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Improving Our Understanding of the Symmetry Energy

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Abstract. Studies if isospin-asymmetric nuclear matter (IANM) are especially important and timely as they support rich on-going and future experimental effort. In this talk, I will review the theoretical framework we adopt to calculate the properties of IANM. I will then demonstrate the crucial role isovector mesons play for the symmetry energy. The importance of a microscopic (rather than phenomenological) approach is highlighted.

1 Introduction

Nuclear matter is a convenient theoretical laboratory for many-body theories. By "nuclear matter" we mean an infinite system of nucleons acted on by their mutual strong forces and no electromagnetic interactions. Nuclear matter is characterized by its energy/particle as a function of density and other thermodynamic quantities, as appropriate (e.g. temperature). Such relation is known as the nuclear matter equation of state (EoS). The translational invariance of the system facilitates theoretical calculations. At the same time, adopting what is known as "local density approximation", one can use the EoS to obtain information on finite systems. This procedure is applied, for instance, in Thomas-Fermi calculations within the liquid drop model, where an appropriate energy functional is written in terms of the EoS [1–3].

Isospin-asymmetric nuclear matter (IANM) simulates the interior of a nucleus with unequal densities of protons and neutrons. The equation of state of (cold) IANM is then a function of density as well as the relative concentrations of protons and neutrons.

The recent and fast-growing interest in IANM stems from its close connection to the physics of neutron-rich nuclei, or, more generally, isospin-asymmetric nuclei, including the very "exotic" ones known as "halo" nuclei. At this time, the boundaries of the nuclear chart are uncertain, with a few hundreds stable nuclides known to exist and perhaps a few thousands believed to exist. The Facility for Rare Isotope Beams (FRIB) has recently been approved for design and construction at Michigan State University. The facility will deliver intense beams of rare isotopes, the study of which can provide crucial information on short-lived elements normally not found on earth. Thus, this new experimental program will have widespread impact, ranging from the origin of elements to the evolution of the cosmos.

It is estimated that the design and construction of FRIB will take ten years. In the meantime, systematic investigations to determine the properties of asymmetric nuclear matter are proliferating at existing facilities. The equation of state of IANM is also the crucial input for the structure equations of compact stars, and thus establishes the connection between nuclear physics and compact astrophysical systems.

In this paper we will present and discuss our approach to the development of the EoS of nuclear and neutron-rich matter. After a brief review of facts and phenomenology about IANM, we will summarize our microscopic approach to calculate the energy/particle in IANM.

Because of the fundamental importance of the symmetry energy in many systems/phenomena, it is of interest to identify the main contributions to its density dependence. We will discuss the contribution of the isovector mesons (π, ρ, and δ) to the symmetry energy and demonstrate the chief role of the pion. Note that the isovector mesons carry the isospin dependence by contributing differently in different partial waves, and that isospin dependence is the crucial mechanism in the physics of IANM. Hence, the relevance of a microscopic model that contains all important couplings of mesons with nucleons.

2 Facts about Isospin-Asymmetric Nuclear Matter

Asymmetric nuclear matter can be characterized by the neutron density, ρ_n, and the proton density, ρ_p, defined as the number of neutrons or protons per unit of volume. In infinite matter, they are obtained by summing the neutron or proton states per volume (up to their respective Fermi momenta, k_F^n or k_F^p) and applying the appropriate degeneracy factor. The result is

$$\rho_i = \frac{(k_F^i)^3}{3\pi^2}, \tag{1}$$

with $i = n$ or p.

It is more convenient to refer to the total density $\rho = \rho_n + \rho_p$ and the asymmetry (or neutron excess) parameter $\alpha = \frac{\rho_n - \rho_p}{\rho}$. Clearly, α=0 corresponds to symmetric matter and α=1 to neutron matter. In terms of α and the average Fermi momentum, k_F, related to the total density in the usual way,

$$\rho = \frac{2k_F^3}{3\pi^2}, \tag{2}$$

the neutron and proton Fermi momenta can be expressed as

$$k_F^n = k_F(1+\alpha)^{1/3} \tag{3}$$

and

$$k_F^p = k_F(1-\alpha)^{1/3}, \tag{4}$$

respectively.

Expanding the energy/particle in IANM with respect to the asymmetry parameter yields

$$e(\rho,\alpha) = e_0(\rho) + \frac{1}{2}\Big(\frac{\partial^2 e(\rho,\alpha)}{\partial\alpha^2}\Big)_{\alpha=0}\alpha^2 + \mathcal{O}(\alpha^4)\;, \tag{5}$$

where the first term is the energy/particle in symmetric matter and the coefficient of the quadratic term is identified with the symmetry energy, e_{sym}. In the Bethe-Weizsäcker formula for the nuclear binding energy, it represents the amount of binding a nucleus has to lose when the numbers of protons and neutrons are unequal. The symmetry energy is also closely related to the neutron β-decay in dense matter, whose threshold depends on the proton fraction. A typical value for e_{sym} at nuclear matter density (ρ_0) is 30 MeV, with theoretical predictions spreading approximately between 26 and 35 MeV.

To a very good degree of approximation, the energy/particle in IANM can be written as

$$e(\rho,\alpha) \approx e_0(\rho) + e_{sym}(\rho)\alpha^2. \tag{6}$$

The effect of a term of fourth order in the asymmetry parameter ($\mathcal{O}(\alpha^4)$) on the bulk properties of neutron stars is very small, although it may impact the proton fraction at high density. More generally, non-quadratic terms are usually associated with isovector pairing, which is a surface effect and thus vanishes in infinite matter [4].

Equation (6) displays a convenient separation between the symmetric and aymmetric parts of the EoS, which facilitates the identification of observables that may be sensitive, for instance, mainly to the symmetry energy. For a recent review and analysis of available constraints, see Ref. [5]. Typically, constraints are extracted from heavy-ion collision simulations based on transport models. Isospin diffusion and the ratio of neutron and proton spectra are among the observables used in these analyses.

These investigations appear to agree reasonably well on the following parametrization of the symmetry energy:

$$e_{sym}(\rho) = 12.5\,MeV\Big(\frac{\rho}{\rho_0}\Big)^{2/3} + 17.5\,MeV\Big(\frac{\rho}{\rho_0}\Big)^{\gamma_i}, \tag{7}$$

where ρ_0 is the saturation density. The first term is the kinetic contribution and γ_i (the exponent appearing in the potential energy part) is found to be between 0.4 and 1.0. Recent measurements of elliptic flows in ^{197}Au + ^{197}Au reactions at GSI at 400-800 MeV/nucleon favor a potential energy term with γ_i equal to 0.9 $\pm$ 0.4.

Isospin-sensitive observables can also be identified among the properties of normal nuclei. The neutron skin of neutron-rich nuclei is a powerful isovector observable, being sensitive to the slope of the symmetry energy, which determines to which extent neutrons will tend to spread outwards to form the skin.

Parity-violating electron scattering experiments are now a realistic option to determine neutron distributions with unprecedented accuracy. The neutron radius of ^{208}Pb is expected to be measured with a precision of 3% thanks to the electroweak program at the Jefferson Laboratory, the PREX experiment in particular, just recently completed at Jefferson Lab. This level of accuracy could not be achieved with hadronic scattering. Parity-violating electron scattering at low momentum transfer is especially suitable to probe neutron densities, as the Z^0 boson couples primarily to neutrons. With the success of this program, reliable empirical information on neutron skins will be able to provide, in turn, much needed *independent* constraint on the density dependence of the symmetry energy.

A measure of the density dependence of the symmetry energy is the symmetry pressure, defined as

$$L = 3\rho_0 \Big(\frac{\partial e_{sym}(\rho)}{\partial \rho}\Big)_{\rho_0} \approx 3\rho_0 \Big(\frac{\partial e_{n.m.}(\rho)}{\partial \rho}\Big)_{\rho_0} , \tag{8}$$

where we have used Eq. (6) with α=1. Thus, L is sensitive to the gradient of the energy per particle in neutron matter ($e_{n.m.}$). As to be expected on physical grounds, the neutron skin, given by

$$S = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle} , \tag{9}$$

is highly sensitive to the same pressure gradient.

Values of L are reported to range from 50 to 100 MeV as seen, for instance, through the numerous parametrizations of Skyrme interactions (see Ref. [6] and references therein), all chosen to fit the binding energies and the charge radii of a large number of nuclei. Heavy-ion data impose boundaries for L at 85 ± 25 MeV, with more stringent constraints being presently extracted. At this time constraints appear to favor lower values of the symmetry pressure. In fact, a range of L values given by 52.7 ± 22.5 MeV has emerged from recent analyses of global optical potentials [7].

Another important quantity which emerges from studies of IANM is the symmetry potential. Its definition stems from the observation that the single-particle potentials experienced by the proton and the neutron in IANM, $U_{n/p}$, are different from each other and satisfy the approximate relation

$$U_{n/p}(k, \rho, \alpha) \approx U_{n/p}(k, \rho, \alpha = 0) \pm U_{sym}(k, \rho)\ \alpha , \tag{10}$$

where the +(-) sign refers to neutrons (protons), and

$$U_{sym} = \frac{U_n - U_p}{2\alpha} . \tag{11}$$

Thus, one can expect isospin splitting of the single-particle potentials to be effective in separating the collision dynamics of neutrons and protons. Furthermore, U_{sym}, being proportional to the gradient between the single-neutron and the single-proton potentials, should be comparable with the Lane potential [8], namely the isovector part of the nuclear optical potential. Optical potential analyses can then help constrain this quantity and, in turn, the symmetry energy.

3 Brief Review of the Theoretical Approach

As stated above, the starting point of our many-body calculation is a realistic nucleon-nucleon (NN) interaction which is then applied in the nuclear medium without any additional free parameters. Thus the first question to be confronted concerns the choice of the "best" NN interaction. After the development of QCD and the understanding of its symmetries, chiral effective theories [9,10] were developed as a way to respect the symmetries of QCD while keeping the degrees of freedom (nucleons and pions) typical of low-energy nuclear physics. However, chiral perturbation theory (ChPT) has definite limitations as far as the range of allowed momenta is concerned. For the purpose of applications in dense matter, where higher and higher momenta become involved with increasing Fermi momentum, NN potentials based on ChPT are unsuitable.

Relativistic meson theory is an appropriate framework to deal with the high momenta encountered in dense matter. In particular, the one-boson-exchange (OBE) model has proven very successful in describing NN data in free space and has a good theoretical foundation. Among the many available OBE potentials, some being part of the "high-precision generation" [11, 12], we seek a momentum-space potential developed within a relativistic scattering equation, such as the one obtained through the Thompson [13] three-dimensional reduction of the Bethe-Salpeter equation [14]. Furthermore, we require a potential that uses the pseudovector coupling for the interaction of nucleons with pseudoscalar mesons. With these constraints in mind, as well as the requirement of a good description of the NN data, Bonn B [15] is a reasonable choice. The mesons included are the pseudoscalar π and η, the scalar σ and δ, and the vector ρ and ω.

As our many-body framework, we choose the Dirac-Brueckner-Hartree-Fock approach. The main strength of the DBHF approach is its inherent ability to account for important three-body forces through its density dependence. In the DBHF approach, one describes the positive energy solutions of the Dirac equation in the medium as

$$u^*(p,\lambda) = \left(\frac{E_p^* + m^*}{2m^*}\right)^{1/2} \begin{pmatrix} \mathbf{1} \\ \frac{\sigma\cdot\vec{p}}{E_p^*+m^*} \end{pmatrix} \chi_\lambda, \qquad (12)$$

where the effective mass, m^*, is defined as $m^* = m + U_S$, with U_S an attractive scalar potential. It can be shown that the description of a single-nucleon via Eq. (12) effectively accounts for an important class of three-body forces, namely those generated by virtual excitation of nucleon-antinucleon pairs. The result is a repulsive effect on the energy/particle in symmetric nuclear matter which depends on the density approximately as

$$\Delta E \propto \left(\frac{\rho}{\rho_0}\right)^{8/3}, \qquad (13)$$

and provides a crucial saturation mechanism missing in conventional Brueckner-Hartree-Fock (BHF) theory. (Alternatively, explicit three-body forces are used along with the BHF method in order to achieve a similar result.) Brown *et al.* showed that the bulk of the desired effect can be obtained as a lowest order (in p^2/m) relativistic correction to the single-particle propagation [16]. See Ref. [17] for a detailed presentation of the formalism adopted in our DBHF method.

4 Contribution of Isovector Mesons to the Equation of State and the Symmetry Energy

Before proceeding to discuss the symmetry energy, we show, for Bonn B, how the various mesons contribute to the energy of symmetric nuclear matter, Figure 1(a), and neutron matter, Figure 1(b). We also note the clear impact of the pion on the saturation density of SNM, demonstrating the remarkable saturating effect generated by the tensor force.

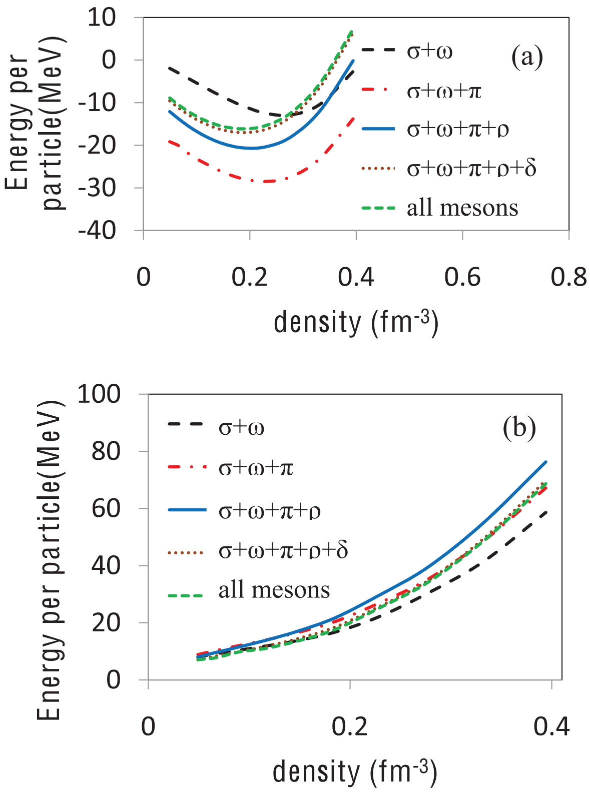

Figure 1. Contribution from the various mesons to the equation of state of symmetric matter (**a**) and neutron matter (**b**).

Table 1. The difference between the potential energy contributions (in MeV) to NM and SNM from isovector mesons.

Potential	$U^{\pi}_{NM} - U^{\pi}_{SNM}$	$U^{\rho}_{NM} - U^{\rho}_{SNM}$	$U^{\delta}_{NM} - U^{\delta}_{SNM}$
Bonn B	20.78	-5.90	-6.78
Bonn A	15.98	-4.68	-2.80
Bonn C	24.42	-5.48	-10.24

In Table 1, we show the difference between the potential energy contributions to NM and SNM from the isovector mesons, as an estimate of the effect of each meson on the potential energy part of the symmetry energy. (The density is taken to be equal to 0.185 fm^{-1}.) Clearly, in a microscopic, meson-theoretic approach the impact of the pion on the symmetry energy is the largest. We find this to be a point of considerable interest, since mean field theories are generally pionless. This is because the bulk of the attraction-repulsion balance needed for a realistic description of nuclear matter can be technically obtained from σ and ω only, an observation that is at the very foundation of Walecka models such as QHD-I [21]. However, in any fundamental theory of nuclear forces, the pion is the most important ingredient. Chiral symmetry is spontaneously broken in low-energy QCD and the pion emerges as the Goldstone boson of this symmetry breaking [10]. Moreover, NN scattering data cannot be described without the pion, which is also absolutely crucial for the two-nucleon bound state, the deuteron.

When moving to nuclear matter (and regardless the possibility of obtaining realistic values of its bulk properties, including the symmetry energy, with a pionless theory), this conceptual problem is not removed. *Isospin dependence is carried by the isovector mesons: Because of their isovector nature, these mesons contribute differently in different partial waves thus giving rise to isospin dependence.* (This is not the case with isoscalar mesons, which tend to contribute similarly in all partial waves.) Thus, an important aspect of the physics is missing in a discussion of isospin dependence that does not include the pion. Also, conclusions concerning the effect of other mesons (particularly ρ and δ) may be distorted due to the absence of the pion. This may include, for instance, observations concerning isospin-sensitive quantities such as the neutron-proton mass splitting in neutron-rich matter.

As mentioned earlier, investigations of ρ and δ contributions to the potential symmetry energy have been reported, such as the one in Refs. [22, 23]. In Figure 6-1 of Ref. [22], for instance, those contributions are shown to be very large in size (about -40 MeV and 50 MeV at saturation density for δ and ρ, respectively). Thus, the interplay between ρ and δ is described as the equivalent, in the isovector channel, of the σ-ω interplay in the isoscalar channel [23].

The dramatic differences between those and our present observations originate from several sources, which include: The absence of the pion; the nature of

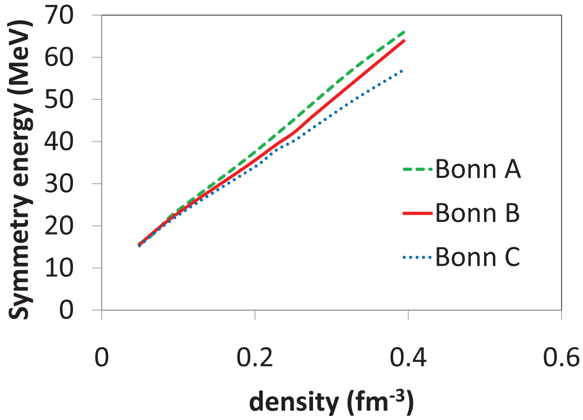

Figure 2. The symmetry energy as predicted with Bonn A, B, and C.

the ρ coupling; the fact that our meson contributions, when iterated, are reduced by the effect of the Pauli projector. As mentioned previously, the role of the δ is important although subtle, and it is found in its different contributions to I=1 and I=0 partial waves, especially the S-waves.

In Figure 2 we show the density dependence of the symmetry energy with Bonn A, B, and C. The potential model dependence comes almost entirely from differences among predictions of the SNM energy. With the three sets of predictions, we mean to estimate the uncertainty to be expected when using different parametrizations for the isovector mesons, while respecting the free-space NN data.

Figure 3 displays the momentum dependence of the single-proton and single-nucleon potentials in IANM, as predicted by the three potentials. Differences are small, at most 10% at the lowest momenta. We recall that the gradient between the potentials shown in Figure 3, closely related to the isovector optical potential, is the crucial mechanism that separates proton and neutron dynamics in IANM.

5 Conclusions

We reviewed our microscopic approach to the calculation of isospin-asymmetric nuclear matter, with particular attention to the symmetry energy. We examined the effect of the isovector mesons on the difference between the potential energies of pure neutron matter and symmetric matter. Our findings are easily understood in terms of the contributions of each meson to the appropriate component of the nuclear force and the isospin dependence naturally generated by isovector mesons.

We found that the pion gives the largest contribution to this difference. The contribution of the pion is often ovelooked, possibly because this meson is missing from some mean field models, which are popular among users of equations

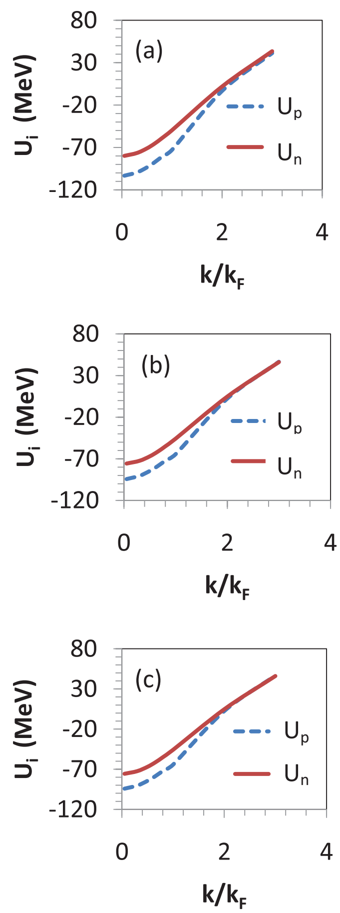

Figure 3. Momentum dependence of the single-nucleon potentials in IANM, U_i (i=p,n), predicted with Bonn A, B, and C. The total density is equal to 0.185 fm^{-3} and the isospin asymmetry parameter is 0.4. The momentum is given in units of the Fermi momentum, which is equal to 1.4 fm^{-1}.

of state. It is our opinion that conclusions regarding the interplay of ρ and δ in phenomenological models must be taken with caution.

Finally, we commented on fundamental differences between our approach and the one of mean field models, particularly pionless QHD theories. First, these differences are of conceptual relevance, since free-space NN scattering and bound state are, essentially, pion physics. Furthermore, they can impact in a considerable way conclusions with regard to isospin dependent systems/phenomena. In order to have a fundamental basis, a microscopic theory of the nuclear many-body problem has to start from the bare NN interaction with all its ingredients.

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