^3_{if}(q), \tag{9}$$

$$M^1_{if}(q) = \sum_{M_J M'_J} \frac{ik}{2\pi} \int d^2\rho \prod_{\nu=1}^{8} dr_\nu \exp(iq\rho)\delta(R_A) \cdot \left\langle \Psi_f^{JM_J} | \Omega_\alpha + 2\Omega_n | \Psi_i^{JM_J} \right\rangle, \tag{10}$$

$M^1_{if}(q)$ – Matrix element of the single scattering, $M^2_{if}(q), M^3_{if}(q)$ – similar matrix elements of double and triple scattering. Substituting them into (8), we obtain the corresponding cross sections, which are shown in Figure 3 for $E = 0.717$ GeV/nucleon. Curves 1, 2, 3 – DS with $M^1_{if}(q)$, $M^3_{if}(q)$, $M^3_{if}(q)$ respectively. Curve 4 – the contribution of all members (9), consistent with their interference. The figure shows that the main contribution at small scattering angles ($\theta < 25^\circ$) is given by single collision with α-cluster and two valence neutrons, but their amplitude decreases rapidly with angle increasing and begin to contribute to the scattering of higher multiplicity. At the point where the curves of single and double scattering cross sections overlap, there is interfer-

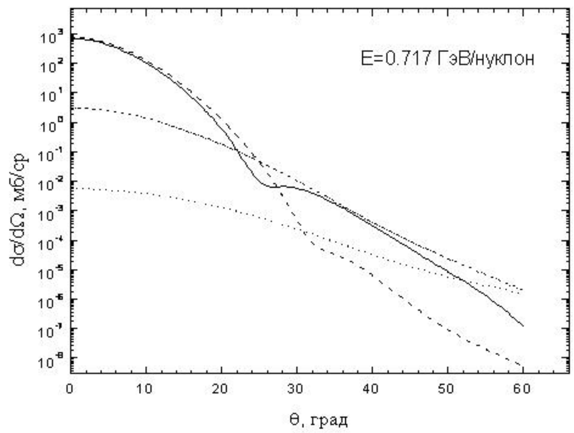


Figure 3. Contributions of different multiplicities of scattering operator Ω at $E = 0.717$ GeV/nucleon. Curves 1, 2, 3 and 4 – a deposit of one-, two-, three-multiple scattering, and their total contribution, respectively.

ence, which is shown as the minimum in the total curve 4, because formula (9) is alternating. After the interference minimum, 2- and 3-multiple collisions start to compete. The total curve from minimum to $\theta \approx 52^\circ$ is lower than the curve of double collisions, and for $\theta > 52^\circ$ it is lower than triple collisions. This is due to destructive interference of double and triple scattering terms, which makes the total cross section be less than partial.

4 Conclusion

We calculated the elastic scattering of ^{6}He within the Glauber theory, the input parameters of which are the elementary NN- and Nα- amplitudes and WF of ^{6}He. Based on these calculations the following conclusions can be made:

1. Diffraction theory adequately reproduces the experimental data at $E = 0.717$ GeV/nucleon and a little bit worse at $E = 0.071$ GeV/nucleon, due to limitations of the theory, not suitable for low energies and large scattering angles.

2. The importance of using the three-particle wave function is confirmed by calculation with the shell wave function, which correctly describes the experimental DCS, both at small and large scattering angles, due to its rapid decrease in the asymptotic behavior and neglect of the correlations of the nucleons in the inner core.

3. The calculation showed that the DS is sensitive to the contribution of the S-and P-waves into the wave function of ^{6}He and to the contribution of different multiplicities of scattering operator Ω. To describe the cross-section over a wide angular range, the contribution of small components of wave function and all multiplicities of scattering should be considered.

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