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Analysis of the ^{11}Li Breakup on a Proton Target

V.K. Lukyanov[1], **D.N. Kadrev**[2], **E.V. Zemlyanaya**[1], **K. Spasova**[2,3], **A.N. Antonov**[2], **K.V. Lukyanov**[1], **M.K. Gaidarov**[2]

[1]Joint Institute for Nuclear Research, Dubna 141980, Russia

[2]Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria

[3]Department of Theoretical and Applied Physics, "Ep. K. Preslavski" University, Shumen 9712, Bulgaria

Abstract. The ^{11}Li breakup effect on ^{11}Li$+p$ scattering at energy of 62 MeV/N is analyzed considering a cluster model for the projectile nucleus with fragments ^{9}Li and $2n$. Predictions for the longitudinal momentum distributions of ^{9}Li fragments produced in the breakup of ^{11}Li at 62 MeV/nucleon on a proton target are given. Calculations of the diffractive and stripping breakup processes are performed.

1 Introduction

The experiments with radioactive ion beams provide the possibility to study the halo nuclei (e.g. [1–3]). A typical example of a halo nucleus is ^{11}Li. The experiments with ^{11}Li nucleus measure the total reaction cross section and the momentum distribution (MD) of the core ^{9}Li and the $2n$ fragments following the breakup of ^{11}Li at high energies.

In general, the study of breakup products following collision processes of the weakly-bound neutron-rich halo nuclei makes it possible to probe their structure. It is known (e.g. [4–6] and references therein) that the MD of the breakup products has a narrow peak, much narrower than that observed in the fragmentation of well bound nuclei. The latter property of MD has been interpreted to be related to the very large extension of the wave function, as compared to that of core nucleons, leading to the existence of the halo of the nucleus [7–9]. As shown in [10], the longitudinal component of the momentum (taken along the beam or z direction) gives the most accurate information on the intrinsic properties of the halo and is insensitive to details of the collision and the size of the target. However, a rather broad components of the MD of neutrons detected in coincidence with the core have been observed in the direction perpendicular to the beam. The transverse distributions of the core are significantly broaden by diffractive effects and by Coulomb scattering [6]. They depend more strongly on the details of the nucleus-nucleus interaction than to the longitudinal MD [10]. The presence of broad components in the perpendicular distributions of the cores turns out to be controversial [4]: the distributions have been found in some cases to be consistent with the superposition of a narrow and a broad component [11, 12], while in other experiments a better fit was obtained in term of a single narrow component [13]. It was emphasized in the review [14] that to understand the measured longitudinal MD one must take into account that the heavy-ion knockout reaction is surface dominated and can probe only the external part of the nuclear wave function. Thus, the shape of the MD reflects the momentum content of this part.

In our previous works the differential cross sections of the elastic scattering of ^{6}He+p [15], ^{8}He+p [16], ^{6}He+^{12}C [17], and ^{11}Li+p [18] are studied using *both the real and imaginary parts of the OP calculated microscopically*.

The aim of the present work is to calculate the diffractive and stripping reaction cross sections for ^{11}Li+p using the microscopic OP obtained in Ref. [19]. Its real part includes the direct and exchange terms calculated by a single-folding procedure using the large-scale shell model (LSSM) density of ^{11}Li [20]. The imaginary part of the OP is derived within the High-Energy Approximation (HEA) [19, 21].

2 Theoretical Scheme

The optical potential used in our calculations has the form

$$U_{opt} = V^F(r) + iW(r). \tag{1}$$

The real part of the nucleon-nucleus OP is assumed to be a result of a single folding of the nuclear density and of the effective NN potential and involves the direct and exchange parts (e.g. Refs. [22–24], see also [15]):

$$V^F(r) = V^D(r) + V^{EX}(r). \tag{2}$$

The direct part $V^D(r)$ is composed by the isoscalar (IS) and isovector (IV) contributions. The energy and the density dependences of the effective NN interaction of CDM3Y6 type are taken in the forms from Refs. [15, 24, 25]. The isoscalar and isovector parts of the exchange ReOP are given in [15, 18]. In the present work we use proton and neutron densities calculated microscopically within the LSSM method using Woods-Saxon basis of single-particle wave functions with realistic exponential asymptotic behavior [20].

In our procedure the Re OP is calculated microscopically using a single-folding procedure. The Im OP is calculated also microscopically within the HEA. Then the obtained OP Eq. (1) is used to calculate the cross sections by means of the code DWUCK4 [26] for solving the Schrödinger equation. We note that we do not apply the Glauber theory to calculate the scattering amplitude at low energies, but use the equivalent HEA OP to solve numerically the respective wave equation. To calculate the HEA OP [19] one can use the definition of the eikonal phase as an integral of the nucleon-nucleus potential over the trajectory of the straight-line propagation, and has to compare it with the corresponding Glauber expression for the phase in the optical limit approximation. In this way, the HEA OP is obtained as a folding of the form factors of the nuclear density and the NN amplitude [19, 21].

In the framework of the ^{9}Li+$2n$ model of ^{11}Li one can estimate the ^{11}Li+p OP as folding of two OP's of interaction of the clusters c and h with protons and the density $\rho_0(s)$, which is corresponding to the wave function of the relative motion of the clusters $\phi_{00}(\mathbf{s})$. In particular, for the s-state the density has the form

$$\rho_0(\mathbf{s}) = |\phi_{00}(\mathbf{s})|^2 = \frac{1}{4\pi}|\phi_0(s)|^2 \tag{3}$$

and it will be used for further calculations of the ground-state matrix elements of breakup processes:

$$\begin{aligned} U^{(b)}(r) &= V^{(b)} + iW^{(b)} \\ &= \int d\mathbf{s}\rho_0(s)\left[U_c\left(\mathbf{r} + (2/11)\mathbf{s}\right) + U_h\left(\mathbf{r} - (9/11)\mathbf{s}\right)\right]. \end{aligned} \tag{4}$$

The potentials U_c and U_h in Eq. (4) are calculated within the microscopic hybrid model of OP [19], in which a single-folding procedure is applied for the real part $V^{(b)}$, while the imaginary part $W^{(b)}$ is derived using the optical limit of the Glauber theory. As is known (see, e.g. [5]), the differential and total cross sections (for elastic scattering, as well as for diffractive breakup and absorption) all require calculations of the probability functions $d^3P(\mathbf{b},\mathbf{k})/d\mathbf{k}$ that depend on the impact parameter $\mathbf{b}$. The general expression for the probability functions can be written as [5]

$$\frac{d^3P_\Omega(\mathbf{b},\mathbf{k})}{d\mathbf{k}} = \frac{1}{(2\pi)^3}\left|\int d\mathbf{r}\phi_k^*(\mathbf{r})\Omega(\mathbf{b},\mathbf{r}_\perp)\phi_0(\mathbf{r})\right|^2, \tag{5}$$

where $\Omega(\mathbf{b},\mathbf{r}_\perp)$ is expressed by means of the two profile functions S_c and S_h of the core and the di-neutron clusters, respectively. According to the eikonal formalism the probability after the collision ($z \to \infty$) the cluster h or c with an impact parameter b to remain in the elastic channel is:

$$|S_i(b)|^2 = e^{-\frac{2}{\hbar v}\int_{-\infty}^{\infty} dz W_i(\sqrt{b^2+z^2})} \quad i = c, h\ , \tag{6}$$

where W is the imaginary part of the OP. Consequently, the probability for the cluster to be removed from the elastic channel is $(1 - |S_i(b)|^2)$. Thus, the common probability of both h and c clusters to leave the elastic channel of the ^{11}Li+p scattering is $(1 - |S_h|^2)(1 - |S_c|^2)$. The total absorbtion cross section can be obtained by averaging the latter by the density $\rho_0(s)$. In the case of a stripping reaction with removing h-cluster from ^{11}Li to the proton target, one should use the probability of h to leave the elastic channel $[1 - |S_h(b_h)|^2]$, and for c to continue its elastic scattering with a probability $|S_c(b_c)|^2$. Then the probability of the whole process is $|S_c(b_c)|^2[1-|S_h(b_h)|^2]$, and to get the total stripping cross section one has to average over $\rho_0(s)$. Similarly, the ^{9}Li transfer can be constructed, and the net contribution of both removal reactions yields the total breakup cross section. The sum of both absorption cross section (σ^{tot}_{abs}) and the breakup cross section (σ^{tot}_{bu}), gives the total reaction cross section σ^{tot}_R.

In the calculation of the probabilities in the case of diffractive scattering one can replace in Eq. (5) the scattering wave by a plane wave. The expression of the probability $d^2P_\Omega(\mathbf{b},\mathbf{k})/dk_L dk_\perp$ has the form [5]:

$$\frac{d^2P_\Omega(\mathbf{b},\mathbf{k})}{dk_L dk_\perp} = \frac{k_\perp}{16\pi^3 k^2}\left|\int dr \int d(\cos\theta)\, g(r)\sin(kr)\int d\varphi_r \Omega(\mathbf{b},\mathbf{r}_\perp)\right|^2 \quad (7)$$

with

$$\Omega(\mathbf{b},\mathbf{r}_\perp) = S_c(\mathbf{b}_c)S_h(\mathbf{b}_h), \quad (8)$$

and $b_c = |\mathbf{r}_\perp - \mathbf{b}_h| = \left[r^2\sin^2\theta + b_h^2 - 2rb_h\sin\theta\cos(\varphi_r-\varphi_h)\right]^{1/2}$, $k=\sqrt{k_L^2+k_\perp^2}$.

In Eq. (7) $g(r)$ is related to the ground-state wave function $\phi_{00}(\mathbf{r})$ and, correspondingly, to the density $\rho(r)$ of ^{11}Li :

$$\phi_{00}(\mathbf{r}) = \frac{1}{\sqrt{4\pi}}\frac{g(r)}{r}\ , \quad (9)$$

$$g(r) = r\sqrt{4\pi\rho(r)} \quad (10)$$

with a normalization

$$4\pi\int r^2\rho(r)dr = 1. \quad (11)$$

Then the diffraction breakup cross section has the form

$$\left(\frac{d\sigma}{dk_L}\right)_{diff} = \int_0^\infty b_h\, db_h \int_0^{2\pi} d\varphi_h \int_0^\infty dk_\perp \frac{d^2P(\mathbf{b},\mathbf{k})}{dk_L dk_\perp}. \quad (12)$$

with $d^2P_\Omega(\mathbf{b},\mathbf{k})/dk_L dk_\perp$ from Eq. (7). The integrations over b_h and φ_h in Eq. (12) mean integration over the impact parameter $\mathbf{b}_h$ of the cluster h with respect to the target.

In the case of the stripping reaction when the h - cluster leaves the elastic channel it can be shown (following [5]) that the cross section takes the form:

$$\begin{aligned}\left(\frac{d\sigma}{dk_L}\right)_{str} &= \frac{1}{2\pi^2}\int b_h db_h d\phi_h\left[1-|S_h(\mathbf{b}_h)|^2\right] \\ &\times \int \rho d\rho d\phi_\rho |S_c(b_c)|^2 \\ &\times \left.\int_0^\infty dz\ \cos(k_L z)\phi_0\left(\sqrt{\rho^2+z^2}\right)\right]^2. \end{aligned} \quad (13)$$

Eq. (13) is obtained in the case when the incident nucleus has spin equal to zero and for the s -state of the relative motion of both clusters in the nucleus $\mathbf{s} = \mathbf{r}_h - \mathbf{r}_c$, $\rho = \mathbf{b}_h - \mathbf{b}_c$, $\mathbf{s} = \rho + \mathbf{z}$.

3 Results and Discussions

The optical potential $U^{(b)}$ (Eq. (4)) constructed in the framework of the ^{9}Li+$2n$ model is applied for calculating the differential cross section of the elastic scattering ^{11}Li+p at 62 MeV/nucleon. For the real part $V^{(b)}$ of this OP we use a single-folding procedure in which the ^{9}Li density is taken from Ref. [20], where it has been microscopically obtained within the LSSM approach. The imaginary part $W^{(b)}$ of the OP is considered like before to be either $W = W^H$ or $W = V^F$. The calculated cross sections are shown in Fig. 1 and are compared with the experimental data from Ref. [27]. For both cases we give in Table 1 the values of the fitted renormalization coefficients N's. One can see from Figure 1 that the angular distributions for both kinds of ImOP are similar to each other and they lead to a good agreement with the empirical data. The values of the absorption cross section and the breakup cross section as well as the total reaction cross section are also given in the Table 1.

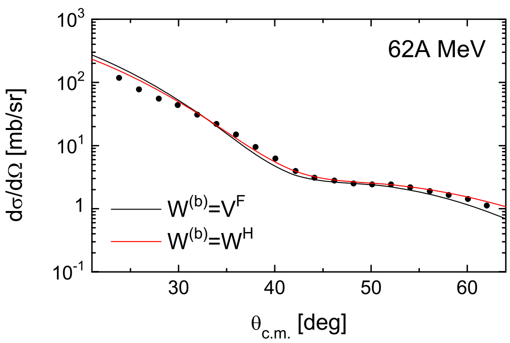

Figure 1. The ^{11}Li+p elastic scattering cross section at $E = 62$ MeV/nucleon using $U^{(b)}$ [Eq. (4)] for values of the parameters N shown in Table 1. Black line: $W^{(b)} = V^F$; red line: $W^{(b)} = W^H$. The experimental data are taken from [27].

Table 1. The values of the N's parameters and HEA estimations of the total cross sections σ^{tot}_{abs}, σ^{tot}_{bu} and σ^{tot}_{R} (in mb) within the ^{9}Li+$2n$ model of ^{11}Li+p at 62 MeV/nucleon.

$W^{(b)}$	N_R	N_I	σ^{tot}_{abs}	σ^{tot}_{bu}	σ^{tot}_{R}
V^F	1.407	1.195	79.0	431.8	510.8
W^H	1.381	1.306	78.6	405.3	483.9

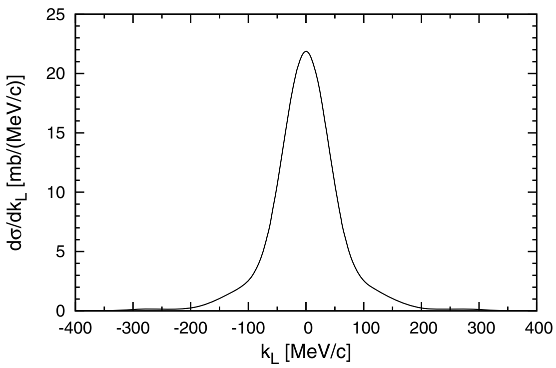

Figure 2. Cross section of diffraction breakup in ^{11}Li$+p$ scattering at $E = 62$ MeV/nucleon.

In Figure 2 and Figure 3 we give as examples the calculated cross sections for the diffractive and stripping (when $h = 2n$ cluster leaves the elastic channel) ^{11}Li $+p$ reactions at $E = 62$ MeV/nucleon, respectively. These results give predictions because there are not experimental data available for such processes at ^{11}Li $+p$ scattering at $E \leq 100$ MeV/nucleon. It can be seen from both Figures that the widths of the both peaks are around 100 MeV/c. We note that the widths obtained in our work are around twice larger than those obtained in the experiments (around 50 MeV/c) for the reactions of ^{11}Li on ^{9}Be, ^{93}Nb and ^{181}Ta at energy 66 MeV/nucleon [28] (that is close to our energy of 62 MeV/nucleon).

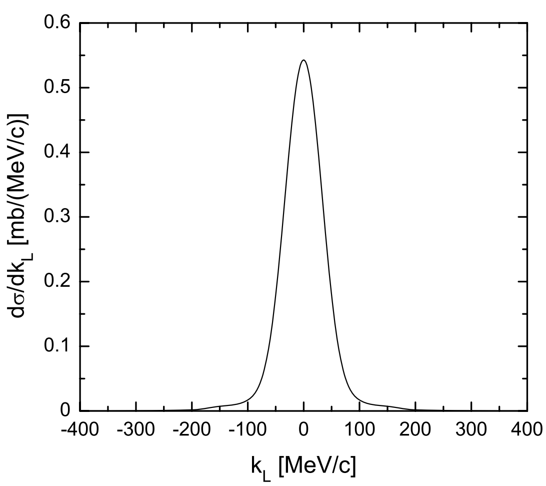

Figure 3. Cross section of stripping in ^{11}Li$+p$ reaction (when $2n$-cluster leaves elastic channel) at $E = 62$ MeV/nucleon.

It is noted in [28] that the width almost does not depend on the target's mass number and thus, it characterizes basically the momentum distribution of the two clusters. From the general point of view the width of around 50 MeV/c is related to a rms radius of about 6 fm for ^{11}Li. This is also discussed in [4,9,29,30]. The problem that arises is that if one considers the rms radius of the ^{11}Li like that obtained from the results on the total cross section of Tanihata (~ 3.1 fm) [3], it is impossible to obtain the value of ~ 50 MeV/c of the width. This is related to the peculiarity of the ^{11}Li nucleus, namely to its large rms radius. Not only for the case of ^{11}Li $+p$ reaction, but also in the cases of ^{11}Li scattering on nuclei, this problem remains open and requires further analysis. Our width for the stripping of $2n$-cluster is similar to the cases of $2n$ stripping from other nuclei (but not from ^{11}Li). It turns out that the account for the $2n$ binding in ^{11}Li is not enough to obtain the observed widths in the scattering of ^{11}Li on nuclei, as well as on proton targets.

4 Concusions

In addition to our previous calculations of ^{11}Li$+p$ reaction at $E < 100$ MeV/N using microscopically calculated OP's in the present work we consider a folding approach that includes ^{11}Li breakup elastic scattering using ^{9}Li $+2n$ cluster model, computing the potentials of the interactions of the two clusters with the proton. Analyzing σ_R^{tot} we observed a more significant role played by the breakup channel of the ^{11}Li$+p$ reaction (around 80%) than in the case of ^{6}He+^{12}C.

Predictions for the longitudinal momentum distributions of ^{9}Li fragments produced in the breakup of ^{11}Li at $E = 62$ MeV/nucleon on a proton target are given and calculations of the diffraction and stripping reaction cross sections are performed. We emphasize that the experiments on the ^{11}Li$+p$ reaction at $E < 100$ MeV/nucleon are desirable.

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