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Elastic p^{15}N-Scattering and Interference Effects of Different Scattering Multiplicities

E. Ibraeva[1], **B. Prmantaeva**[3], **O. Imambekov**[2], **I.K.Tleulessova**[1], **A. Kozlovskiy**[3]

[1]Institute of Nuclear Physics NNC RK, Almaty, Kazakhstan

[2]Al-Farabi Kazakh National University, Almaty, Kazakhstan

[3]L.N. Gumilyov Eurasian National University, Astana, Kazakhstan

Abstract. The interest in the scattering of protons in nuclei with an excess number of neutrons (or protons) is due to the possibility of studying their exotic structures, such as halo or skin. In ^{15}N nucleus the number of neutrons is more than the number of protons just on one nucleon. With the low prevalence (only 0.36%) this nucleus is not sufficiently investigated experimentally. The purpose of this paper is to calculate the differential cross section (DCS) of the elastic scattering of protons on nucleons of the ^{15}N within the Glauber diffraction theory, taking into account single and double collisions and their interference. It is shown that the double-scattering approximation using shell wave function of ^{15}N, the amplitude of ^{15}N-process can be calculated analytically [1]. We have calculated the DCS at three incident proton energies of 0.2, 0.6 and 1.0 GeV in the $40^\circ > \theta > 0^\circ$ angular range, where the Glauber theory is applicable. Interference effects in the amplitudes of DCS are shown in the whole angular range. In the front scattering angles the single collisions are dominant, the accounting of double collisions reduces the total cross section. In the region where the amplitudes are comparable in magnitude, their interference leads to a minimum in the cross section, since the Glauber multiple scattering row is alternating, and the amplitude of single and double collisions have opposite signs. At large scattering angles, the DCS is completely determined by the amplitude of double collisions. Because of accounting of the nuclear structure the partial (single and double) DCS themselves have a complicated form, with alternating maxima and minima. The position of the maxima and minima of the DCS depends on the initial energy of incident particles: the higher it is, the more minima in the cross section.

1 Introduction

Interest to the scattering of protons on nuclei with excessive number of neutrons (or protons) in inverse kinematics is caused by possibility of studying of their exotic structure, such as halo or skin. Abnormal the big material radius, a long tail of density distributions, small binding energy and narrow longitudinal momentum distribution of components in their basic state – these are some characteristic features of these nuclei.

In the nucleus ^{15}N the number of neutrons exceeds number of protons for one. Having small prevalence (only 0.36%), this nucleus is still insufficiently studied experimentally.

The diffraction theory of Glauber allows to not only calculate the differential cross-section (DCS), but also to make the idle time coordinated, the scattering analysis free from parameters in which we consider correlations between nuclear structure and interaction model. The purpose of the present work is the calculation DS of elastic scattering of protons on the nucleons of ^{15}N in frameworks of Glauber diffraction theory with the account of one- and double impacts and their interferences. It is shown [1] that in approach of double scattering with use of shell wave function ^{15}N, the amplitude r^{15}N-process can be calculated analytically that raises accuracy of calculation. Impacts of the higher frequency rates were not considered, as a number of repeated scattering converges quickly and at small angles the contribution of the higher multiplicity on usages less than the unitary.

We have calculated DCS at three energies of flying protons 0.2, 0.6 and 1.0 GeV in a range of angles $40^\circ > \theta > 0^\circ$, there, where the Glauber theory is applicable. It is shown that effects of an interference of amplitudes in DCS take place at all energies and in all angular range.

2 Brief Formalism

The matrix element (amplitude) of scattering in Glauber diffraction theory [2] registers as follows:

$$M_{if}(\vec{q}_\perp) = \sum_{M_j M_j'} \frac{ik}{2\pi} \int d^2\vec{\rho} \prod_{\nu=1}^{A} d\vec{r}_\nu \exp(i\vec{q}_\perp \vec{\rho}) \left\langle \Psi_f^{JMj} \left|\Omega\right| \Psi_i^{JM_j'} \right\rangle, \quad (1)$$

where Ψ_i^{JMj} and $\Psi_f^{J'M_j'}$ WF of initial and final conditions, $\vec{\rho}$ – the aim parameter corresponding to the projection of radius-vector of dissipating $\vec{r}$ particles on the surface, perpendicular to the direction of their distribution; A – number of nucleons in the target, $\vec{q}_\perp$ – momentum transferred in reaction $\vec{\rho} = \vec{k} - \vec{k}'$, in case of elastic scattering and $\left|\vec{k}\right| = \left|\vec{k'}\right|$, $q_\perp = 2k\sin(\theta/2)$, θ – angle of scattering, $\hbar = c = 1$.

For the description of internal structure of an isotope ^{15}N we used multiparticle shell model in which the basic condition is level of negative parity $J^{pi} = 1/2^-$ with a configuration $(1s)4(1)11$.

We present the shell WF in the following form:

$$\Psi_{i,f}^{JM_j}(\vec{r}_i) = \Psi_{n_0 l_0 m_0}(\vec{r}_1 \ldots \vec{r}_4)\Psi_{n_1 l_1 m_1}(\vec{r}_5 \ldots \vec{r}_{15}), \quad (2)$$

where $n_i l_i m_i$ there are quantum numbers (main, orbital and magnetic) of corresponding shell: $n_0 = 0, l_0 = 0, m_0 = 0$; $n_1 = 1, l_1 = 1, m_1 = \pm 1$,

$\Psi_{nim}(\vec{r}_1, \vec{r}_2, \ldots) = \prod_\nu \Psi_{nim}(\vec{r_\nu})$ is a product of one-partial functions, $\vec{r}_\nu$ – one-partial co-ordinates of nucleons.

The wave function calculated in spherically symmetric potential, factorized on radial $R_{ni(r_\nu)}$ and angular $Y_{im}(r_\nu)$ ($r_\nu \equiv (\theta_\nu, \varphi_\nu)$) parts:

$$\Psi_{nim}(\vec{r_\nu}) = R_{ni}(r_\nu) Y_{im}(r_\nu). \tag{3}$$

Considering that radial WF in the potential of harmonious oscillator are expressed through the Gauss function $R_{00} = C_{00} \exp\left(-r^2/2r_0^2\right)$, $C_{00} = r^2/(\pi^{1/4} r_0^{3/2})$, $R_{11} = C_{11}(r/r_0) \exp\left(-r^2/2r_0^2\right)$, where $C_{11} = \sqrt{\frac{2}{3}} C_{00}$ (for nucleus of 1r-shell $r_0 = 1.62$ fm), integration of a matrix element (1) can be calculated analytically.

The operator Ω in Glauber theory is expressed in the form of multiple scattering row:

$$\begin{aligned} \Omega &= 1 - \prod_{\nu=1}^{A} (1 - \omega_\nu(\vec{\rho} - \vec{\rho}_\nu)) \\ &= \sum_{\nu=1}^{A} \omega_\nu - \sum_{\nu<\mu} \omega_\nu \omega_\mu + \sum_{\nu<\mu<\eta} \omega_\nu \omega_\mu \omega_\eta - \cdots (-1)^{A-1} \omega_1 \omega_2 \ldots \omega_A, \end{aligned} \tag{4}$$

where the first member is responsible for the single impacts, the second – for double impacts, etc., to the last member which is responsible for A-multiple impacts. We are limited to the two first members of the row as it is known [2] that the row converges quickly also each following member gives the contribution to section on some usages less than the previous. Profile functions are expressed through elementary rN – amplitudes $f_{pN}(q)$:

$$\omega_\nu(\vec{\rho} - \vec{\rho}_\nu) = \frac{1}{(2\pi i k)} \int d^2 \vec{q}_{\perp\nu} \exp(-i \vec{q}_{\perp\nu}(\vec{\rho} - \vec{\rho_\nu})) f_{pN}(q_\nu) \tag{5}$$

The elementary amplitude registers in the standard form:

$$f_{pN}(q_\nu) = \frac{k \sigma_{pN}}{4\pi} (i + \epsilon_{pN}) \exp\left(-\frac{\beta_{pN}^2 q_\nu^2}{2}\right) \tag{6}$$

Here σ_{pN}-full cross-section of scattering of a proton on a nucleon, ϵ_{pN} – the relation of the real part of amplitude to the imaginary part, β_{pN} – parameter of an inclination of a cone of amplitude. Parameters of amplitude for pN- scattering are presented in the table. Having substituted first two members of the formula (4) in a matrix element (1), we receive:

$$M_{if}(\vec{q}_\perp) = M_{if}^{(1)}(\vec{q}_\perp) - M_{if}^{(2)}(\vec{q}_\perp), \tag{7}$$

where the first member corresponds for single, the second – to double scattering:

$$M_{if}^{(1)}(\vec{q}_{\perp}) = \sum_{M_j M_j^{'}} \frac{ik}{2\pi} \int d^2\vec{\rho} \prod_{\nu=1}^{A} d\vec{r}_{\nu} \exp(i\vec{q}_{\perp}\vec{\rho}) \left\langle \Psi_f^{JM_j} \middle| \sum_{\nu=1}^{A} \omega_{\nu} \middle| \Psi_i^{JM_j^{'}} \right\rangle, \quad (8)$$

$$M_{if}^{(2)}(\vec{q}_{\perp}) = \sum_{M_j M_j^{'}} \frac{ik}{2\pi} \int d^2\vec{\rho} \prod_{\nu=1}^{A} d\vec{r}_{\nu} \exp(i\vec{q}_{\perp}\vec{\rho}) \left\langle \Psi_f^{JM_j} \middle| \sum_{\nu<\tau} \omega_{\nu}\omega_{\tau} \middle| \Psi_i^{JM_j^{'}} \right\rangle. \quad (9)$$

The calculation of amplitudes (8) and (9) with WF (2) is presented in work [1], we will write down their definitive expressions:

$$M_{if}^{(1)}(\vec{q}) = 4\sqrt{\frac{\pi}{2q}} \int_0^{\infty} |R_{00}(r)|^2 J_{1/2}(qr) r^{3/2} dr + 11\pi \sqrt{\frac{2}{q}} \sqrt{\frac{6}{5}} B_{112}(q)(Y_{22}(q) + Y_{2-2}(q)) + \frac{1}{\sqrt{4\pi}} B_{110}(q), \quad (10)$$

where

$$B_{11\lambda}(q) = \int_0^{\infty} |R_{11}(r)|^2 J_{\lambda+1/2}(qr) r^{5/2} dr. \quad (11)$$

$$M_{if}^{(2)}(\vec{q}) 6C_1 F_{\lambda=0}^{s}(q) + 44C_2 \sum_{\lambda\mu} (i)^{\lambda} Y_{\lambda\mu}(q) \Omega_{\lambda\mu}^{sp} F_{\lambda\mu}^{sp}(q) + 55C_2 \sum_{\lambda\mu} (i)^{\lambda} Y_{\lambda\mu}(q) \Omega_{\lambda\mu}^{p} F_{\lambda\mu}^{p}(q), \quad (12)$$

where

$$C_1 = \frac{1}{4\sqrt{2\pi}}, \;\; F_{\lambda=0}^{s}(q) = \frac{1}{\sqrt{q}} \int_0^{\infty} |R_{00}|^4 J_{1/2}(qr) r_{3/2} dr, \quad (13)$$

$$C_2 = (2\pi)^{3/2}, \;\; F_{\lambda\mu}^{sp}(q) = \frac{1}{\sqrt{q}} \int_0^{\infty} |R_{00}|^2 |R_{11}|^2 J_{\lambda+1/2}(qr) r_{3/2} dr, \quad (14)$$

$$\Omega_{\lambda\mu}^{sp} = \frac{1}{4\pi} \sum_{mm'} \int Y_{1m}^{*}(r) Y_{\lambda\mu}(r) Y_{1m}(r) d\Omega_r, \quad (15)$$

$$F_{\lambda\mu}^{p}(q) = \frac{1}{\sqrt{q}} \int_0^{\infty} |R_{11}|^4 J_{\lambda+1/2}(qr) r^{3/2} dr, \quad (16)$$

$$\Omega_{\lambda\mu}^{p} = \frac{1}{4\pi} \sum_{mm'} \int |Y_{1m}(r) Y_{\lambda\mu}(r)|^2 |Y_{1m'}(r)|^2 Y_{\lambda\mu} d\Omega_r. \quad (17)$$

The differential cross section of scattering is defined as a square of amplitude (1):

$$\frac{d\sigma}{d\Omega} = \frac{1}{2J+1} \left| M_{if}(\vec{q}) \right|^2 . \tag{18}$$

Having substituted in (18) formula (7), we will receive:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2J+1} \left| M_{if}^{(1)}(\vec{q}) - M_{if}^{(2)}(\vec{q}) \right|^2 . \tag{19}$$

Here the first member defines the single scattering, the second - double scattering, $M_{if}^{(1)}(\vec{q})$ is calculated using formulas (10), (11), $M_{if}^{(2)}(\vec{q})$ – using formulas (12)-(17). The further calculations were made on the computer by using the program MAPLE.

3 Analysis of the Results

Using the formulas from the previous part, we have calculated the DCS depending on energies of protons and have considered the contribution to cross section from different frequency rates of impacts in Ω operator. Results of calculations are shown in Figures 1–4.

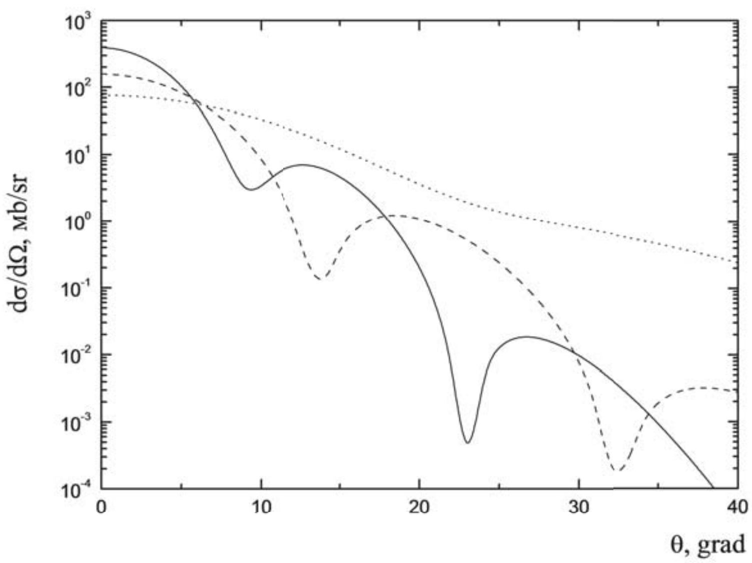

Figure 1. Differential sections at different energies of flying protons. Dotted curve $E = 0.2$ GeV, dashed – $E = 0.6$ GeV, solid – $E = 1.0$ GeV, with sets of parameters of elementary amplitudes from the table: for $E = 0.2$ and 0.6 GeV set 1, for $E = 1$ GeV set 2.

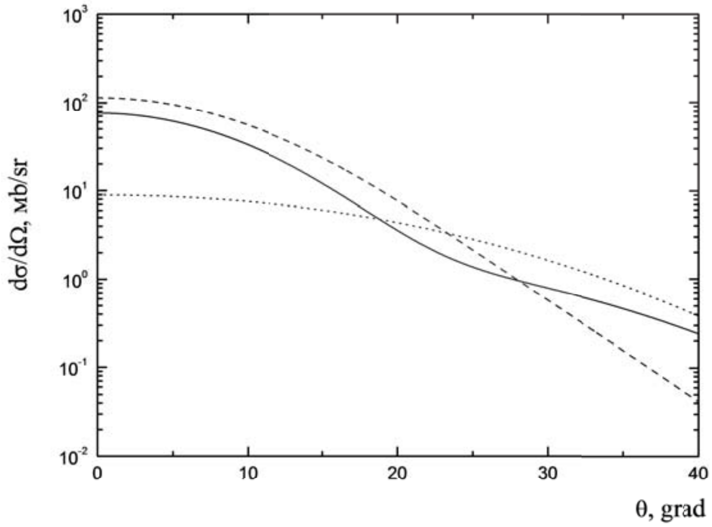

Figure 2. Contribution to differential section of different multiplicy of scattering in the operator Ω at $E = 0.2$ GeV. Dotted curve - cross section of single scattering, dashed curve – cross section of double scattering, solid curve – total single and double cross sections, taking into account the interference.

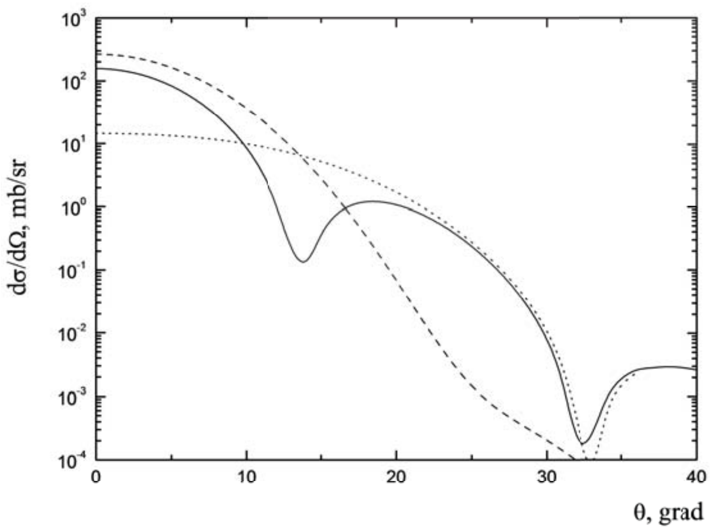

Figure 3. Contribution to differential section of different multiplicy of scattering in the operator Ω at $E = 0.6$ GeV. Dotted curve – cross section of single scattering, dashed curve - cross section of double scattering, solid curve – total single and double cross sections, taking into account the interference.

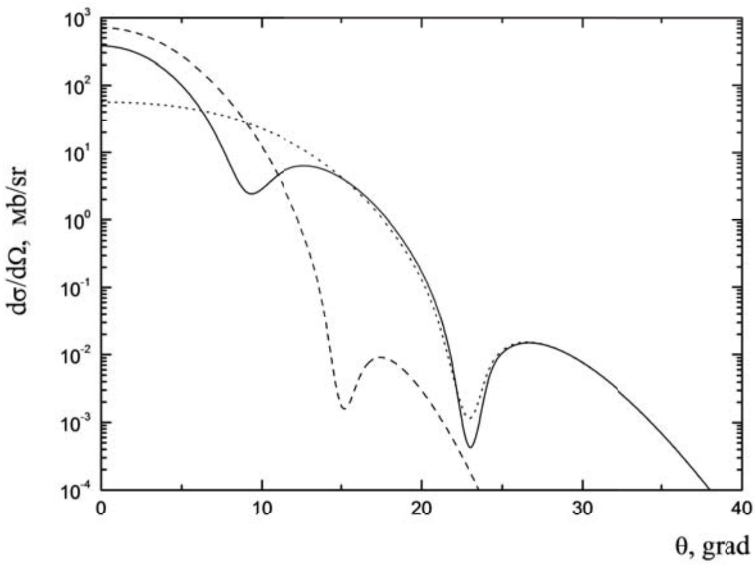

Figure 4. Contribution to differential section of different multiplicy of scattering in the operator Ω at $E = 1$ GeV. Dotted curve – cross section of single scattering, dashed curve – cross section of double scattering, solid curve – total single and double cross sections, taking into account the interference.

Lets consider the dependence of DCS from energies of flying protons. In Figure 1, dotted, dashed and solid curves correspond to energy 0.2, 0.6 and 1.0 GeV, with following sets of parameters of elementary amplitudes from the table: for $E = 0.2$ and 0.6 GeV set 1, for $E = 1$ GeV set 2.

From Figure 1 it is visible that with increase of energy of flying protons diffraction picture in cross section (alternation of well expressed minima and maxima) becomes more distinct. So, at $E = 0.2$ GeV the cross section is

Table 1. Parametres of pN-amplitudes

E_p, GeV	σ_{pN}, fm^2	ε_{pN}	β_{pN}, fm^2	Ref.	Set number
0.2	3.28	0.93	0.67	[3]	1
0.2	4.2	0.71	0.68	[3]	2
0.2	3.45	0.953	0.131	[4]	3
0.6	3.7	-0.1	0.12	[5]	1
0.55	3.62	0.04	0.125	[4]	2
0.65	4.0	-0.095	0.16	[4]	3
1.0	4.75	-0.5	0.218	[4]	1
1.0	4.356	-0.3	0.187	[6]	2
1.0	4.32	-0.275	0.21	[4]	3

smoothly falling down curve with a small excess at $\theta \sim 24°$. At $E = 0.6$ GeV DCS represent distinct diffraction picture with two minima at both $\theta \sim 14°$ and $32°$ and two maxima at $\theta \sim 0°$ and $20°$. At $E = 1.0$ GeV the diffraction picture of cross- section amplifies, minima at $\theta \sim 9°$ both $20°$ and maxima at $\theta \sim 0°$, $14°$ and $20°$ there is a third maximum. The absolute size of cross sections at zero angle increases with energy.

Lets consider the behaviour of differential section depending on the contribution one- and double impacts in the operator. Calculation by the formula (19) is presented in Figures 2-4 at energies $E = 0.2$ GeV (Figure 2), $E = 0.6$ GeV (Figure 3), $E = 1.0$ GeV (Figure 4). The dashed curve is considered only – single scattering (the first member of the formula (19)), dotted curve – double dispersion (the second member of the formula (19)), solid curve – their sum is considered only, taking into account an interference (all members of the formula (19)). At all energies the single scattering dominates at small angles. Double scattering, which 10 times less than single at zero angle, decreases not so quickly, as single one, (as with increase of scattering angle the flying particle gets into internal area of a nuclei where density of nucleons above and the probability of repeated impacts raises more deeply) and the defining contribution at the big angles ($\theta > 25°$ at $E = 0.2$ GeV, $\theta > 15°$ at $E = 0.6$ GeV, $\theta > 10°$ at $E = 1.0$ GeV). At some angle of cross section one- and double impacts are compared ($\theta = 24°$ for $E = 0.2$ GeV, $\theta = 14°$ for $E = 0.6$ GeV and $\theta = 9°$ for $E = 1.0$ GeV) and in this point in total section there are minima because of interferential member in the formula (19), because the multiple scattering row is sign-variable. Minima which are observed in single scattering at $\theta \sim 15°$ ($E = 1.0$ GeV) and in double scattering at $\theta \sim 23°$ ($E = 1.0$ GeV) and $\theta \sim 32°$ ($E = 0.6$ GeV), arise because of polynoms which appear in matrix elements after integration of formulas (13), (14), (16).

4 Conclusions

The calculation carried out by us has shown that application to Glauber diffraction theory to a nucleus 15N, WF of which is presented in multiparticle shell model, and in the operator members of single and double impacts are considered, allows to calculate amplitude analytically that raises the accuracy of calculation.

Effects of the interference of amplitudes in DCS are shown in all angular range. In the field of forward angles of scattering single impacts dominate, the account of double impacts reduces total section a little. In area where amplitudes are compared on size, their interference leads to a minimum in cross section as the Glauber row of multiple scattering is sign-variable, both amplitudes of single and double impacts have different signs. At the big corners of scattering DCS it is completely defined by amplitude of double impacts. Because of the account of the structure of a nuclei the partial (single and double) DCS have difficult appearance, with alternating maxima and minima. Position of maxima and minima of DCS depends on initial energy of flying particles: the more the energy, the more number of minima in cross section.

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