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Boson Excitations and Different Parity Bands in Even-Even Deformed Nuclei

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Abstract.
We analyze the behavior of different parity low laying bands. We observe that the first excited 0^- band, which is always considered together with the gsb actually runs in paralel with some of the excited 0^+ bands. Their energies are well reproduced by means of a simple generalization of the rigid rotor expression for them. The same results are very accurately reproduced within the symplectic extension of the Interacting Boson Model (IVBM). The relations between the phenomenological parameters of the model Hamiltonian and the generalized rotor are obtained. It is shown that the value of the moment of inertia of the considered collective bands, depends on the number of monopole bosons building their band heads, obtained from the energy distributions of the 0^+ states.

1 Introduction

A lot of new experimental data on excited states and γ transitions between them are observed in the even-even rare earth and lantanides (see for example [1]) nuclei. In some of them they are a lot of collective bands both with positive and negative parities, which are observed up to very high spins. This abundance of collective states makes the task of evaluating the states energy ordering in bands and their multipolarities a very hard task. That is why the model analysis of the spectra of these nuclei is a very important task and at the same time a good testing ground for the reliability of the theoretical nuclear structure models.

In this work we aim to review again the application of one of the dynamical symmetries [2] of the symplectic extension of the Interacting Vector Boson Model (IVBM) [3], which was used for the description of the behavior of the ground and octupole bands in the heavy even-even nuclei. Unfortunately the consideration of other excited bands [4] in this framework, required new assignment of the quantum numbers defining the basis states, as well as the introduction of new parameters in the model Hamiltonian, which complicated their description and interpretation. In this work we present a simplified approach to the description of these bands, which includes the consideration of some additional excited 0^+ bands, which as is empirically observed strongly influence the behavior of the ground and octupole bands. They cross at some point the ground state band and so form the second part of the yrast band. In many cases the negative parity 0^- band is parallel to these bands, which simplifies to a large extend their description, which is analogous to the one of the positive parity bands. We show that the energies of the considered collective bands are strongly dependant on the number of bosons that build their band head configurations. This once again outlines the advantage of the symplectic extension of the IVBM, which is based on the construction of the basis states from different number of boson excitations.

2 Theoretical background – U(6) limit of the Interacting Vector Boson Model

The U(6) limit of the Interacting Vector Boson Model (IVBM) [3] has been rather successful in describing collective bands both with possitive and negative parities within the framework of the boson representation of the $Sp(12, R)$ algebra [5]. All the possible irreducible representations: $[N]$ of $U(6)$, (λ, μ) of $SU(3)$ and KLM, which define the multiplicity of the the angular momentum L and its projection M in the final reduction to the $SO(3)$ representations are determined uniquely through all possible sets of the eigenvalues of the Hermitian operators $\mathbf{N}$, $\mathbf{T^2}$, $\mathbf{T_0}$ and $\mathbf{L^2}$ reducing the symplectic extension $sp(12, R) \supset u(6)$ [2] to "the unitary" limit of the model [6].

$$\begin{array}{ccccl} sp(12,R) & \supset & u(6) & \supset & su(3) \otimes u(2) \\ & & & & \cup \\ & & & & so(3) \supset so(2) \end{array} \tag{1}$$

A good description of the energies up to rather high angular momenta was achieved for the states with positive and negative parity from the ground and octupole rotational bands respectively in [2], where a comprehensive description of the algebraic model may be found.

Here we present only the necessary expressions for the energy spectrum and the decomposition rules for the representations in the dynamical symmetry chain (1), needed to extend its description to higher spins and to include in the considerations other collective bands.

The energy spectrum produced by the IVBM Hamiltonian acting on the basis states of the boson representations of $Sp(12, R)$, labelled by the quantum numbers of the subgroups of group-subgroup chain (1) is given by:

$$E((N,T); KLM; T_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + \alpha_1 T_0^2 \tag{2}$$

The decomposition rules used to obtain the representation labels of the basis states $N = 0, 2, 4, 6...$ – even, are: $T = \frac{N}{2}, \frac{N}{2} - 1, \frac{N}{2} - 2, ..., 0$ or 1 and $T_0 = -T, -T+1, ..., T-1, T$.

The following relations between the labels of the $su(3)$ and the $su(2)$ complimentary subalgebras are implied:

$$\lambda = 2T, \qquad \mu = \frac{N}{2} - T, \tag{3}$$

which facilitates the well known final reduction to the $SO(3)$ algebra of the angular momentum.

$$K = \min(\lambda,\mu), \quad \min(\lambda,\mu)\ -2,\ldots,0 \text{ or } 1$$

$$\begin{array}{llllll} K=0 \longrightarrow & L=\max(\lambda,\mu), & L=\max(\lambda,\mu)\ -2, & ..., & 0, & 1 \\ K\neq\ 0 \longrightarrow & L=\max(\lambda,\mu), & L=\max(\lambda,\mu)-1, & ..., & 0, & 1 \end{array}$$

The index K appearing in this reduction is related to the projection of L in the body fixed frame and is used with the parity to label the different bands in the energy spectra of the nuclei. The parity of the states in this application of IVBM is defined as $\pi = (-1)^T$. This allows us to describe both positive and negative parity bands.

Further as the spectrum generating algebra of the IVBM is noncompact we can change the number of bosons N, that build the states of a given band. In this paper we modify the earlier application of the IVBM [7] for the description of the first excited even and odd parity bands in order to reach much higher angular momentum states in both band types. The other aim is to investigate the relationships, if such exist of the of the low laying collective bands and to improve their description. We apply the model to even- even deformed nuclei, which exhibit a low-lying negative parity band next to the ground band traditionally considered to be an octupole band [8]. In particular we would like to investigate the influence of the other excited 0^+ bands on the behavior of yrast and the negative parity parity bands.

In order to do this we first have to identify theirs experimentally observed states to the ones of the symplectic basis:

$$\mid [N]_6;(\lambda,\mu);K,L,M;T_0\rangle = \mid (N,T);K,L,M;T_0\rangle \tag{4}$$

for the even representation $N-even$ of $Sp(12,R)$. The ground state of the system is:

$$\mid 0\rangle = \mid (N=0,T=0);K=0,L=0,M=0;T_0=0\rangle \tag{5}$$

which is the vacuum state for the $Sp(12,R)$ group. From this state starts the ground state band (gsb), which obviously has $T=T_0=0$. Further we identify the states of the band as a sequence of states with $\lambda = 0$ (3) and $\mu = L = 0,2,4....$ with $K=0^+$. Hence for the gsb our choice are the $SU(3)$ multiplets $(0,\mu)$ with $\mu = L = N/2$. Thus, taking into account the reduction rules (3) the energy (2) can be rewritten only in terms of the angular momentum L in the following way:

$$E_g(L) = \beta_g L(L+\omega_g), \tag{6}$$

where

$$\beta_g = 4b + \beta_3, \tag{7}$$

which gives the inertia parameter of the band, and

$$\omega_g = \frac{2a + \beta_3}{4b + \beta_3} \tag{8}$$

is a geometrical parameter, describing the deviation of the bands'energies from the rigid rotor behavior.

The expression (6) for the energies of the gsb is analogous to the one obtained empirically in [9] on the basis of the classification scheme, introduced in [10]. The geometrical parameter (8) is related there to the experimental ratio:

$$R_2(\omega_g) = \frac{E_g(4)}{E_g(2)} = 2 + \frac{4}{2 + \omega_g} \tag{9}$$

and is proven to be a good indicator for the nuclear collectivity [9] incorporating the limiting cases of the rigid rotor $\omega_g = 1$ $(a = 2b)$ and harmonic oscillator $\omega_g \to \infty$. Here the model parameters of the IVBM are related to the generalized rotor parameters (7), (8) and in this way reduced to only two for the gsb.

Further we generalize the above identification of the ground state band to the other excited 0^+ bands, with $T = 2, 4, 6...-$even and $\lambda_0 = 2T-$fixed. We introduce an initial value of $\mu = \mu_0 + i$, where $\mu_0 = \lambda_0 = 2T$ and $i = L = 0, 2, 4, ...$ Hence $N = \lambda_0 + 2\mu_0 + 2i = N_0 + 2L = 6T + 2L$ or $L = \frac{1}{2}(N - N_0) = \frac{1}{2}(N - 6T)$.

If we substitute the last relations in (2) we obtain for the energies of the other excited 0^+ bands the expression:

$$\begin{aligned} E_{\beta_T}(L) &= E_g(L) + \alpha_3 T(T+1) + aN_0 + bN_0^2 + 4bN_0 L \\ &= E_g(L) + \alpha_{\beta_T} T(T + \omega_{\beta_T}) + 24bTL, \end{aligned} \tag{10}$$

where $\alpha_{\beta_T} = 4b + \alpha_3$ and $\omega_{\beta_T} = \frac{6a+\alpha_3}{\alpha_{\beta_T}}$. Obviously these energies are shifted in respect to the ground state band with a constant energy, which depends on the fixed for the band values of T or N_0 and the parameter α_3 fitted for the considered even-even nucleus. The lowest such band can cross the ground state band, because of the last term $4bN_0L$ $(24bTL)$ and form the second higher part of the yrast band. At the crossing point of the two bands a backbending is observed or a sudden change in ω_g, depending on N_0 or T.

For the octupole band with $T = 1$ or other odd number and $\lambda_0 = 2T-$fixed, we choose $\mu_0 = \lambda_0 + 1 > \lambda_0$. Then for $\mu = \mu_0 + i$ with $i = 0, 2, 4, ...$ $L = \mu = \frac{N}{2} - T$ or $N = \lambda_0 + 2\mu_0 + 2i = N_0 + 2L - 2 = 2T + 2L$.

$$E_{oct}(L) = E_g(L) + \alpha_{oct} T(T + \omega_{oct}) + 8bTL \tag{11}$$

where $\alpha_{oct} = 4b + \alpha_3$ and $\omega_{oct} = \frac{2a+\alpha_3}{\alpha_{oct}}$.

From the expressions (10) and (11) it is easy to see that $\alpha_{oct} = \alpha_{\beta_T}$ and respectively $\omega_{\beta_T} = \omega_{oct}$ and they depend on the parameter b in front of N^2 in (2). On the same parameter b depend as well the the last terms in (10) and (11), which give the interaction of these excited bands with the ground state band. The later are distinguished from each other by the number of bosons N_0 that build their band head configurations. Hence we introduce here a dependence of this parameter on N_0 of the following type:

$$b \equiv b_{band} = \frac{b}{\frac{N_{0band}}{5} + 1} \tag{12}$$

The dependence of the parameter b_{band} on the number of bosons that build the band head configurations of the considered excited bands is motivated microscopically by the relations of the nuclear surface oscillation model [11] giving the density distribution of the nucleus, in n-boson excited state [12] as a function of the number of monopole bosons n, that build it. In the framework of the IVBM, there is also a dynamical symmetry chain $Sp(12, R) \supset Sp(4, R) \otimes SO(3)$ [13], which gives the energy distribution of states with fixed angular momentum L as function of the number of vector bosons that build them. This chain is strongly related to the explored here dynamical symmetry (1) and further proves the importance of the boson structure of the band head's configurations.

Further we investigate the fine structure effects in the collective rotational spectra of deformed even-even nuclei. The odd-even staggering patterns between ground and octupole bands have been investigated previously in [2], by meanse of the staggering function [14]:

$$\begin{aligned} Stg(L) = 6(E_L - E_{L-1}) - 4(E_{L-1} - E_{L-2}) - 4(E_{L+1} - E_L) \\ + E_{L+2} - E_{L+1} + E_{L-2} - E_{L-3}, \end{aligned} \tag{13}$$

This function is a finite difference of of fifth order in respect to energy $E(L)$ and is characteristic for the deviation of the rotational behavior from that of the rigid rotor. The function (13) is sensitive to the interaction between the two bands and at the point where they cross the so called "beat" patterns are observed. A much simpler test illustrating the behavior of the positive and negative parity bands in respect to each other is the second order staggering function [15]:

$$S(L) = \frac{|E_{L+1} - E_L| - |E_L - E_{L-1}|}{E_2} \tag{14}$$

also called a signature splitting index for an alternating parity band [16]. Such a band by definition should provide an equal spacing of the levels, so from (14) is clear that in the case when the positive and negative parity bands are parallel and close to each other the "amplitudes" of the function are almost constant and close to zero. Hence in the present investigation we use (14) as a measure of the strong interaction between the positive and negative parity bands forming the collective alternating parity band.

3 Results and Discussion

The aim of our present investigation is to explore the high spin behavior of the alternating parity bands and the interaction between the positive and negative parity bands that form them. Such an investigation should be performed systematically as in [17] for the nuclei from the rare-earth and actinides but here we choose only to illustrate our interpretation of these bands on the examples of the nuclei: ^{232}Th, ^{238}U and ^{158}Yb. In Figures 1–3, respectively, we present the comparison between the experimental data and theoretical calculations within the $U(6)$ limit of the IVBM [2], reviewed in the previous sections, of the energies (2) (left side of the figures) and the signature splitting functions the lowest negative parity band (14) with the gsb and one of the excited 0^+ bands (right side of the figures). A rather good agreement between the theory and experiment is obtained in all the presented examples. The parameters of IVBM obtained in the overall fitting of the energies of the assigned levels to the experiment [18] are given in the figure captions.

Important characteristic introduced in this work is the dependance of the model parameter b in (2) on the number of bosons N_0 that build the band head of the excited bands. The interaction between the excited and the ground state band according (10) and (11) also depends on N_0. As in all the three considered nuclei these characteristics are the same the difference in the behavior of the ^{158}Yb follows from the differen values of the parameters a, b, α_3, β_3 and α_1 of the model Hamiltonian. They differ significantly from the ones for the actinides,

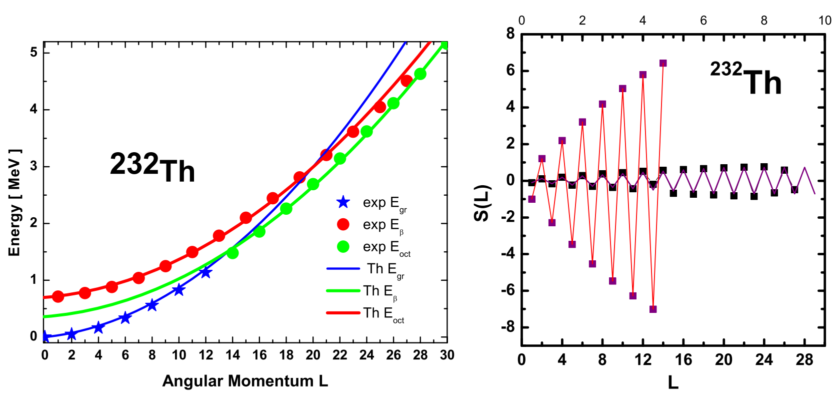

Figure 1. Left: Experimental (symbols) and theoretical (lines) excitation energies of the gsb Egr, first excited 0^+ band E_β and the octupole band $Eoct$ calculated with the parameters: $a = 0.0076507, b = 0.00084069, \alpha_3 = 0.0499193, \beta_3 = 0.00295902, \alpha_1 = 0.299516$;
Right: The experimental (black shapes) and theoretical (lines) values of the stagering functions $S(L)$ (14) between the ground and octupole bands (red) and the excited 0^+ with $N_0 = 12$ and the octupole bands $N_0 = 8$ (blue) in ^{232}Th.

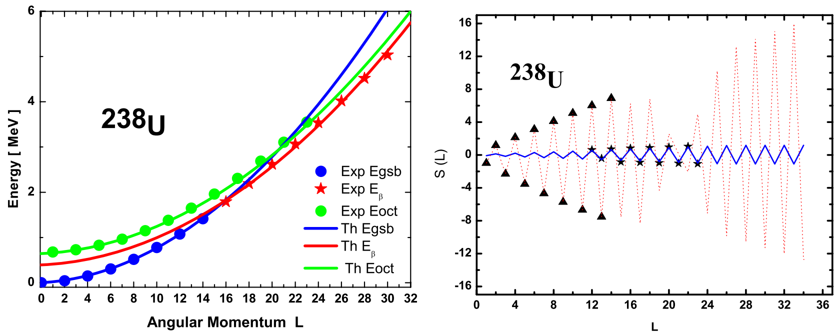

Figure 2. Left: Experimental (symbols) and theoretical (lines) excitation energies of the gsb $Egsb$, first excited 0^+ band E_β and the octupole band E_{oct} calculated with the parameters: $a = 0.00566353$, $b = 0.000834171$, $\alpha_3 = 0.0487449$, $\beta_3 = 0.00291152$, $\alpha_1 = 0.292469$;
Right: The experimental (black shapes) and theoretical (lines) values of the stagering functions $S(L)$ (14) between the ground and octupole bands (red) and the excited 0^+ with $N_0 = 12$ and the octupole bands (blue) $N_0 = 8$ in ^{238}U.

e.g. $\alpha_3 < 0$. For this nucleus the considered 0^- band starts from rather high spin at $L = 7$ and then runs very close to the high spin states of the yrast band. Hence it is not possible to investigate its behavior in respect to the gsb. The investigation of the negative parity bands in the rare earths needs to be more thorough and related to the microscopic structure [16]. A good characteristic of

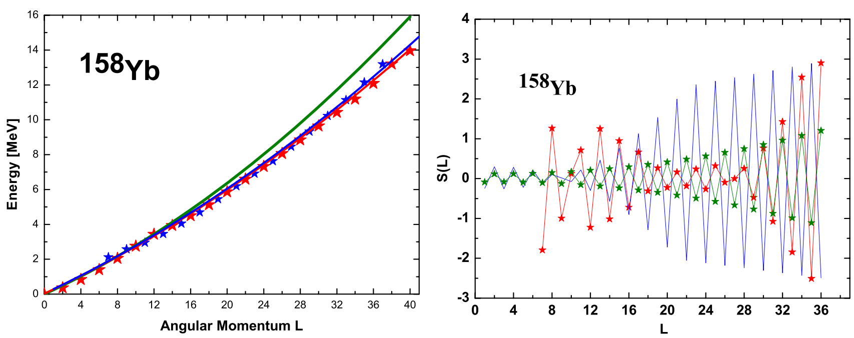

Figure 3. Left: Experimental (symbols) and theoretical (lines) excitation energies of the yrast band (red), first excited 0^+ band (green) and the octupole band (blue) calculated with the parameters: $a = 0.118092$, $b = 0.000479904$, $\alpha_3 = -0.252192$, $\beta_3 = 0.00206851$, $\alpha_1 = 0.6$;
Right: The experimental functions $S(L)$ (14) between the yrast and octupole bands (joined red shapes), the theoretical values of the $S(L)$ (14) between the ground and octupole bands (blue lines) and between excited 0^+ with $N_0 = 12$ and the octupole band $N_0 = 12$ (joined green shapes) in ^{158}Yb.

Table 1. Values of the parameters ω, β and C for each of the considered bands of the nuclei ^{232}Th, ^{238}U, and ^{158}Yb

Nucleus/band	ω	β	C
^{232}Th gsb	2.476726	0.006230	0
0^+ β-band	5.950906	0.004254	0.505817
0^- oct.band	5.335211	0.004507	0.686615
^{238}U gsb	2.252941	0.004523	0
0^+ β-band	5.763204	0.004259	0.4390493
0^- oct.band	5. 134381	0.004523	0.603257
^{158}Yb gsb	52.68245	0.00448	0
0^+ β-band	94.843072	0.002588	0.102592
0^- oct.band	86.2887	0.002831	0.041319

the behavior of the considered bands is the expression of their energies in terms of the generalized rotor $E(L) = \beta L(L + \omega) + C$. These parameters for each band are given in Table 1. The parameter ω reveals the collectivity of the bands and clearly shows the deviation from the quadrupole rotational character of the bands. In all the considered cases $\omega > 1$, but in the ^{158}Yb it reveals an almost vibrational character of all the bands, caused by the octupole deformation of the nuclear shapes.

For the excited 0^+ bands in the actinides ^{232}Th, ^{238}U and also ^{158}Yb in Figures 1–3 $N_0 = 8$, which is actually the first band of this kind that appears in the classification of the nuclear states. For these bands, we use as a band head configuration – the $SU(3)$-multiplet $(4, 4)$ with $N_0 = 12$. From the behavior of the energies and the staggering functions given in Figures 1–3, it could be seen that with this choice the octupole band runs in parallel with the considered excited 0^+ bands, but not with the ground state band. These two bands actually form the alternating parity band. The later band crosses at some point the ground state band and from there the respective β-band becomes the second part of the yrast band. At the crossing point a beat pattern in the staggering (13) is observed. The crossing point of the excited bands with the ground state bands appears at much higher spins – around $L = 12-16$ for the actinides and much lower for the rare earth nuclei. Its occurrence obviously depends on the boson structure of the band head's configuration, as the less collective states, build by smaller number of bosons shift the critical point, where the excited alternating parity bands are crossing the gsb to higher angular momenta. The behavior of the considered bands at hight spins is dominated by the behavior of the excited bands and their band head structure.

In conclusion we could summarize that in the framework of the IVBM the properties of the first important 0^+ and 0^- bands of the heavy even-even nuclei from the regions of the rare-earth and the actinides are very well reproduced and interpreted in a rather simple, but physically meaningful way. It is important to proceed systematically in this way, in order to relate the observed phenomena to the microscopic and geometrical structure of the nuclei. The employed group-theoretical approach is a rather convenient tool to achieve these goals.

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