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Nuclear Forces from Chiral Effective Field Theory: Achievements and Challenges

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Abstract. In the past decade, there has been substantial progress in the derivation of nuclear forces from chiral effective field theory. Accurate two-nucleon forces have been constructed at next-to-next-to-next-to-leading order (N^3LO) and applied (together with three-nucleon forces at NNLO) to nuclear few- and many-body systems—with a good deal of success. This may suggest that the 80-year old nuclear force problem has finally been cracked. Not so! Some pretty basic issues are still unresolved. In this talk, I focus on the two most pressing ones, namely, the proper renormalization of the two-nucleon potential and subleading many-body forces.

1 Introduction

The problem of a proper derivation of nuclear forces is as old as nuclear physics itself, namely, almost 80 years [1]. The modern view is that, since the nuclear force is a manifestation of strong interactions, any serious derivation has to start from quantum chromodynamics (QCD). However, the well-known problem with QCD is that it is non-perturbative in the low-energy regime characteristic for nuclear physics. For many years this fact was perceived as the great obstacle for a derivation of nuclear forces from QCD—impossible to overcome except by lattice QCD.

The effective field theory (EFT) concept has shown the way out of this dilemma. For the development of an EFT, it is crucial to identify a separation of scales. In the hadron spectrum, a large gap between the masses of the pions and the masses of the vector mesons, like $\rho(770)$ and $\omega(782)$, can clearly be identified. Thus, it is natural to assume that the pion mass sets the soft scale, $Q \sim m_\pi$, and the rho mass the hard scale, $\Lambda_\chi \sim m_\rho \sim 1$ GeV, also known as the chiral-symmetry breaking scale. This is suggestive of considering a low-energy expansion arranged in terms of the soft scale over the hard scale, $(Q/\Lambda_\chi)^\nu$, where Q is generic for an external momentum (nucleon three-momentum or pion four-momentum) or a pion mass. The appropriate degrees of freedom are, obviously, pions and nucleons, and not quarks and gluons. To make sure that this EFT is not just another phenomenology, it must have a firm link with QCD. The link is established by having the EFT observe all relevant symmetries of the underlying theory, in particular, the broken chiral symmetry of low-energy QCD [2].

The early applications of chiral perturbation theory (ChPT) focused on systems like $\pi\pi$ [3] and πN [4], where the Goldstone-boson character of the pion guarantees that the expansion converges. But the past 15 years have also seen great progress in applying ChPT to nuclear forces [5–23]. As a result, nucleon-nucleon (NN) potentials of high precision have been constructed, which are based upon ChPT carried to next-to-next-to-next-to-leading order (N^3LO) [19, 21, 23], and applied in nuclear structure calculations with great success.

However, in spite of this progress, we are not done. Due to the complexity of the nuclear force issue, there are still many subtle and not so subtle open problems. We will not list and discuss all of them, but instead just focus on the two open issues, which we perceive as the most important ones:

- The proper renormalization of chiral nuclear potentials and
- Subleading chiral few-nucleon forces.

2 Renormalization of Chiral Nuclear Forces

2.1 The chiral NN potential

In terms of naive dimensional analysis or "Weinberg counting", the various orders of the irreducible graphs which define the chiral NN potential are given by:

$$V_{\rm LO} = V_{\rm ct}^{(0)} + V_{1\pi}^{(0)} \tag{1}$$

$$V_{\rm NLO} = V_{\rm LO} + V_{\rm ct}^{(2)} + V_{1\pi}^{(2)} + V_{2\pi}^{(2)} \tag{2}$$

$$V_{\rm NNLO} = V_{\rm NLO} + V_{1\pi}^{(3)} + V_{2\pi}^{(3)} \tag{3}$$

$$V_{\rm N^3LO} = V_{\rm NNLO} + V_{\rm ct}^{(4)} + V_{1\pi}^{(4)} + V_{2\pi}^{(4)} + V_{3\pi}^{(4)} \tag{4}$$

where the superscript denotes the order ν of the low-momentum expansion. LO stands for leading order, NLO for next-to-leading order, *etc*. Contact potentials carry the subscript "ct" and pion-exchange potentials can be identified by an obvious subscript. For more details concerning the above potentials, see ref. [23].

2.2 Nonperturbative renormalization of the NN potential

The two-nucleon system is characterized by large scattering lengths and shallow (quasi) bound states which require a nonperturbative treatment. Following Weinberg's prescription [5], this is accomplished by inserting the potential V into the Lippmann-Schwinger (LS) equation:

$$T(\vec{p}\,',\vec{p}) = V(\vec{p}\,',\vec{p}) + \int d^3p''\, V(\vec{p}\,',\vec{p}\,'')\, \frac{M_N}{p^2 - p''^2 + i\epsilon}\, T(\vec{p}\,'',\vec{p})\,, \tag{5}$$

where M_N denotes the nucleon mass.

In general, the integral in the LS equation is divergent and needs to be regularized. One way to achieve this is by multiplying V with a regulator function

$$V(\vec{p}\,',\vec{p}) \longmapsto V(\vec{p}\,',\vec{p})\, e^{-(p'/\Lambda)^{2n}}\; e^{-(p/\Lambda)^{2n}}\,. \tag{6}$$

Typical choices for the cutoff parameter Λ that appears in the regulator are $\Lambda \approx 0.5$ GeV $< \Lambda_\chi \approx 1$ GeV.

In field theories, divergent integrals are not uncommon and methods have been designed to deal with them. One regulates the integrals and then removes the dependence on the regularization parameters (scales, cutoffs) by "renormalization". In the end, the theory and its predictions do not depend on cutoffs or renormalization scales. So-called renormalizable quantum field theories, like QED, have essentially one set of prescriptions that takes care of renormalization through all orders. In contrast, EFTs are renormalized by "counter terms" (contact terms) that are introduced order by order in increasing numbers.

Naively, the most perfect renormalization procedure is the one where the cutoff parameter Λ is taken to infinity while stable and quantitative results are maintained through the adjustment of counter terms. This was accomplished at LO in the work by Nogga *et al.* [24]. At NNLO, the infinite-cutoff renormalization procedure has been investigated in [25] for partial waves with total angular momentum $J \leq 1$ and in [26] for all partial waves with $J \leq 5$. However, for a quantitative chiral NN potential one needs to advance all the way to N^{3}LO. At N^{3}LO, the $^1S_0$ state was considered in Ref. [27], and all states up to $J = 6$ were investigated in Ref. [28]. From all of these works, it is evident that no counter term is effective in partial-waves with short-range repulsion and only a single counter term can constructively be used in partial-waves with short-range attraction. Thus, for the $\Lambda \to \infty$ renormalization prescription, even at N^{3}LO, there exists either one or no counter term per partial-wave state. This is inconsistent with any reasonable power-counting scheme and prevents an order-by-order improvement of the predictions.

To summarize: In the infinite-cutoff renormalization scheme, the potential is admitted up to unlimited momenta. However, the EFT this potential is derived from has validity only for momenta smaller than the chiral symmetry breaking scale $\Lambda_\chi \approx$ GeV. The lack of order-by-order convergence and discrepancies in lower partial-waves demonstrate that the potential should not be used beyond the limits of the effective theory [28] (see Ref. [29] for a related discussion). The conclusion then is that cutoffs should be limited to $\Lambda \lesssim \Lambda_\chi$ (but see also Ref. [30]).

Crucial for an EFT are regulator independence (within the range of validity of the EFT) and a power counting scheme that allows for order-by-order improvement with decreasing truncation error. The purpose of renormalization is to achieve this regulator independence while maintaining a functional power counting scheme.

Thus, in the spirit of Lepage [31], the cutoff independence should be examined for cutoffs below the hard scale and not beyond. Ranges of cutoff indepen-

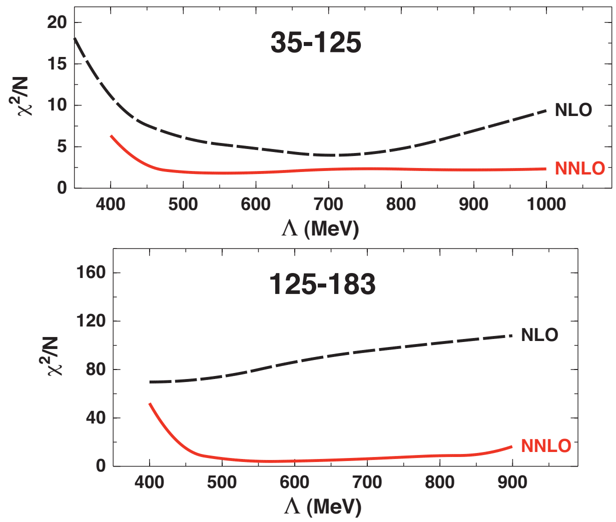

Figure 1. χ^2/datum for the reproduction of the np data in the energy range 35-125 MeV (upper frame) and 125-183 MeV (lower frame) as a function of the cutoff parameter Λ of the regulator function Eq. (6). The (black) dashed curves show the χ^2/datum achieved with np potentials constructed at order NLO and the (red) solid curves are for NNLO.

dence within the theoretical error are to be identified using 'Lepage plots' [31]. Recently, we have started a systematic investigation of this kind. In our work, we quantify the error of the predictions by calculating the χ^2/datum for the reproduction of the neutron-proton (np) elastic scattering data as a function of the cutoff parameter Λ of the regulator function Eq. (6). We have investigated the predictions by chiral np potentials at order NLO and NNLO applying Weinberg counting for the counter terms (NN contact terms). We show our results for the energy range 35-125 MeV in the upper frame of Figure 1 and for 125-183 MeV in the lower frame. It is seen that the reproduction of the np data at these energies is generally poor at NLO, while at NNLO the χ^2/datum assumes acceptable values (a clear demonstration of order-by-order improvement). Moreover, at NNLO one observes "plateaus" of constant low χ^2 for cutoff parameters ranging from 450 to 850 MeV. This may be perceived as cutoff independence (and, thus, successful renormalization) in the relevant range of cutoff parameters.

3 Few-Nucleon Forces and What is Missing

We will now discuss the other issue we perceive as unfinished and important, namely, subleading chiral few-nucleon forces.

Nuclear three-body forces in ChPT were initially discussed by Weinberg [7]. The three-nucleon force (3NF) at NNLO, was derived by van Kolck [10] and applied, for the first time, in nucleon-deuteron scattering by Epelbaum *et al.* [32].

The leading 4NF (at N^3LO) was constructed by Epelbaum [33] and found to contribute in the order of 0.1 MeV to the ^{4}He binding energy (total ^{4}He binding energy: 28.3 MeV) in a preliminary calculation [34], confirming the traditional assumption that 4NF are essentially negligible. **Therefore, the focus is on 3NF.**

For the order of a 3NF, we have

$$\nu = 2 + 2L + \sum_i \Delta_i \,, \tag{7}$$

where L denotes the number of loops and Δ_i is the vertex index. We will use this equation to analyze 3NF contributions order by order. The first non-vanishing 3NF occurs at $\nu = 3$ (NNLO), which is obtained when there are no loops ($L = 0$) and $\sum_i \Delta_i = 1$, i.e., $\Delta_i = 1$ for one vertex while $\Delta_i = 0$ for all other vertices. There are three topologies which fulfill this condition, known as the two-pion exchange (2PE), one-pion exchange (1PE), and contact graphs.

The 3NF at NNLO has been applied in calculations of few-nucleon reactions [35], structure of light- and medium-mass nuclei [36–41], and nuclear and neutron matter [42–45] with a great deal of success. However, the famous 'A_y puzzle' of nucleon-deuteron scattering [32] and the analogous problem with the analyzing power in p-^{3}He scattering [46] is not resolved. Furthermore, the spectra of light nuclei leave room for improvement [36]. Since we are dealing with a perturbation theory, it is natural to turn to the next order when looking for improvements.

The next order is N^3LO, where we have loop and tree diagrams. For the loops, we have $L = 1$ and, therefore, all Δ_i have to be zero to ensure $\nu = 4$. Thus, these one-loop 3NF diagrams can include only leading order vertices, the parameters of which are fixed from πN and NN analysis. One sub-group of these diagrams (the 2PE graphs) has been calculated by Ishikawa and Robilotta [47], and the other topologies have been evaluated by the Bochum-Bonn group [48, 49]. The N^3LO 2PE 3NF has been applied in the calculation of nucleon-deuteron observables in Ref. [47] causing little impact. Very recently, the long-range part of the chiral N^3LO 3NF has been tested in the triton [50] and in three-nucleon scattering [51] yielding only moderate effects. The long- and short-range parts of this force have been used in neutron matter calculations (together with the N^3LO 4NF) producing relatively large contributions from the 3NF [52]. Thus, the ultimate assessment of the N^3LO 3NF is still outstanding and will require more few- and many-body applications.

In the meantime, it is of interest to take already a look at the next order of 3NFs, which is N^4LO or $\nu = 5$ (of the Δ-less theory to which the present discussion is restricted because of lack of space). The loop contributions that occur at this order are obtained by replacing in the N^3LO loops one vertex by a $\Delta_i = 1$ vertex (with LEC c_i), which is why these loops may be more sizable than the N^3LO loops. The 2PE topology has already been evaluated [53] and turns out to be of modest size; moreover, it can be handled in a practical way by summing it up together with the 2PE topologies at NNLO and N^3LO [53]. However, there are four more loop topologies, which are very involved and that have not been worked out yet. Finally, a tree topology at N^4LO provides a new set of 3N contact interactions, which have recently been derived by the Pisa group [54]. Contact terms are typically simple (as compared to loop diagrams) and their coefficients are unconstrained (except for naturalness). *Therefore, it would be an attractive project to test some terms (in particular, the spin-orbit terms) of the N^4LO contact 3NF [54] in calculations of few-body reactions (specifically, the p-d and p-^{3}He A_y) and spectra of light nuclei.*

4 Conclusions and Outlook

The past 15 years have seen great progress in our understanding of nuclear forces in terms of low-energy QCD. Key to this development was the realization that low-energy QCD is equivalent to an effective field theory (EFT) which allows for a perturbative expansion that has become known as chiral perturbation theory (ChPT). In this framework, two- and many-body forces emerge on an equal footing and the empirical fact that nuclear many-body forces are substantially weaker then the two-nucleon force is explained automatically.

In spite of the great progress and success of the past 15 years, there are still some unresolved issues. One problem is the proper renormalization of the chiral two- and many-nucleon potentials, where systematic investigations are already under way (cf. Sec. 2).

The other unfinished business are the few-nucleon forces beyond NNLO ("sub-leading few-nucleon forces") which are needed to hopefully resolve some important outstanding nuclear structure problems. At orders N^3LO and N^4LO very many new 3NF structures appear, some of which have already been tested. However, in view of the multitude of 3NF topologies it will take a while until we will have a proper overview of impact and convergence of these contributions.

If the open issues discussed in this paper will be resolved within the next few years, then, after 80 years of desperate struggle, we may finally claim that the nuclear force problem is essentially under control. The greatest beneficiaries of such progress will be the fields of exact few-nucleon calculations and *ab initio* nuclear structure physics.

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