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Effects of Core Polarization and Pairing Correlations on Magnetic Moments of Deformed Odd Nuclei

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Abstract. In self-consistent mean-field approaches to odd-mass nuclei, the time-reversal symmetry of the underlying one-body Hamiltonian is broken. This induces a polarization of the even-even core to which the odd nucleon is added. In addition pairing correlations in these nuclei are quenched for the nucleons in odd number. To take this effect into account, a particle-number conserving formalism is necessary (as opposed to Hartree–Fock–Bogolyubov or Hartree–Fock–BCS calculations). In this context we use the Highly Truncated Diagonalization Approach applied to describe pairing correlations and discuss the first results of intrinsic magnetic moments in well-deformed odd-mass nuclei. Finally, in the framework of the Bohr and Mottelson unified model, the collective contribution to the magnetic moment is calculated from the Hartree–Fock–BCS solution of the underlying even-even core. We present preliminary results for the total magnetic moment, which are found to be in fair agreement with experimental data.

1 Introduction

In previous papers [1,2], we have studied the removal of the Kramers degeneracy occuring in the single particle (s.p.) spectrum of a Skyrme-Hartree-Fock solution describing an odd-mass nucleus, assuming axial symmetry. The breaking of the time-reversal symmetry inherent to the proper microscopic description of a system involving an odd number of fermions generates polarizing potentials in the corresponding mean-field Hamiltonian. It has then been found, when the odd-particle wave function possesses a small spin mixing, that the spin part of the time-odd fields is dominating. The Kramers degeneracy is thus suppressed in such a way that the energetically favored s.p. state is the one corresponding

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to a spin alignment with the spin field. It has further been shown [2, 3] that the phenomenological quenching factor for the spin gyromagnetic factors g_s (around 30%) can be ascribed essentially to the spin polarization of the core. This polarization is thus expected to have an effect on intrinsic magnetic dipole moments.

In this context, the main goal of this paper is to investigate the effect of the core polarization in the presence of pairing correlations, on the magnetic dipole moment in well-deformed odd-mass nuclei. To implement these correlations and account in a proper way for the blocking effect, we make use of the particle-number conserving Highly Truncated Diagonalization Approach (HTDA) [4, 5], with a residual δ interaction. Since our study includes nuclei around ^{48}Cr, we consider proton-neutron pairing correlations in the $T = 1$ as well as $T = 0$ isospin channels. To calculate the magnetic dipole moment in the ground state of well-deformed odd-mass nuclei, we rely on the Bohr and Mottelson unified model picture and calculate the collective gyromagnetic ratio of the underlying even-even core within the HFBCS approach. This contribution is outlined as follows. In Section 2 we describe the theoretical framework of our calculations, namely the treatment of odd-mass nuclei in the Skyrme–Hartree–Fock–BCS and HTDA approaches and the microscopic calculation of the magnetic dipole moment. Then we present and discuss in Section 3 the calculated results of spin quenching factors and magnetic moments, which we compare with available experimental data. Finally we draw the main conclusions and give some perspectives in Section 4.

2 Theoretical Framework

2.1 Self-consistent mean-field solutions in odd-mass nuclei

Self-consistent mean-field ground-state solutions are obtained in the Skyrme–Hartree–Fock–BCS framework, briefly recalled below with an emphasis on the approximations made and the peculiarities stemming from the time-reversal symmetry breaking at the one-body level.

The nuclear Hamiltonian $\hat{H}$ considered is the sum of the intrinsic kinetic energy $\hat{K}$ and the nuclear interaction $\hat{V}$. As often done, we make the approximation that neutrons and protons have equal masses and neglect the two-body contribution to $\hat{K}$, so that the intrinsic kinetic energy becomes a one-body operator [6] written as $\hat{K} \approx \sum_{i=1}^{A} \left(1 - \frac{1}{A}\right) \frac{\hat{\mathbf{p}}_i^2}{2m}$. . The nuclear interaction $\hat{V}$ is chosen to be the Skyrme density-dependent two-body interaction defined by the sum of the central $\hat{V}_{\rm c}$, density-dependent $\hat{V}_{\rm DD}$ (to mock up three-body effects), spin-orbit $\hat{V}_{\rm s.o.}$ and Coulomb $\hat{V}_{\rm Coul}$ contributions given, in coordinate representation, by

$$V_{\rm c}(\mathbf{r}_1, \mathbf{r}_2) = t_0(1 + x_0 P_\sigma)\, \delta(\mathbf{r}_1 - \mathbf{r}_2) + \frac{t_1}{2}(1 + x_1 P_\sigma) \Big[\delta(\mathbf{r}_1 - \mathbf{r}_2)\, \mathbf{k}^2 + \mathbf{k}^{\dagger\, 2} \delta(\mathbf{r}_1 - \mathbf{r}_2)\Big] + t_2(1 + x_2 P_\sigma)\, \mathbf{k}^\dagger \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2)\, \mathbf{k}\,, \qquad (1)$$

where $P_\sigma = \frac{1}{2}(1+\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$ is the spin-exchange operator and $\mathbf{k} = \frac{i}{2}(\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2)$,

$$V_{\rm DD}(\mathbf{r}_1, \mathbf{r}_2) = \frac{t_3}{6}(1 + x_3 P_\sigma)\, \rho^\alpha \Big(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\Big)\, \delta(\mathbf{r}_1 - \mathbf{r}_2)\,, \tag{2}$$

where ρ is the nucleon density, and

$$V_{\rm s.o.}(\mathbf{r}_1, \mathbf{r}_2) = i\, W_0\, (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{k}^\dagger \times \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{k}\,. \tag{3}$$

As well known, the expectation value E of the above Hamiltonian $\hat{H}$ calculated for a Slater determinant $|\Phi\rangle$ is a time-even functional

$$E = \langle \Phi | \hat{H} | \Phi \rangle = \int d\mathbf{r}\, \Big(\mathcal{H}_{\rm kin}(\mathbf{r}) + \mathcal{H}_{\rm c}(\mathbf{r}) + \mathcal{H}_{\rm DD} + \mathcal{H}_{\rm s.o.}(\mathbf{r}) + \mathcal{H}_{\rm Coul}(\mathbf{r}) \Big) \tag{4}$$

of local densities, where $\mathcal{H}_{\rm kin}(\mathbf{r})$, $\mathcal{H}_{\rm c}(\mathbf{r})$, $\mathcal{H}_{\rm DD}$, $\mathcal{H}_{\rm s.o.}(\mathbf{r})$ and $\mathcal{H}_{\rm Coul}(\mathbf{r})$ are the kinetic, central, density-dependent, spin-orbit and Coulomb energy-density contributions. These local densities are classified in two categories according to their behavior under time-reversal symmetry, represented by an antiunitary operator $\mathcal{T}$:

- time-even densities, which commute with $\mathcal{T}$ and are scalar or rank-2 tensor quantities: nucleon density $\rho_q(\mathbf{r})$, kinetic-energy density $\tau_q(\mathbf{r})$, spin-current tensor $J_q^{\mu\nu}(\mathbf{r})$;

- time-odd densities, which anticommute with $\mathcal{T}$ and are vector quantities: spin density $\mathbf{s}_q(\mathbf{r})$, current (or momentum) density $\mathbf{j}_q(\mathbf{r})$, spin-kinetic-energy density $\mathbf{T}_q(\mathbf{r})$.

The subscript q denotes the considered charge state, namely $q = n$ for neutrons and $q = p$ for protons. It is omitted when the sum of neutron and proton contributions is implied. The definition of the above listed densities can be found, *e.g.*, in Refs. [7–9]. Note that additional densities come into play when the nuclear interaction includes tensor terms (see, *e.g.*, Ref. [10]). The above energy-density contributions have the following expressions in terms of the local densities (whose $\mathbf{r}$-dependence is omitted to alleviate the expressions)

$$\mathcal{H}_{\rm kin}(\mathbf{r}) = \Big(1 - \frac{1}{A}\Big) \frac{\hbar^2}{2m}\, \tau\,, \tag{5}$$

$$\begin{aligned}\mathcal{H}_{\rm c}(\mathbf{r}) = {} & B_1\, \rho^2 + B_{10}\, \mathbf{s}^2 + B_3(\rho\, \tau - \mathbf{j}^2) + B_{14}(\mathbf{s} \cdot \mathbf{T} - \overleftrightarrow{\mathbf{J}}^2) \\ & + B_5\, \rho\, \Delta\rho + B_{16}\, \mathbf{s} \cdot \Delta\mathbf{s} + \sum_q \Big[B_2\, \rho_q^2 + B_{11}\, \mathbf{s}_q^2 + B_4(\rho_q \tau_q - \mathbf{j}_q^2) \\ & + B_{15}(\mathbf{s}_q \cdot \mathbf{T}_q - \overleftrightarrow{\mathbf{J}}_q^2) + B_6\, \rho_q\, \Delta\rho_q + B_{17}\, \mathbf{s}_q \cdot \Delta\mathbf{s}_q \Big]\,,\end{aligned} \tag{6}$$

with $\overleftrightarrow{\mathrm{J}}_q^{\,2} = \sum_{\mu,\nu} (J_q^{\mu\nu})^2$,

$$\mathcal{H}_{\rm DD}(\mathbf{r}) = \rho^\alpha \left[B_7\, \rho^2 + B_{12}\, \mathbf{s}^2 + \sum_q \left(B_8\, \rho_q^2 + B_{13}\, \mathbf{s}_q^2\right)\right], \tag{7}$$

$$\mathcal{H}_{\rm s.o.} = B_9 \left[\rho\, \boldsymbol{\nabla}\cdot\mathbf{J} + \mathbf{j}\cdot\boldsymbol{\nabla}\times\mathbf{s} + \sum_q \Big(\rho_q\, \boldsymbol{\nabla}\cdot\mathbf{J}_q + \mathbf{j}_q\cdot\boldsymbol{\nabla}\times\mathbf{s}_q\Big)\right], \tag{8}$$

where $\mathbf{J}_q$ is the antisymmetric part of the spin-current tensor $J_q^{\mu\nu}$ [8], and

$$\mathcal{H}_{\rm Coul}(\mathbf{r}) \approx \frac{1}{2}\,\rho_p(\mathbf{r})\, V_{\rm CD}(\mathbf{r}) - \frac{3}{4} e^2 \Big(\frac{3}{\pi}\Big)^{\frac{1}{3}} \rho_p^{\frac{4}{3}}(\mathbf{r}) \tag{9}$$

within the Slater approximation for the exchange terms [11, 12]. In Eqs. (5) to (8), the coefficients B_i are functions of the Skyrme parameters t_i, x_i and W_0 as defined in Ref. [9].

The pairing correlations are then included in the many-body ground state within the BCS framework. As explained in detail in Ref. [7], this amounts to simply extend the definition of the local densities by incorporating the BCS occupation factors and to add the so-called pairing energy to the energy-density functional given by Eq. (4). The resulting energy is then varied with respect to the single-particle wave functions (in terms of which the local densities are expressed), with a normalization constraint enforced by Lagrange multipliers interpreted as single-particle energies e_i, and with respect to the occupation factors. The former variation yields the Hartree–Fock equations $\hat{h}_{\rm HF}|\phi_i\rangle = e_i|\phi_i\rangle$ with a Hartree–Fock one-body Hamiltonian $\hat{h}_{\rm HF}$ of the following form in coordinate representation for the charge state q

$$\begin{aligned} h_{\rm HF}^{(q)}(\mathbf{r}) = &- \nabla.\Big(\frac{\hbar^2}{2m_q^*(\mathbf{r})}\Big)\nabla + U_q(\mathbf{r}) + \delta_{q\,p} V_{\rm Coul}(\mathbf{r}) \\ &+ \frac{1}{2i}\Big(W_q^{\mu\nu}(\mathbf{r})\nabla_\mu\sigma_\nu + \nabla_\mu\sigma_\nu W_q^{\mu\nu}(\mathbf{r})\Big) \\ &+ \mathbf{S}_q(\mathbf{r})\cdot\boldsymbol{\sigma} - \frac{i}{2}\Big(\mathbf{A}_q(\mathbf{r})\cdot\nabla + \nabla\cdot\mathbf{A}_q(\mathbf{r})\Big) - \nabla\cdot\big(\mathbf{C}_q(\mathbf{r})\cdot\boldsymbol{\sigma}\big)\nabla\,. \end{aligned} \tag{10}$$

In this expression, the time-even fields $m^*(\mathbf{r})$, $U_q(\mathbf{r})$, $V_{\rm Coul}(\mathbf{r})$ and $W_q^{\mu\nu}(\mathbf{r})$ denote the effective mass, the central-plus-density-dependent field, the Coulomb field and the spin-orbit field respectively, whereas $\mathbf{S}_q(\mathbf{r})$, $\mathbf{A}_q(\mathbf{r})$ and $\mathbf{C}_q(\mathbf{r})$ are time-odd fields. They are functions of the above local densities. In the following we call $\mathbf{S}_q(\mathbf{r})$ the spin field and $\mathbf{C}_q(\mathbf{r})$ the spin-gradient field. For the discussion which follows, we give only the expressions of the spin-orbit and time-odd fields:

$$W_q^{\mu\nu}(\mathbf{r}) = -B_9\, \epsilon^{\kappa\mu\nu}\nabla_\kappa(\rho+\rho_q) + 2\big(B_{14}J^{\mu\nu} + B_{15}J_q^{\mu\nu}\big) \tag{11}$$

$$\begin{aligned} \mathbf{S}_q(\mathbf{r}) = &\, 2\big(B_{10} + B_{12}\rho^\alpha\big)\mathbf{s} + 2\big(B_{11} + B_{13}\rho^\alpha\big)\mathbf{s}_q + B_9\,\nabla\times(\mathbf{j}+\mathbf{j}_q) \\ &+ 2\big(B_{16}\,\Delta\mathbf{s} + B_{17}\,\Delta\mathbf{s}_q\big) + B_{14}\,\mathbf{T} + B_{15}\,\mathbf{T}_q \end{aligned} \tag{12}$$

$$\mathbf{A}_q(\mathbf{r}) = -2\big(B_3\,\mathbf{j} + B_4\,\mathbf{j}_q\big) + B_9\,\nabla\times(\mathbf{s}+\mathbf{s}_q) \tag{13}$$
$$\mathbf{C}_q(\mathbf{r}) = B_{14}\,\mathbf{s} + B_{15}\,\mathbf{s}_q\,. \tag{14}$$

The expressions of the other time-even fields can be found, *e.g.*, in Refs. [7, 9].

In this work we choose the SIII parametrization [13] of the Skyrme functional for its fairly good bulk as well as spectroscopic properties [14, 15] across the nuclide chart. In this parametrization, the coefficients B_{14} and B_{16} identically vanish. Therefore the quantities $\mathbf{T}(\mathbf{r})$, $\overleftrightarrow{\mathbf{J}}^2$ and $\Delta\mathbf{s}(\mathbf{r})$ do not need to be computed neither in the central energy density $\mathcal{H}_c(\mathbf{r})$ nor in the spin-orbit, spin and spin-gradient fields. In contrast, the coefficients B_{15} and B_{17} are non zero and the corresponding quantities $\mathbf{T}_q(\mathbf{r})$, $\overleftrightarrow{\mathbf{J}_q}^2$ and $\Delta\mathbf{s}_q(\mathbf{r})$ should be calculated in $\mathcal{H}_c(\mathbf{r})$, $W_q^{\mu\nu}(\mathbf{r})$, $\mathbf{S}_q(\mathbf{r})$ and $\mathbf{C}_q(\mathbf{r})$. However for simplicity we neglect as in, *e.g.*, Ref. [9], these densities, which suppresses the spin-gradient field contribution in the Hartree–Fock Hamiltonian and reduces the role of the spin-current tensor to the one of its antisymmetric part only.

We close this subsection with a discussion of two peculiar aspects of the above approach in odd-mass nuclei.

When neutrons or protons are in odd number, the appearance of time-odd fields in the Hartree–Fock Hamiltonian (10) removes the Kramers degeneracy in the single-particle spectrum. In the BCS and HTDA treatments of pairing correlations, this poses the problem of properly defining the notion of pairs. In this work we define the conjugate state $|\widetilde{i}\rangle$ of a given neutron or proton single-particle state $|i\rangle$ as the one that has the same charge state and the largest overlap in absolute value with the time-reversed state $|\bar{i}\rangle = \mathcal{T}|i\rangle$. In practice, it turns out that this overlap is equal to 1 within 1% or less for all cases encountered. One can similarly define a neutron-proton pair without ambiguity.

Moreover, we study here well-deformed nuclei (in order for their mean-field description to be relevant) with axial and left-right symmetric shapes in their ground state. Therefore the single-particle states $|i\rangle$ have a definite projection Ω_i of the angular momentum on the symmetry axis and a definite parity π_i. In these rigidly deformed nuclei the unified model picture [16] for the ground state applies and allows one to equate the total angular momentum I and parity $\pi_{\rm tot}$ in the ground state with the third angular-momentum projection K and parity π of the odd nucleon, namely $I = K$ and $\pi_{\rm tot} = \pi$ in the absence of Coriolis coupling effects. When solving the Hartree–Fock–BCS equations, we start from a converged solution for the underlying even-even core (assuming axial and left-right symmetries) and implement the self-consistent blocking procedure. More precisely, the single-particle state which has the desired quantum numbers K and π and the lowest energy above the underlying even-even core is imposed to have an occupation factor equal to 1 and does not participate in pair excitations. The BCS equations are thus solved for the remaining single-particle spectrum.

2.2 Highly Truncated Diagonalization Approach

The Highly Truncated Diagonalization Approach (HTDA) can be viewed as a highly truncated shell model based on a mean-field solution [4, 17].

We begin with the Hamiltonian $\hat{H}$ written as the sum of the intrinsic kinetic energy $\hat{K}$ and the nuclear-plus-Coulomb interaction $\hat{V}$. Then we consider an attractive one-body potential $\hat{U}$, obtained in this work by a Skyrme–Hartree–Fock–BCS calculation as presented in the previous subsection, and the associated one-body Hamiltonian $\hat{H}_0$ defined by $\hat{H}_0 = \hat{K} + \hat{U}$, whose single-particle eigenstates $|i\rangle$, corresponding to the eigenvalues e_i, form an orthonormal basis of the one-body space (including position, spin and isospin degrees of freedom). The lowest eigenstate of $\hat{H}_0$ is a quasi-particle vacuum of particle-hole type, in other words a Slater determinant, denoted by $|\Phi_0\rangle$. In the following this state is called merely quasi-vacuum and serves as a reference state in the definition of all normal products. We can express the Hamiltonian $\hat{H}$ as the sum of a one-body Hamiltonian, which we refer to as the independent quasiparticle Hamiltonian $\hat{H}_{\text{iqp}}$, a residual-interaction operator $\hat{V}_{\text{res}}$ and the expectation value of $\hat{H}$ in $|\Phi_0\rangle$. The independent quasiparticle Hamiltonian $\hat{H}_{\text{iqp}}$ is the normal-product form of $\hat{H}_0$

$$\hat{H}_{\text{iqp}} = : \hat{H}_0 : = \hat{H}_0 - \langle \Phi_0 | \hat{H}_0 | \Phi_0 \rangle \, . \tag{15}$$

In the single-particle basis $\{|i\rangle\}$ associated with $\hat{H}_0$, the operator $\hat{H}_{\text{iqp}}$ takes the second-quantization form $\hat{H}_{\text{iqp}} = \sum_i e_i \; : a_i^\dagger \hat{a}_i :$, where the operator $a_i^\dagger$ creates a nucleon in a state $|i\rangle$ whose isospin is implicitly specified in the label i, whereas a_i annihilates a nucleon in the state $|i\rangle$. The residual interaction $\hat{V}_{\text{res}}$ is defined by

$$\hat{V}_{\text{res}} = : \hat{V} : + : \overline{V} - \hat{U} : \, , \tag{16}$$

where $\overline{V}$ denotes the one-body reduction of $\hat{V}$ for $|\Phi_0\rangle$ and $\hat{V} = : \hat{V} : + : \overline{V} : + \langle \Phi_0 | V | \Phi_0 \rangle$ according to the Wick theorem applied to the two-body operator $\hat{V}$, so that $: \hat{V} : = \hat{V} - \overline{V} + \langle \Phi_0 | V | \Phi_0 \rangle$. Because the potential $\hat{U}$ comes from a Hartree–Fock–BCS calculation, it is expected to slightly differ from $\overline{V}$ when the solution to the BCS equations corresponds to non-vanishing pairing gaps. In the HTDA framework we neglect the contribution $: \overline{V} - \hat{U} :$ to the residual interaction $\hat{V}_{\text{res}}$.

A proper account of pairing correlations beyond $|\Phi_0\rangle$ requires one to use in $\hat{V}_{\text{res}}$ a nucleon-nucleon interaction having satisfactory particle-particle matrix elements. Very few Skyrme parametrizations provide a good description of mean-field and pairing properties simultaneously. Those which do (such as, e.g., the SkP parametrization [18]) are unfortunately not fitted to reproduce pairing properties in $N = Z$ nuclei. Therefore we resort to replacing the nuclear interaction in $\hat{V}_{\text{res}}$ by a contact interaction in the form of a density-independent δ interaction $\hat{V}_\delta$. Moreover we neglect the Coulomb contribution to the residual interaction, hence $\hat{V}_{\text{res}} \approx : \hat{V}_\delta :$. Because of the space-symmetric character of the

δ interaction, $\hat{V}_\delta$ can be decomposed as $\hat{V}_\delta = \sum_{T=0}^{1} V_0^{(T)}\,\delta(\mathbf{r}_1 - \mathbf{r}_2)\,\hat{\Pi}_S\,\hat{\Pi}_T$, where $\hat{\Pi}_S$ ($\hat{\Pi}_T$) is the spin (isospin) projection operator in the two-body subspace of the Fock space and $S = 1 - T$ (see, *e.g.*, Ref. [19]).

In practical HTDA applications, the many-body basis in which the Hamiltonian is diagonalized has to be truncated. Here we consider a model space including single-pair (SP) and double-pair (DP) excitations – with respect to the quasi-vacuum $|\Phi_0\rangle$–whose particle-hole excitation energy does not exceed a given cutoff energy $E_{\rm cut} = 3\,\hbar\omega(A)$ with the empirical inter-shell energy $\hbar\omega(A) = 41\,A^{-1/3}$ (in MeV). Since the correlations considered in this work are of pairing type in both $T = 1$ and $T = 0$ channels, all combinations of nn, np and pp pairs in the excited configurations are taken into account.

2.3 Magnetic dipole moment

In the unified model of Bohr and Mottelson [16], the magnetic dipole moment μ of the axially-symmetric ground state of an odd-mass nucleus is the sum of an intrinsic contribution $\mu_{\rm intr}$ and a contribution from the collective degrees of freedom $\mu_{\rm coll}$. The former is proportional to the projection $\hat{\mu}_z$ of the magnetic dipole moment operator on the symmetry axis (chosen to be z) according to $\mu_{\rm intr} = \frac{K}{K+1}\langle\Psi|\hat{\mu}_z|\Psi\rangle$, where $|\Psi\rangle$ is the normalized nuclear state with good K quantum number. The one-body operator $\hat{\mu}_z$ is defined by $\hat{\mu}_z = g_\ell \hat{\ell}_z + g_s \hat{s}_z$, where $\hat{\ell}_z$ and $\hat{s}_z$ are the corresponding projections of the single-particle orbital and spin angular-momentum operators.

Upon writing $\langle\Psi|\hat{\mu}_z|\Psi\rangle$ in the same form as in the single-particle model (without core polarization), one can define an effective spin gyromagnetic ratio $g_s^{(\rm eff)}$ by the relation $\langle\Psi|\hat{\mu}_z|\Psi\rangle = g_\ell^{(q)}\langle\hat{\ell}_z\rangle_{\rm odd} + g_s^{(\rm eff)}\langle\hat{s}_z\rangle_{\rm odd}$, where q is the charge state of the odd nucleon (the other charge state will be noted $\overline{q}$) and $\langle\hat{\ell}_z\rangle_{\rm odd}$ (resp. $\langle\hat{s}_z\rangle_{\rm odd}$) is the expectation value of $\hat{\ell}_z$ (resp. $\hat{s}_z$) for the odd nucleon. The ratio $g_s^{(\rm eff)}/g_s^{(q)}$ is called the spin quenching factor.

The collective contribution $\mu_{\rm coll}$ is proportional to the collective gyromagnetic ratio g_R according to $\mu_{\rm coll} = \frac{K}{K+1}\,g_R$. In this work we calculate g_R microscopically in the cranking model with BCS pairing correlations as in Ref. [20]

$$g_R = \frac{\sum_{k,\ell}\langle\ell|\hat{\mu}_-|k\rangle\langle k|\hat{j}_+|\ell\rangle\,(u_k v_\ell - u_\ell v_k)^2/(E_k + E_\ell)}{\sum_{k,\ell}\langle\ell|\hat{j}_-|k\rangle\langle k|\hat{j}_+|\ell\rangle\,(u_k v_\ell - u_\ell v_k)^2}\,, \tag{17}$$

where the sums run over all Hartree–Fock–BCS single-particle states, $\hat{j}_+ = \hat{j}_x + i\,\hat{j}_y$ is the usual raising angular-momentum operator, $\hat{j}_- = \hat{j}_+^\dagger$ (similar expressions hold for $\hat{\mu}_\pm$), E_k is the quasi-particle energy of the single-particle state $|k\rangle$ and u_k (resp. v_k) is the BCS vacancy (resp. occupation) factor of the single-particle state $|k\rangle$.

3 Results and Discussion

We have performed Skyrme–Hartree–Fock–BCS calculations in twelve well-deformed odd-mass nuclei from the $A \sim 50$ region to the actinide region, upon blocking the lowest K^π state above the underlying even-even core. The Skyrme functional has been chosen in its SIII parametrization and the seniority force of Ref. [15] has been used to solve the BCS equations. To solve the Hartree–Fock equations we have expanded the single-particle wave functions onto a cylindrical hamonic-oscillator basis with basis parameters optimized for the Skyrme–Hartree–Fock–BCS solution of the underlying even-even core.

For each ground-state solution we have calculated the intrinsic magnetic dipole moment and deduced the g_s quenching factor as defined in the previous section. Overall we have found the same g_s^{eff}-values as for the pure Hartree–Fock solutions of Ref. [2]. The nn and pp pairing correlations have thus no impact on the intrinsic magnetic moment, at least in the BCS framework.

To check this conclusion against a particle-number conserving approach to pairing correlations, we have performed HTDA calculations for these nuclei. As it turns out the difference with the Hartree–Fock results for the g_s quenching factor are of the order of only 10^{-3} for all nuclei except those with $N \approx Z$. Selected results obtained with $V_0^{(T=1)} = -250 \text{ MeV} \cdot \text{fm}^3$ are presented in Table 1.

For ^{49}Cr the HTDA results on the second line are obtained with $V_0^{(T=0)} = V_0^{(T=1)}$, and the results on the third line are obtained by setting to 0 the matrix elements between $T_z = 0$ two-body states. These results show that the np pairing correlations only have an effect on the intrinsic magnetic moment. This is consistent with the fact that the $T = 0$ residual interaction plays a role in the intrinsic magnetic moment as already observed in Ref. [3].

To better understand the np pairing mechanism at work in μ_{intr}, we consider the following two-level model for ^{49}Cr. Let us assume that the Kramers

Table 1. Quenching factor of g_s calculated within the HTDA framework, together with the main components of the corresponding ground-state solution.

Nucleus	K^π	$\langle \hat{s}_z \rangle_{\text{odd}}$	Quenching factor		isospin channel	Weight in wave function [%]			
						1 pair			2 pairs
			HF	HTDA		qq	$\overline{qq}$	np	
^{49}Cr	$5/2^-$	0.426	0.77	0.56	$T = 1$	3.2	11.0	8.2	1.3
				0.60	$T = 1, 0$	3.0	16.6	10.2	2.5
				0.77	$T_z = \pm 1$	3.9	15.3	0.0	1.0
^{177}Lu	$7/2^+$	-0.480	0.78	0.78	$T = 1$	3.8	31.6	0.0	3.6
^{177}Hf	$7/2^-$	-0.415	0.71	0.71		13.4	11.6	0.1	2.2
^{179}Hf	$9/2^+$	0.438	0.73	0.73		7.1	13.1	0.0	1.4
^{179}Ta	$9/2^-$	0.478	0.82	0.82		5.0	25.4	0.1	3.0
^{235}U	$7/2^-$	0.364	0.72	0.72		7.3	24.9	0.0	3.7

degeneracy and isospin symmetry are realized with a very good approximation for the last occupied $|n\rangle$ (resp. $|p\rangle$) and first unoccupied $|\nu\rangle$ (resp. $|\pi\rangle$) single-particle levels in ^{48}Cr for neutrons (resp. protons). Then we describe the unperturbed ground-state of ^{49}Cr by the Slater determinant $|\Phi_0\rangle = a^\dagger_\nu|^{48}\mathrm{Cr}\rangle$, where $|^{48}\mathrm{Cr}\rangle$ denotes the lowest-energy Slater determinant representing the unperturbed ground-state of ^{48}Cr. The HTDA many-body basis for ^{49}Cr, limited to single-pair excitations, is thus made of the four Slater determinants: $|\Phi_0\rangle$, $|\Phi_{pp}\rangle = a^\dagger_\pi a^\dagger_{\overline{\pi}} a_{\overline{p}} a_p|\Phi_0\rangle$ (one-pp-pair excitation), $|\Phi_{np}\rangle = a^\dagger_{\overline{\nu}} a^\dagger_\pi a_p a_{\overline{n}}|\Phi_0\rangle$ and $|\Phi'_{np}\rangle = a^\dagger_{\overline{\nu}} a^\dagger_\pi a_{\overline{p}} a_n|\Phi_0\rangle$ (one-np-pair excitations). The HTDA solution has thus the form $|\Psi\rangle = \chi_0|\Phi_0\rangle + \chi_{pp}|\Phi_{pp}\rangle + \chi_{np}|\Phi_{np}\rangle + \chi'_{np}|\Phi'_{np}\rangle$, where the χ coefficients are determined by diagonalization of the HTDA hamiltonian in the above basis.

The contribution from pairing correlations to the expectation value of $\hat{\mu}_z$ for the HTDA solution, defined by $\langle\hat{\mu}_z\rangle_{\mathrm{corr}} = \langle\hat{\mu}_z\rangle_{\mathrm{HTDA}} - \langle\hat{\mu}_z\rangle_{\mathrm{HF}}$, reads

$$\begin{aligned}\langle\hat{\mu}_z\rangle_{\mathrm{corr}} = \big(\chi'^2_{np} + \chi^2_{np}\big)\Big[K + \big(g_s^{(p)} - 1\big)\langle\pi|\hat{s}_z|\pi\rangle - g_s^{(n)}\langle\nu|\hat{s}_z|\nu\rangle\Big] \\ + \big(\chi'^2_{np} - \chi^2_{np}\big)\Big[K_{\mathrm{hole}} + \big(g_s^{(p)} - 1\big)\langle p|\hat{s}_z|p\rangle - g_s^{(n)}\langle n|\hat{s}_z|n\rangle\Big],\end{aligned} \quad (18)$$

where K denotes the angular-momentum projection for the blocked state $|\nu\rangle$ (as well as for the analog proton state $|\pi\rangle$) and K_{hole} is the angular-momentum projection for the hole levels $|n\rangle$ and $|p\rangle$. This shows that in the absence of np pairing correlations the HTDA value of μ_{intr} does not differ from the Hartree–Fock value. If both kinds of np-pair excitations are equally populated, which is approximately the case when the $T = 0$ channel of the residual interaction is negligible with respect to the $T = 1$ channel, and if one neglects the charge symmetry breaking, the correlation contribution to $\langle\hat{\mu}_z\rangle$ becomes:

$$\langle\hat{\mu}_z\rangle_{\mathrm{corr}} \approx \big(\chi'^2_{np} + \chi^2_{np}\big)\Big[K + \big(g_s^{(p)} - g_s^{(n)} - 1\big)\langle\hat{s}_z\rangle_{\mathrm{odd}}\Big], \quad (19)$$

with $g_s^{(p)} - g_s^{(n)} - 1 \approx 8.412$. This expression explains the large pairing-correlation contribution to $\langle\hat{\mu}_z\rangle$ when $\langle\hat{s}_z\rangle_{\mathrm{odd}} > 0$, hence the large differences

Table 2. Comparison of the total magnetic moment μ with its experimental values from Ref. [21] when available.

Nucleus	$(J^\pi)_{\mathrm{exp}}$	$(K^\pi)_{\mathrm{th}}$	g_R	$\mu = \mu_{\mathrm{coll}} + \mu_{\mathrm{intr}}$			Exp.
				HF	HTDA	isospin channel	
^{49}Cr	$5/2^-$	$5/2^-$	0.535	-0.51	-0.28	$T=1$	$\pm 0.476(3)$
					-0.32	$T=1,0$	
^{179}Hf	$9/2^+$	$9/2^+$	0.378	-0.65			$-0.6409(13)$
^{179}Ta	$7/2^+$	$9/2^-$	0.378	5.37		$T=1$	
^{235}U	$7/2^-$	$7/2^-$	0.235	-0.59			$-0.38(3)$

obtained in HTDA and Hartree–Fock g_s quenching factors for ^{49}Cr as seen in Table 1. In the presence of $T = 0$ pairing, the second term in Eq. (18) comes into play but numerical calculations show that the above explanation still holds.

Finally we have calculated the collective gyromagnetic ratio and compared the total magnetic moment with the experimental value when available. As can be seen from Table 2, the agreement is fairly good.

4 Conclusion

We have studied the effect of core polarization on single-particle states particularly on intrinsic magnetic moments of odd-mass nuclei in the presence of $T = 0$ and $T = 1$ pairing correlations. Overall we have obtained a quenching of the spin contribution to intrinsic magnetic moments of about 0.78, which is close to the empirical value.

Moreover we have found that, away from the $N = Z$ line, the pairing correlations have a negligible effect on magnetic moments, whereas they play a major role in $N \sim Z$ odd-mass nuclei. We have traced back this effect to the np pairing correlations which are active only close to the $N = Z$ line, in both $T = 0$ and $T = 1$ isospin channels, whereas the nn and pp pairing correlations are spectators.

Finally, when including the collective contribution to magnetic moments, we obtain a fairly good agreement with experimental data for well-deformed odd mass nuclei in mass regions $A \sim 50$, rare-earth nuclei around $A = 178$ and actinide nuclei (around $A = 236$).

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