NUCLEAR THEORY, Vol. 31 (2012)
eds. A. Georgieva, N. Minkov, Heron Press, Sofia

Role of the SU(3)-Symmetry-Breaking Terms of the Hamiltonian in the Pairing-Plus-Quadrupole Model for Two Oscillator Shells

K.P. Drumev, **A.I. Georgieva**

Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria

Abstract. An extended pairing-plus-quadrupole model for two oscillator shells, realized in the framework of the Elliott SU(3) scheme, is used to study the combined effects of the quadrupole-quadrupole, pairing, and single-particle interactions on energy spectra and lowest-J state shapes of nuclear systems. The pairing part of the Hamiltonian consists of pp-, nn- and pn-pairing terms and terms describing the pair-scattering between two oscillator shells. Results for systems of different mass are calculated in the $ds + fp$ shells for a reasonable choice of values for the interaction parameter strengths.

1 Introduction

Intruder levels of opposite parity from the upper shell are key ingredient for the building of a complete shell-model theory. Indeed, their inclusion in the model space gives the opportunity for interpretation of many interesting features in nuclei. Specifically, the size of the model space under consideration allows the description of experimentally observed high-spin and high-energy states and the abundance of 0^+ states in a certain energy interval while the addition of opposite-parity levels is key for the simultaneous description of both positive-parity and negative-parity states. The intruder levels are present in heavy deformed nuclei where the strong spin-orbit interaction destroys the underlying harmonic oscillator symmetry of the nuclear mean-field potential. The role they play for the overall dynamics of the system has been the topic of many discussions [1–4]. Until now, the problem has been either approached within the framework of a truncation-free toy model [1] or by just considering the role of the single intruder level detached from its like-parity partners [2, 4]. It was argued in [1] that particles in these levels contribute in a complementary way to building the collectivity in nuclei. However, some mean-field theories suggest that these particles play the dominant role in inducing deformation [3].

The purpose of the present work is to improve the predictive power of the extended SU(3) realization of the pairing-plus-quadrupole model in two oscillator shells introduced in [5] by adding the single-particle terms in the Hamiltonian of the system. This approach for the first time explicitly included particles from the complete unique-parity sector. After a short introduction for the basics of the model - basis states and Hamiltonian used - we present some results for the ground-state and/or lowest-J state properties of identical-particle nuclear systems in the full $ds + fp$ shell model space. We conclude with the outcome for the proton-neutron system of 2 protons and 2 neutrons which in the $ds+fp$ shell is the prototype of the nucleus ^{20}Ne.

2 Pairing-Plus-Quadrupole Model in Two Oscillator Shells

The pairing-plus-quadrupole model, first introduced by Bohr and Mottelson and Pines [6], and Belyaev [7], has been widely used to reproduce both few-particle non-collective and many-particle collective features of nuclei [8–10]. First of all, the model incorporates the features most important in nuclear mean-field theories: the interaction between particles can be summed up, in a first approximation, to an average spherical single-particle potential; and long range particle-hole correlations and short-range particle-particle correlations can be taken into account by a deformation of the field and a pairing potential, respectively [11]. The model has never been applied for full-space calculations in more than one of the low-lying oscillator shells or for restricted number of particles in higher-lying shells.

The SU(3) realization of the model was initially introduced in one [12, 13] and two spaces [14](for proton-neutron systems) and later extended to four spaces [5] giving an opportunity to explore proton-neutron systems in model spaces including more than one shell. In this contribution we use this novel concept where the SU(3) basis states are of the type

$$|\{i_\pi; i_\nu\}\rho(\lambda,\mu)\kappa L, \{S_\pi, S_\nu\}S; JM_J\rangle \,. \tag{1}$$

They are built as SU(3) proton (π) and neutron (ν) strongly-coupled configurations with well-defined particle number and good total angular momentum J. In (1) the quantum numbers indicated by $i_\sigma = \{i_{\sigma N}, i_{\sigma U}\}\rho_\sigma(\lambda_\sigma,\mu_\sigma)$, where the $i_{\sigma\tau} = N_{\sigma\tau}[f_{\sigma\tau}]\alpha_{\sigma\tau}(\lambda_{\sigma\tau},\mu_{\sigma\tau})$ are the basis-state labels for the four spaces in the model (σ stands for π or ν, and τ stands for normal (N) or unique (U) parity levels). First, the particles from the normal and the unique spaces are coupled for both protons and neutrons. Then, the resulting proton and neutron irreps are coupled to a total final set of irreps. The total angular momentum J results from the coupling of the total orbital angular momentum L with the total spin S. The ρ and κ are, respectively, the multiplicity indices for the different occurrences of (λ,μ) in $\{(\lambda_\pi,\mu_\pi)\times(\lambda_\nu,\mu_\nu)\}$ and L in (λ,μ).

The SU(3) classification of many-body states has the advantage of allowing for a geometrical analysis of the eigenstates of a nuclear system via the relations between the microscopic parameters (λ,μ) and the parameters (β,γ) of the collective model [15] and hence it gives an insight into phenomena associated with nuclear deformation. The parameter β is a measure of the degree of axial deformation while the triaxiality parameter γ takes the values of 0 degrees for a prolate shape of the nucleus, 60 degrees for an oblate shape and 30 degrees in case of maximal triaxiality. The parameters β and γ are given by the following expressions [16]:

$$k\beta = \frac{2}{3}\sqrt{C_2+3}$$
$$\gamma = \frac{1}{3}cos^{-1}\Big(\frac{C_3}{2\sqrt{(C_2+3)^3}}\Big). \tag{2}$$

Here the constant $k = \sqrt{\frac{5}{9\pi}}A < r^2 >$, where A is the total number of nucleons, $< r^2 >$ - the nuclear mean square radius of the system, and $C_2 = \lambda^2 + \mu^2 + 3(\lambda+\mu)+\lambda\mu$ and $C_3 = (\lambda-\mu)(\lambda+2\mu+3)(2\lambda+\mu+3)$ are the Casimir operators of second and third order, respectively. These relations can only be used for a state with good SU(3) symmetry. When this is not the case, one should calculate the expectation values of these expressions instead.

In this paper we use a Hamiltonian more general than the ones used in the earlier one-shell SU(3) realizations of the pairing-plus-quadrupole model for identical particle [12, 13] and proton-neutron [14] systems. We consider additionally identical-particle pair-scattering terms ($\tau \neq \tau'$), proton-neutron terms ($\pi\nu$) and isoscalar terms which is obviously a more sophisticated form of the well-known pairing-plus-quadrupole model. The Hamiltonian has the form

$$H = H^{s.p.} - \frac{\chi}{2}Q.Q - G\Big\{\sum_{\sigma,\tau}(S^+)^{J=0T=1}_{\sigma\tau}(S^-)^{J=0T=1}_{\sigma\tau} + \sum_{\sigma,\tau\neq\tau'}(S^+)^{J=0T=1}_{\sigma\tau}(S^-)^{J=0T=1}_{\sigma\tau'} + \sum_{JT}\sum_{\tau,\tau'}(S^+)^{JT}_{\pi\nu,\tau}(S^-)^{JT}_{\pi\nu,\tau'}\Big\} \tag{3}$$

where, for simplicity, all pairing terms are taken with the same strength. The standard pair-creation and annihilation operators are given by

$$(S^+)^{JT}_{\sigma\tau} = \frac{1}{2}\sum_{\eta l j m_j}(-)^{l+j-m_j}[(a^\dagger_{(\eta,0)l\frac{1}{2}jm_j})_{\sigma\tau} \times (a^\dagger_{(\eta,0)l\frac{1}{2}j-m_j})_{\sigma\tau}]^{JT} \tag{4}$$

and

$$(S^-)^{JT}_{\sigma\tau} = ((S^+)^{JT}_{\sigma\tau})^\dagger, \tag{5}$$

with $a^\dagger_{(\eta,0)l\frac{1}{2}jm_j}$ and $a_{(\eta,0)l\frac{1}{2}jm_j}$ being the creation and annihilation operators, respectively. In equation (3), $L=0$ identical-particle pairing and pair-scattering are given by the first and the second term, respectively. The third term in the braces describes the isovector $((JT) = (01))$ and the isoscalar $((JT) = (10))$ proton-neutron pairing and pair-scattering. The single-particle part of the Hamiltonian is

$$H^{s.p.} = \sum_j \epsilon_j \hat{n}_j \tag{6}$$

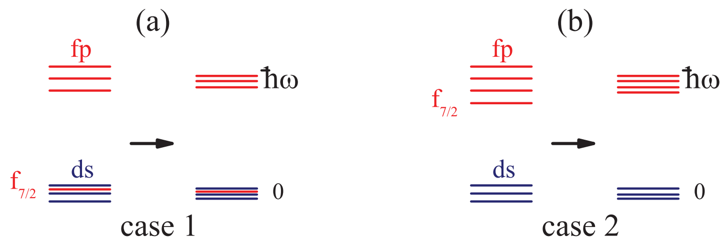

Figure 1. The single-particle states for case 1 (a) and case 2 (b).

where $\hat{n}_j = \sum_{m_j} a^{\dagger}_{(\eta 0)ljm_j} a_{(\eta 0)ljm_j}$ is the number operator and the sum goes over the single-particle levels of energy ϵ_j for both oscillator shells involved in the construction of the total model space (see below).

3 Results

Calculations are performed in a model space composed of two - ds and fp - shells which we call the $ds + fp$ shell, where no restrictions on the basis have been imposed and two distinct cases have been explored (see Figure 1). Case 1 represents the situation when the energy of the $f_{7/2}$ level is close to the one of the levels from the ds shell, so it is considered to be an intruder level which penetrates the lower-lying ds-shell levels of opposite parity. In case 2 the single-particle energy of the $f_{7/2}$ level is similar to that of its like parity partners from the fp shell and is not an intruder level. Hence, the two cases correspond to two different deformations in the Nilsson basis.

Our main goal is to explore the behavior of the systems implied by the use of diferent values for the parameters $\hbar\omega$, G and χ. The values for $\hbar\omega$ are chosen to include the one, typical for the ds shell nuclei of about 15 MeV. Specifically, this parameter varies in steps between zero and 20 MeV. For simplicity, pairing strengths are taken equal for identical-particle pairing and the proton-neutron terms and are 0.05 MeV(mild value) and 0.2 MeV(a reasonable value, which is somewhat reduced from the one of 0.45 MeV used in one-shell calculations because of the new pairing terms included in the Hamiltonian). Finally, we vary the values of the parameter χ in small steps between 0 and 0.3 MeV.

3.1 System of 2, 3 and 4 particles in the $ds + fp$ shells

These systems can be considered as the prototypes for the following three pairs of mirror nuclei: ^{18}O and ^{18}Ne, ^{19}O and ^{19}Na, and ^{20}O and ^{20}Mg. The even-even nuclei from the list above have a ground state with $J = 0^+$. In the case of ^{19}O(^{19}Na) we calculated the lowest-J state with characteristics $J = \frac{1}{2}^+$.

First, results for the wave-function contents in the ground state of the system of 4 particles are shown for the case 1 (Figure 2) and case 2 (Figure 3), respectively. The sum contribution of configurations with the three possible dis-

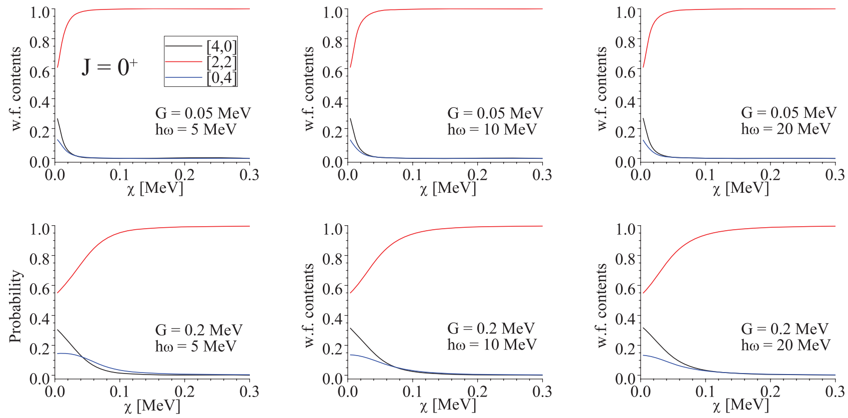

Figure 2. Wave-function contents of the ground state for the system of 4 particles using various sets of parameters G and $\hbar\omega$ as a function of the quadrupole parameter with the single-particle energies according to case 1.

tributions of nucleons between the ds and fp shells – [4,0], [2,2] and [0,4] – is displayed. The symbol $[N, U]$ stands for the type of configurations where N is the number of particles in the ds shell and U - the one in the fp shell. Qualitatively, the situation in the two cases is very distinct. In case 1, all three types of configurations compete for lower χ values – up to $\chi = 0.04$ MeV for $G = 0.05$ MeV and $\chi = 0.1$ MeV for $G = 0.2$ MeV independently of the value of $\hbar\omega$. The dominant configurations for any reasonable value of the parameters

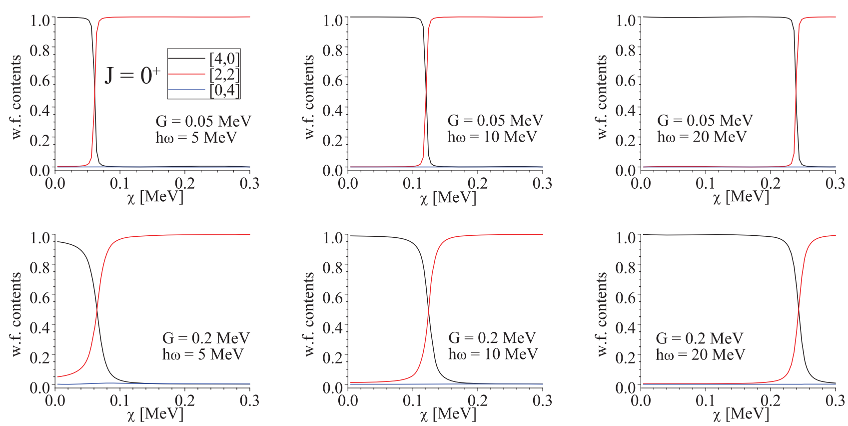

Figure 3. Wave-function contents of the ground state for the system of 4 particles using various sets of parameters G and $\hbar\omega$ as a function of the quadrupole parameter with the single-particle energies according to case 2.

$\hbar\omega$, G and χ are of the type [2,2] where two particles are in the ds shell and 2 – in the fp shell. With the rise of the values of χ parameter and the fall of G parameter, only configurations of this type are present. In case 2, the type of configuration [0,4] is not present for any values of χ parameter. There is a point, at which the type of dominant configurations changes from [4,0] to [2,2] with the rise of χ. For a choice of bigger interval between the two oscillator shells (higher $\hbar\omega$ values) this point moves to larger χ values. The role of the pairing interaction (higher G values) is to soften the transition which occurs over a

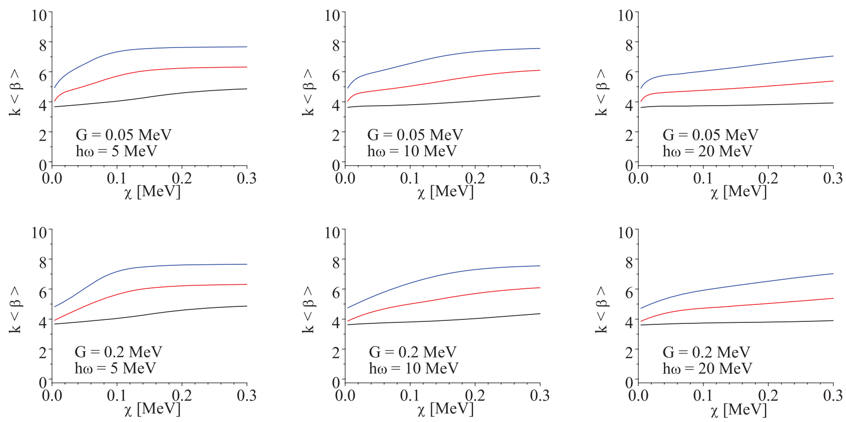

Figure 4. Expectation value of the shape parameter β for the lowest-J state of the systems of 2 ($J = 0^+$, black), 3($J = \frac{1}{2}^+$, red) and 4($J = 0^+$, blue) particles using various sets of parameters G and $\hbar\omega$ with the single-particle energies according to case 1.

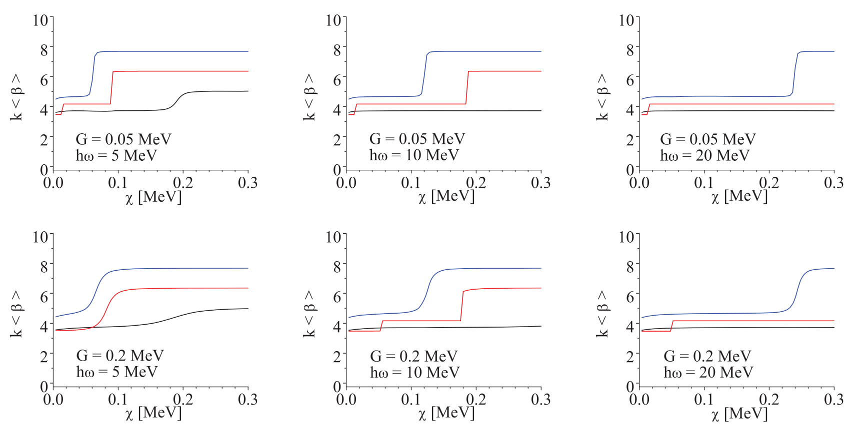

Figure 5. Expectation value of the shape parameter β for the lowest-J state of the systems of 2 ($J = 0^+$, black), 3($J = \frac{1}{2}^+$, red) and 4($J = 0^+$, blue) particles using various sets of parameters G and $\hbar\omega$ with the single-particle energies according to case 2.

broader interval of χ values.

Next, we use Eq.(2) and the eigenfunction expansion over the basis states to calculate the expectation values $<\beta>$ and $<\gamma>$. Figures 4 and 5 show the results for the expectation values of the deformation parameter β in the lowest-J states for the systems of 2, 3 and 4 particles. In case 1, the value of β goes up with the rise of χ, while the behavior looks very similar for all the chosen values of $\hbar\omega$ and G. In case 2, there is a rapid jump in the value which happens at the same χ value, where the change in the character of the dominant configurations

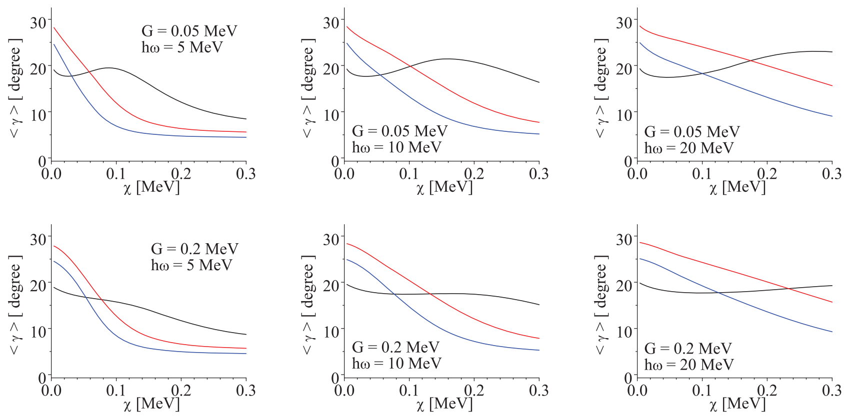

Figure 6. Expectation value of the shape parameter γ for the lowest-J state of the systems of 2 ($J = 0^+$, black), 3($J = \frac{1}{2}^+$, red) and 4($J = 0^+$, blue) particles using various sets of parameters G and $\hbar\omega$ with the single-particle energies according to case 1.

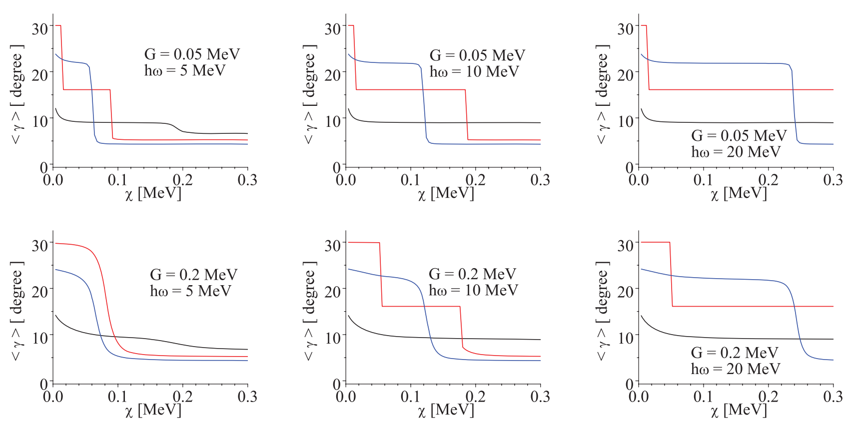

Figure 7. Expectation value of the shape parameter γ for the lowest-J state of the systems of 2 ($J = 0^+$, black), 3($J = \frac{1}{2}^+$, red) and 4($J = 0^+$, blue) particles using various sets of parameters G and $\hbar\omega$ with the single-particle energies according to case 2.

occurs (see Figure 3). Heavier systems are normally characterized by larger values of the β parameter which is indeed the case for higher values of the χ parameter. The situation for the expectation value of the parameter γ is similar, so we only point out the differences. This parameter has a smaller value for heavier systems. The case of $\chi = 0$ gives an almost perfect triaxial shape for the case of 3 particles which is the result of a single irrep with $\lambda = \mu$. For lower $\hbar\omega$ values the transition to a less triaxial shape occurs more rapidly in case 1 for small-χ values.

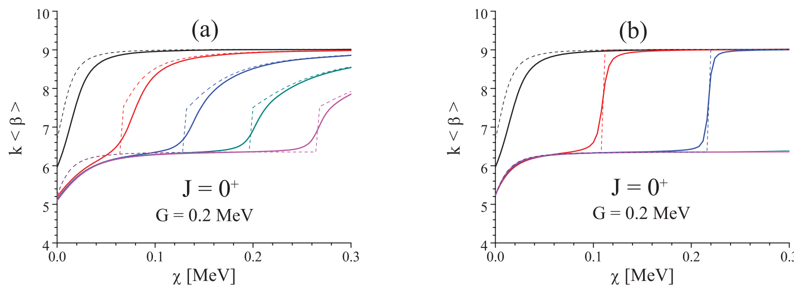

Figure 8. Expectation value of the shape parameter β for the ground state of a system of 2p+2n using the total Hamiltonian (solid line) and without the use of the scattering pairing terms (dashed line). In this calculation the single-particle energies are chosen according to case 1 (a) and case 2 (b) with $\hbar\omega = 0$ MeV (black), 5 MeV (red), 10 MeV (blue), 15 MeV (cyan), and 20 MeV (magenta).

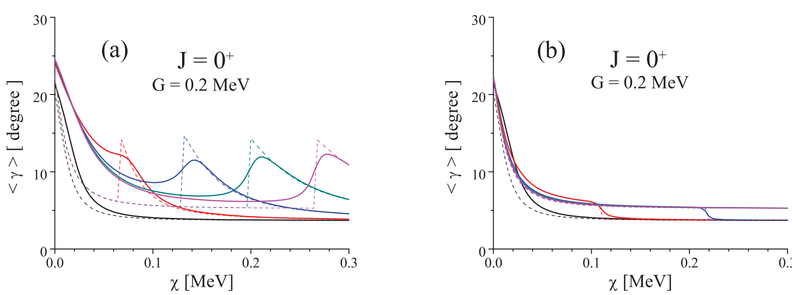

Figure 9. Expectation value of the shape parameter γ for the ground state of a system of 2p+2n using the total Hamiltonian (solid line) and without the use of the scattering pairing terms (dashed line). In this calculation the single-particle energies are chosen according to case 1 (a) and case 2 (b) with $\hbar\omega = 0$ MeV (black), 5 MeV (red), 10 MeV (blue), 15 MeV (cyan), and 20 MeV (magenta).

3.2 The system $2p + 2n$ in the $ds + fp$ shells

Finally, we present results from calculations performed for a N=Z nuclear system of two protons and two neutrons which we have not labeled as ^{20}Ne, since no comparison with experimental data has been shown. Compared to Figures 4 and 5, Figure 8 illustrates a slightly higher saturation value for the deformation parameter β with respect to the result for four particles. An analogous comparison of Figure 9 with Figures 6 and 7 shows slightly lower saturation value for the triaxiality parameter γ. This change is due to the consideration of proton neutron interactions in two shells. For both β and γ the role of the pair-scattering terms in the Hamiltonian is also shown. Namely, the lack of these terms (result given by the dashed lines in both Figures 8 and 9) implies a rapid rise in both β and γ. This (as well the sharp jump obtained in the case 1 for γ, illustrated in Figure 6 (a)) is a result of the transition in the eigenstate contents from the leading representation of the dominant type of configuration to a mixture of representations of the other type. This effect is not observed in case of transition from the dominance of a single irrep of one type to a single one of another type as is the situation for case 2 in Figures 8 (b) and 9 (b). The addition of all the pair-scattering terms (solid line) leads to softening of the rapid jump in the value for a certain strength χ. This effect is more prominent than the sum effect of the proton-neutron pairing and pair-scattering terms.

4 Conclusion

In this work, calculations for the systems of 2, 3 and 4 particles and the system of 2 protons and 2 neutrons were performed in a free of truncation model space built over the combination of two oscillator shells. The effects of the quadrupole, pairing and single-particle terms of the Hamiltonian were studied for the two extreme cases for the position of the $f_{7/2}$ single-particle level. The results indicate that while the pairing interaction mostly softens the effects, the strength of the single-particle energies drives the main (rapid) changes in the behavior of the systems.

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