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A Model for Quasi Parity-Doublet Spectra with Strong Coriolis Mixing

N. Minkov[1,2], **S. Drenska**[1], **M. Strecker**[2]

[1]Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Tzarigrad Road 72, BG-1784 Sofia, Bulgaria

[2]Institut für Theoretische Physik der Justus-Liebig-Universität, Heinrich-Buff-Ring 16, D–35392 Giessen, Germany

Abstract. The model of coherent quadrupole and octupole motion (CQOM) is combined with the reflection-asymmetric deformed shell model (DSM) in a way allowing fully microscopic description of the Coriolis decoupling and K-mixing effects in the quasi parity-doublet spectra of odd-mass nuclei. In this approach the even-even core is considered within the CQOM model, while the odd nucleon is described within DSM with pairing interaction. The Coriolis decoupling/mixing factors are calculated through a parity-projection of the single-particle wave function. Expressions for the Coriolis mixed quasi parity-doublet levels are obtained in the second order of perturbation theory, while the K-mixed core plus particle wave function is obtained in the first order. Expressions for the B(E1), B(E2) and B(E3) reduced probabilities for transitions within and between different quasi-doublets are obtained by using the total K-mixed wave function. The model scheme is elaborated in a form capable of describing the yrast and non-yrast quasi parity-doublet spectra in odd-mass nuclei.

1 Introduction

The presence of qudrupole-octupole degrees of freedom in the odd-mass nuclei and the coupling between the even-even core and the odd nucleon lead to the appearance of a quasi parity-doublet structure of the spectrum [1, 2]. It is considered that the mutual disposition of the parity-doublet counterparts (states with the same angular momentum and opposite parity) depends on the qudrupole-octupole motion of the core. At the same time the rotation behaviour of the energy levels with increasing angular momentum is influenced by the Coriolis interaction between the core and the single particle (s.p.).

Some years ago a collective model assuming coherent quadrupole-octupole motion (CQOM) was developed and applied to the yrast alternating-parity bands in even-even nuclei and quasi parity-doublet spectra in odd-mass nuclei [3, 4]. Recently the model scheme was extended to non-yrast quadrupole-octupole spectra in even-even [5] and odd-mass [6, 7] nuclei. Regarding the odd-mass nuclei, the further development of this model approach needs to take into account the complicated character of the interaction between the collective modes and the

s.p. motion in the case of quadrupole-octupole deformation which so far was considered on a phenomenological level.

The purpose of this work is to extend the model scheme in microscopic direction by coupling the collective CQOM vibration-rotation modes in the core to the one-quasiparticle (q.p.) states corresponding to the odd-nucleon motion and described within the reflection-asymmetric deformed shell model (DSM) [8] with pairing interaction of BCS type. In this framework the collective CQOM formalism and the intrinsic DSM+BCS approach are combined in conjunction providing a possibility to describe the yrast and non-yrast quasi parity-doublet sequences in odd-mass nuclei on a deeper microscopic level. The proposed model mechanism suggests a strong coupling between the coherent quadrupole-octupole vibrations and the intrinsic q.p. states in the formation of excited quasi-doublet spectra. It is considered that the core induces a reflection-asymmetric deformation in the s.p. potential which leads to the appearance of parity-mixed s.p. states, while the total core plus particle wave function has a good parity. In [9] it was shown that the model wave function can be taken in a way that the mixed parity of the s.p./q.p state is projected on the fixed parity of the collective state providing a good parity for the total model state. On this basis a preliminary work on the connecting of the CQOM model to the DSM+BCS approach was recently done in [10]. In the present work this unified model scheme is completed in a form providing a fully microscopic description of the Coriolis decoupling and K mixing effects in the energy levels and B(E1), B(E2) and B(E3) reduced transition probabilities in the quasi parity-doublet spectra.

The paper is organized as follows. In Sec. 2 the structure of the total model Hamiltonian based on CQOM and DSM with pairing and Coriolis interaction is presented. The separate solutions of the collective and s.p./q.p. parts of the model are given also there. In Sec. 3 the core plus particle coupling scheme is described. The unified model solution for quasi parity-doublet spectra with microscopic Coriolis interaction is also given there. In Sec. 4 expressions for the model energies and B(E1)–B(E3) transition probabilities between K-mixed doublet states are presented. Also, the possibilities for future applications of the model scheme are discussed. In Sec. 5 concluding remarks are given.

2 Model Hamiltonian

The Hamiltonian of quadrupole-octupole vibrations and rotations coupled to the s.p. motion with Coriolis interaction and pairing correlations can be written in the form

$$H = H_{\text{s.p.}} + H_{\text{pair}} + H_{\text{qo}} + H_{\text{Coriol}}. \tag{1}$$

Here $H_{\text{s.p.}}$ is the s.p. Hamiltonin with the Woods-Saxon potential for axial quadrupole, octupole and higher multipolarity deformations [8], while H_{pair} is the standard BCS pairing Hamiltonian [11]. H_{qo} represents oscillations of the even–even core with respect to the quadrupole (β_2) and octupole (β_3) axial

deformation variables mixed through a centrifugal (rotation-vibration) interaction [4]

$$H_{\rm qo} \;=\; -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + U(\beta_2,\beta_3,I)\,, \tag{2}$$

where

$$U(\beta_2,\beta_3,I) = \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{d_0+\hat{I}^2-\hat{I_z}^2}{2(d_2\beta_2^2+d_3\beta_3^2)}, \tag{3}$$

and $\hat{I}$ and $\hat{I_z}$ are the operators of the total angular momentum and its third projection, respectively. Here B_2 (B_3), C_2 (C_3) and d_2 (d_3) are quadrupole (octupole) mass, stiffness and inertia parameters, respectively, while d_0 determines the potential core at zero angular momentum. The last term in (1) represents the Coriolis interaction

$$H_{\rm Coriol} = -\frac{(\hat{I}_+\hat{j}_- + \hat{I}_-\hat{j}_+)}{2(d_2\beta_2^2+d_3\beta_3^2)}\,, \tag{4}$$

where $\hat{I}_\pm = \hat{I}_x \pm i\hat{I}_y$ and $\hat{j}_\pm = \hat{j}_x \pm i\hat{j}_y$ are the spherical components of the total nuclear and s.p. angular momenta, respectively. The quantity $\mathcal{J}(\beta_2,\beta_3) = (d_2\beta_2^2+d_3\beta_3^2)$ can be associated to the moment of inertia of an axially symmetric quadrupole-octupole deformed shape.

The s.p. Hamiltonian is diagonalized in the axially deformed harmonic oscillator (ADHO) basis $|Nn_z\Lambda\Omega\rangle$ and the wave function for the odd particle is obtained as [8]

$$\mathcal{F}_\Omega = \sum_{Nn_z\Lambda} C^\Omega_{Nn_z\Lambda}|Nn_z\Lambda\Omega\rangle, \tag{5}$$

where Ω is equal to the z-projection K of the total angular momentum I. The obtained s.p. state is with mixed-parity and the wave function $\mathcal{F}_K$ can be divided up [9]

$$\mathcal{F}_K = \sum_{\pi_{\rm sp}=\pm1} \mathcal{F}_K^{(\pi_{\rm sp})} = \mathcal{F}_K^{(+)} + \mathcal{F}_K^{(-)}, \tag{6}$$

where $\mathcal{F}_K^{(+)}$ contains the positive-parity components, while $\mathcal{F}_K^{(-)}$ contains the negative-parity ones with $\hat{\pi}_{\rm sp}\mathcal{F}_K^{(\pm)} = \pm\mathcal{F}_K^{(\pm)}$. (Here $\hat{\pi}_{\rm sp}$ is the s.p. parity operator.)

The pairing effect is taken into account through a BCS procedure for $H_{\rm pair}$ [11] with constant pairing interaction applied to the DSM s.p. levels. The pairing constants $G_{n/p}$ for neutrons(n)/protons(p) are taken in the form [13] (see page 311):

$$G_{n/p} = \left(g_0 \mp g_1\frac{N-Z}{A}\right)/A\,, \tag{7}$$

where the parameters $g_0 = 17.8$ MeV and $g_1 = 7.4$ MeV are taken as in [12]. The BCS procedure is applied, as suggested in [13], within energy windows including $(15\,N)^{1/2}$ orbitals for neutrons and $(15\,Z)^{1/2}$ orbitals for protons below and above the Fermi surface. The phenomenological value $\Delta = 12 \cdot A^{-1/2}$ and the energy of the last occupied orbital are used for the pairing gap and the chemical potential λ, respectively, at the starting point in the numerical solution of the gap equation. In this procedure the orbital occupied by the single particle should be blocked. The one-q.p. energy is obtained through the s.p. energy $E^K_{\rm sp}$ as

$$\epsilon^K_{\rm qp} = \sqrt{(E^K_{\rm sp} - \lambda)^2 + \Delta^2}\,. \tag{8}$$

The q.p. energies $\epsilon^K_{\rm qp}$ are eigenvalues of the Hamiltonian $H_{\rm qp} = H_{\rm s.p.} + H_{\rm pair}$. Then the s.p. wave functions $\mathcal{F}_K$, (5), transform to the wave functions of the BCS q.p. states. The matrix elements of s.p. operators (such as $\hat{j}_\pm$) between two one-q.p. BCS-states are related to the matrix elements between the corresponding s.p. states by factors involving the BCS occupation amplitudes (see below). Then one can write

$$H = H_{\rm qp} + H_{\rm qo} + H_{\rm Coriol}. \tag{9}$$

The core Hamiltonian $H_{\rm qo}$ is diagonal in the collective rotation space and the potential (3) can be taken as [3]

$$U(\beta_2, \beta_3, I) = \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{X(I)}{d_2\beta_2^2 + d_3\beta_3^2}\,, \tag{10}$$

with $X(I) = [d_0 + I(I+1)]/2$ and $K = 0$. After introducing ellipsoidal coordinates $\beta_2 = p\eta\cos\phi$ and $\beta_3 = q\eta\sin\phi$, with $p = \sqrt{d/d_2}$, $q = \sqrt{d/d_3}$, $d = (d_2 + d_3)/2$ and by assuming coherent quadrupole-octupole oscillations with a frequency $\omega = \sqrt{C_2/B_2} = \sqrt{C_3/B_3} \equiv \sqrt{C/B}$, the spectrum of $H_{\rm qo}$ is obtained in the form [3]

$$E_{nk}(I) = \hbar\omega\left[2n + 1 + \sqrt{k^2 + bX(I)}\right], \tag{11}$$

where $n = 0, 1, 2, ...$, $k = 1, 2, 3, ...$ and $b = 2B/(\hbar^2 d)$. The quadrupole-octupole vibration wave function is

$$\Phi^\pi_{nkI}(\eta, \phi) = \psi^I_{nk}(\eta)\varphi^{\pi{\rm c}}_k(\phi), \tag{12}$$

where the radial part

$$\psi^I_{nk}(\eta) = \sqrt{\frac{2c\Gamma(n+1)}{\Gamma(n+2s+1)}}e^{-c\eta^2/2}(c\eta^2)^s L^{2s}_n(c\eta^2) \tag{13}$$

involves generalized Laguerre polynomials in the variable η with $c = \sqrt{BC}/\hbar$ and $s = (1/2)\sqrt{k^2 + bX(I)}$. The angular part in the variable ϕ appears with a

positive or negative parity, $\pi_{\rm c} = (\pm)$, of the even-even core as follows

$$\varphi_k^{\pi_{\rm c}=(+)}(\phi) = \sqrt{2/\pi}\cos(k\phi)\,, \qquad k=1,3,5,\ldots\,, \tag{14}$$

$$\varphi_k^{\pi_{\rm c}=(-)}(\phi) = \sqrt{2/\pi}\sin(k\phi)\,, \qquad k=2,4,6,\ldots\,. \tag{15}$$

In [4] the Coriolis term (4) was incorporated into the potential (10) by extending $X(I)$ for the case of odd nuclei with $K=1/2$ as

$$X(I,K) = \frac{1}{2}\left[d_0 + I(I+1) - K^2 + \pi a \delta_{K,\frac{1}{2}}(-1)^{I+1/2}\left(I+\frac{1}{2}\right)\right]\,. \tag{16}$$

Also, in the odd-mass nuclei the total angular momentum I includes the odd-nucleon momentum and therefore takes half-integer values. In [4] the decoupling factor a was taken as a fitting parameter. Below the microscopic contents of the Coriolis interaction will be completely taken into account.

3 Core Plus Particle Coupling Scheme

The eigenfunction corresponding to the Hamiltonian part $H_{\rm qp}+H_{\rm qo}$ can be taken in the form [9]

$$\Psi^{\pi}_{nkIMK}(\eta,\phi) = \frac{1}{2N_K}\sqrt{\frac{2I+1}{16\pi^2}}(1+\mathcal{R}_1)D^I_{M\,K}(\boldsymbol{\theta})(1+\pi\hat{P})\Phi^{\pi_c}_{nkI}(\eta,\phi)\mathcal{F}_K, \tag{17}$$

where N_K is a normalization factor determined below and $D^I_{M\,K}(\boldsymbol{\theta})$ is the Wigner rotation function. The core function $\Phi^{\pi_c}_{nkI}(\eta,\phi)$ is given by (12). $\hat{P} = \hat{\pi}_c\cdot\hat{\pi}_{\rm sp}$ is the operator of the total parity with eigenvalue π ($\pi = \pi_c\cdot\pi_{\rm sp}$). The operator $\mathcal{R}_1$ represents a rotation by an angle π about an axis perpendicular to the intrinsic z-axis. The actions of $\mathcal{R}_1$ and $\hat{\pi}_{\rm sp}$ on the respective parts of the wave function are

$$\mathcal{R}_1 D^I_{M\,K} = (-1)^{I+K}D^I_{M\,-K},\ \mathcal{R}_1\Phi^{\pi_c}_{nkI} = \pi_c\Phi^{\pi_c}_{nkI} \tag{18}$$

$$\mathcal{R}_1\mathcal{F}_K = \mathcal{F}_{-K},\ \ \hat{\pi}_{\rm sp}\mathcal{F}_K = \mathcal{F}^{(+)}_K - \mathcal{F}^{(-)}_K\,. \tag{19}$$

The core parity π_c is fixed so as $\pi_c = (+)$ for the doublet counterparts with $\varphi^+(\phi)$ [Eq. (14)] containing the ground or bandhead state (states shifted down), and $\pi_c = (-)$ for the counterparts with $\varphi^-(\phi)$ [Eq. (15)] which are shifted up. The operator $(1+\pi\hat{P})$ in (17) projects out the component $\mathcal{F}^{(+)}_K$ or $\mathcal{F}^{(-)}_K$ from $\mathcal{F}_K$, providing a good total parity of the states in the split parity-doublet spectrum. Thus, if the lowest (ground or bandhead) state of the doublet has a positive total parity, $\pi = \pi^b = \pi_{\rm sp} = (+)$, the projected component is $\mathcal{F}^{(+)}_K$, while for a negative-parity $\pi = \pi^b = \pi_{\rm sp} = (-)$, the projected component is $\mathcal{F}^{(-)}_K$. Therefore, the wave function (17) can be characterized by the parity π of the considered doublet state and the experimentally determined parity of the

bandhead state $\pi^b = \pm$ on which the parity doublet is built and with respect to which the DSM wave function (5) is projected. By taking the action of the operators in (18), (19) and the condition for a good total parity $\pi = \pi_c \cdot \pi^b$ (or $\pi_c = \pi \cdot \pi^b$) the function (17) can be written in the following simplified form

$$\Psi^{\pi,\pi^b}_{nkIMK}(\eta,\phi) = \frac{1}{N_K}\sqrt{\frac{2I+1}{16\pi^2}}\Phi^{\pi\cdot\pi^b}_{nkI}(\eta,\phi) \times \left[D^I_{M\,K}(\theta)\mathcal{F}^{(\pi^b)}_K + \pi\cdot\pi^b(-1)^{I+K}D^I_{M\,-K}(\theta)\mathcal{F}^{(\pi^b)}_{-K}\right]. \quad (20)$$

Here the index b labels the ground or the excited bandhead state as a member of the DSM s.p. (or q.p.) spectrum, while $\mathcal{F}^{(\pi^b)}_K = \mathcal{F}^{(\pm)}_K$ are the respectively projected components of the s.p./q.p. wave function. (In the CQOM notations the considered bandhead states are labeled by the quantum number $n = 0, 1, 2$, i.e. we have a unique correspondence between b and n.) The normalization factor in (20) is

$$N_K = \left[\left\langle \mathcal{F}^{(\pi^b)}_K \middle| \mathcal{F}^{(\pi^b)}_K \right\rangle\right]^{\frac{1}{2}}, \quad (21)$$

and the orthonormalization property of the wave function (20) reads

$$\left\langle \Psi^{\pi',\pi^{b'}}_{n'k'I'M'K_{\nu'}} \middle| \Psi^{\pi,\pi^b}_{nkIMK_\nu} \right\rangle = \delta_{nn'}\delta_{kk'}\delta_{II'}\delta_{MM'}\delta_{K_\nu K_{\nu'}}\frac{1}{2}(1+\pi\pi'\pi^b\pi^{b'}) \times \frac{P_{K_{\nu'}K_\nu}}{N_{K_{\nu'}}N_{K_\nu}}\left\langle \mathcal{F}_{K_{\nu'}}{}^{(\pi^{b'})} \middle| \mathcal{F}_{K_\nu}{}^{(\pi^b)} \right\rangle. \quad (22)$$

The quantity $P_{K_{\nu'}K_\nu} = U_{K_{\nu'}}U_{K_\nu} + V_{K_{\nu'}}V_{K_\nu}$ (with $P_{K_\nu K_\nu} = 1$) involves the usual BCS occupation factors coming from the pairing interaction, i.e. after the q.p. states corresponding to the wave functions (20) are considered (as mentioned after Eq. (8)). Hereafter the occupation factor $P_{K_{\nu'}K_\nu}$ will be considered in all matrix elements and/or scalar products involving the functions (20).

It should be noted that in the case of mixed parity ($\beta_3 \neq 0$) the overlap integral $\left\langle \mathcal{F}_{K_{\nu'}}{}^{(\pi^{b'})} \middle| \mathcal{F}_{K_\nu}{}^{(\pi^b)} \right\rangle$ is not zero for different s.p. states ($\nu' \neq \nu$) contrary to the case of a good parity. This is due to the circumstance that the projected parts of the q.p. wave functions do not contain the complete set of expansion components appearing in (5). Nevertheless, the orthogonality of the wave function (20) for the states belonging to different doublets, i.e. states built on different s.p./q.p. states is provided by the orthogonality of the collective quadrupole-octupole part of the wave function through the Kronecker factor $\delta_{nn'}$. Also, the factor $\left(1+\pi\pi'\pi^b\pi^{b'}\right)$ does not contribute to the orthogonality of states having the parity of the bandheads (one may check different combinations of the parities in this factor), and again the orthogonality is provided by the factors $\delta_{nn'}\delta_{kk'}$ coming from the collective part of the function.

In the case of a good s.p. parity ($\beta_3 = 0$) the total wave function (20) includes the full s.p./q.p. wave function $\mathcal{F}_K^{(\pi^b)} = \mathcal{F}_K$. The overlap integral $\left\langle \mathcal{F}_{K_{\nu'}}{}^{(\pi^{b'})} \middle| \mathcal{F}_{K_\nu}{}^{(\pi^b)} \right\rangle$ in (22) gives $\delta_{\nu\nu'}\delta_{bb'}$ and then one has

$$\left\langle \Psi^{\pi',\pi^{b'}}_{n'k'I'M'K_{\nu'}} \middle| \Psi^{\pi,\pi^b}_{nkIMK_\nu} \right\rangle = \delta_{nn'}\delta_{kk'}\delta_{II'}\delta_{MM'}\delta_{K_\nu K_{\nu'}}\delta_{\nu\nu'}\delta_{bb'}\frac{1}{2}(1+\pi\pi'). \quad (23)$$

The orthonormality in (23) is provided by the both, collective and s.p. parts of the wave function.

The Coriolis Hamiltonian (4) can be diagonalized through the contact transformation [14] or treated by the perturbation theory in the basis of the wave functions (20). The result is obtained up to the second order in the following form, which is pre-defined for an intrinsic state with a given quantum number K_b on which the parity doublet is built:

$$H^{\rm c}_{K_b} = \frac{1}{2\mathcal{J}(\beta_2,\beta_3)}(-1)^{I+1/2}\left(I+\frac{1}{2}\right)a_{1/2}\delta_{K_b,\frac{1}{2}} - \frac{1}{[2\mathcal{J}(\beta_2,\beta_3)]^2}\sum_{\substack{\nu\neq b \\ (K_\nu = K_b \pm 1,\frac{1}{2})}} \frac{\left[\widetilde{a}^{(\pi^b)}_{K_\nu K_b}(I)\right]^2}{\epsilon^{K_\nu}-\epsilon^{K_b}}, \quad (24)$$

where

$$\widetilde{a}^{(\pi^b)}_{K_\nu K_b}(I) = \begin{cases} \sqrt{(I-K_b)(I+K_b+1)}\,a^{(\pi^b)}_{K_\nu K_b}\,, & K_\nu = K_b+1 \\ \sqrt{(I+K_b)(I-K_b+1)}\,a^{(\pi^b)}_{K_b K_\nu}\,, & K_\nu = K_b-1 \\ \pi\cdot\pi^b(-1)^{(I+\frac{1}{2})}a^{(\pi^b)}_{\frac{1}{2},-\frac{1}{2}}\,, & K_\nu = K_b = \frac{1}{2}\,, \end{cases} \quad (25)$$

with the Coriolos mixing factors defined as

$$a^{(\pi^b)}_{K_{\nu'}K_\nu} = \frac{P_{K_{\nu'}K_\nu}}{N_{K_{\nu'}}N_{K_\nu}}\langle \mathcal{F}^{(\pi^b)}_{K_{\nu'}}|\hat{j}_+|\mathcal{F}^{(\pi^b)}_{K_\nu}\rangle = \frac{P_{K_\nu K_{\nu'}}}{N_{K_\nu}N_{K_{\nu'}}}\langle \mathcal{F}^{(\pi^b)}_{K_\nu}|\hat{j}_-|\mathcal{F}^{(\pi^b)}_{K_{\nu'}}\rangle. \quad (26)$$

The decoupling factor $a_{1/2}$ is

$$a_{1/2} = \pi\pi_b a^{(\pi_b)}_{\frac{1}{2}-\frac{1}{2}}, \quad (27)$$

where $\pi_b = \pm$ is the parity of the doublet bandhead (or ground) state and π is the total parity of the given state with angular momentum I. The sum in (24) runs over q.p. states with energies ϵ^{K_ν} above the Fermi level. Since the operators $\hat{j}_\pm$ do not change the parity, the K-mixing factors (26) connect the considered

bandhead state with the projected parts of the s.p./q.p. states in the sum of (24) which have the same parity π_b (see below). In the case of a good s.p. parity (27) and (26) reduce to the standard decoupling factor and K-mixing matrix elements, respectively.

Now the rotation factor $X(I,K)$, Eq. (16), can be replaced by

$$\widetilde{X}(I,K_b) = \frac{1}{2}\Bigg[d_0 + I(I+1) - K_b^2 + (-1)^{I+\frac{1}{2}}\left(I+\frac{1}{2}\right)a_{\frac{1}{2}}\delta_{K_b,\frac{1}{2}} - \frac{1}{2\mathcal{J}(\beta_2,\beta_3)}\sum_{\substack{\nu\neq b\\(K_\nu=K_b\pm1,\frac{1}{2})}}\frac{[\widetilde{a}^{(\pi^b)}_{K_\nu K_b}(I)]^2}{\epsilon^{K_\nu}-\epsilon^{K_b}}\Bigg]. \quad (28)$$

Then the general Hamiltonian (9) is redefined in a form where the angular momentum factor $X(I)$ in (10) is replaced by the above microscopically determined term $\widetilde{X}(I,K)$. In such a way the solution of the s.p. problem for the odd nucleon and its influence on the collective motion through the Coriolis interaction is included into the centrifugal part of the collective problem for the potential (10). The solution of this problem is not straightforward due to the deformation dependence of $\widetilde{X}(I,K)$ through the factor $1/[2\mathcal{J}(\beta_2,\beta_3)]$. However, we remark that in (28) this factor is joined, through (26), to the matrix elements $\langle\mathcal{F}^{\pm}_{K'}|\hat{j}_+|\mathcal{F}^{\pm}_{K}\rangle$. The latter depend on β_2 and β_3, which enter the DSM procedure as parameters. Then one can suppose that in (28) both, $1/(2\mathcal{J})$ and $\langle\mathcal{F}^{\pm}_{K'}|\hat{j}_+|\mathcal{F}^{\pm}_{K}\rangle$, depend on the same fixed set of deformation parameters (β_2^0,β_3^0). In this case the factor

$$A \equiv 1/[2\mathcal{J}(\beta_2,\beta_3)] = 1/[2\mathcal{J}(\beta_2^0,\beta_3^0)] \quad (29)$$

is a constant, which in the simplest approach can be treated as an adjustable parameter. In such a case the explicit deformation dependence of the kinematic factor (28) is removed.

By solving the problem under the above conditions the eigenfunctions of the general Hamiltonian (9) corresponding to Coriolis perturbed/mixed parity-doublet states built on a q.p. state with $K = K_b$ are obtained in the form

$$\widetilde{\Psi}^{\pi,\pi^b}_{nkIMK_b} = \frac{1}{\widetilde{N}_{K_b}}\left[\Psi^{\pi,\pi^b}_{nkIMK_b} + \sum_{\substack{\nu\neq b\\(K_\nu=K_b\pm1,\frac{1}{2})}} C^I_{K_\nu K_b}\Psi^{\pi,\pi^b}_{nkIMK_\nu}\right], \quad (30)$$

where the expansion coefficients are given by

$$C^I_{K_\nu K_b} = A\frac{\widetilde{a}^{(\pi^b)}_{K_\nu K_b}(I)}{\epsilon^{K_\nu}-\epsilon^{K_b}}. \quad (31)$$

The normalization factor $\widetilde{N}_{K_b}$ in the Coriolis perturbed function (30) is given by

$$\begin{aligned}\widetilde{N}^2_{K_b} &= \left\langle \widetilde{\Psi}^{\pi,\pi^b}_{nkIMK_b} \middle| \widetilde{\Psi}^{\pi,\pi^b}_{nkIMK_b} \right\rangle \\ &= 1 + 2 \sum_{\substack{\nu\neq b \\ (K_\nu = K_b = \frac{1}{2})}} C_{K_\nu K_b} \delta_{K_\nu K_b} \frac{P_{K_\nu K_b}}{N_{K_\nu} N_{K_b}} \left\langle {\mathcal{F}_{K_\nu}}^{(\pi^b)} \middle| {\mathcal{F}_{K_b}}^{(\pi^b)} \right\rangle \\ &\quad + \sum_{\substack{\nu_1\neq b \\ (K_{\nu_1} = K_b \pm 1, \frac{1}{2})}} \sum_{\substack{\nu_2\neq b \\ (K_{\nu_2} = K_b \pm 1, \frac{1}{2})}} C_{K_{\nu_1} K_b} C_{K_{\nu_2} K_b} \delta_{K_{\nu_1} K_{\nu_2}} \\ &\qquad \times \frac{P_{K_{\nu_1} K_{\nu_2}}}{N_{K_{\nu_1}} N_{K_{\nu_2}}} \left\langle {\mathcal{F}_{K_{\nu_1}}}^{(\pi^b)} \middle| {\mathcal{F}_{K_{\nu_2}}}^{(\pi^b)} \right\rangle . \quad (32)\end{aligned}$$

In the case of good parity ($\beta_3 = 0$) the overlap integral $\left\langle {\mathcal{F}_{K_\nu}}^{(\pi^b)} \middle| {\mathcal{F}_{K_b}}^{(\pi^b)} \right\rangle$ gives $\delta_{\nu b}$ which together with the summation condition $\nu \neq b$ makes the first term in (32) zero, while the last term is reduced to $\delta_{\nu_1 \nu_2}$ which lifts the second summation and thus one has

$$\widetilde{N}^2_{K_b} = 1 + \sum_{\substack{\nu\neq b \\ (K_\nu = K_b \pm 1, \frac{1}{2})}} C^2_{K_\nu K_b} . \quad (33)$$

4 Energies and Transition Probabilities

4.1 Energies

According to the formalism developed in the previous section the unified model expression for the energy levels of a quasi parity-doublet built on a Coriolis mixed q.p. state with angular momentum projection K_b has the general form

$$E_{nk}(I, K_b) = \epsilon^{K_b} + \hbar\omega \left[2n + 1 + \sqrt{k^2 + b\widetilde{X}(I, K_b)} \right] , \quad (34)$$

where the term $X(I)$ in the CQOM expression (11) is replaced by $\widetilde{X}(I, K)$ from (28) and ϵ^{K_b} is the q.p. energy obtained by DSM+BCS. The expression (34) suggests that the quasi parity-doublet bands can be considered as the result of collective quadrupole-octupole rotations and vibrations superposed on the top of s.p./q.p. excitations. On the other hand, it can be assumed that the excited doublet levels have a completely collective rotation-vibration origin described by CQOM. This means that the ϵ^{K_b} energies in (34) can be discarded, while the collective levels still remain coupled to the q.p. excitations through the Coriolis mixing factors entering $\widetilde{X}(I, K)$ in (28). Such an assumption is made in the preliminary work on this approach done in [10].

In all cases the structure of the model quasi parity-doublet spectrum is obtained by (34) by considering different rotation-vibration modes characterized

by the "radial" oscillator quantum number $n = 0, 1, 2, ...$, and pairs of odd and even values of the "angular" quantum number k, $k_n^{(+)}$ and $k_n^{(-)}$ ($k_n^{(+)} < k_n^{(-)}$) labeling the opposite parity states (with $I^{\pm}$ =half-integer) within a given doublet. The yrast doublet with $n = 0$ is coupled to the ground state, while the non-yrast doublets with $n = 1, 2, ...$ are coupled to higher q.p. states determined in the DSM+BCS model. For given n the k-values are determined so that $k = k_n^{(+)}$ for $I^{\pi=\pi^{(n)}}$ and $k = k_n^{(-)}$ for $I^{\pi=-\pi^{(n)}}$, where $\pi^{(n)}$ is the experimentally observed parity of the ground state ($n = 0$) or the excited doublet bandhead ($n > 0$). The difference between $k_n^{(+)}$ and $k_n^{(-)}$ generates the splitting of the parity-doublet. It should be noted that coherent quadrupole-octupole oscillation modes with $n > 0$ were used to describe the non-yrast alternating-parity bands in even-even nuclei [5] and quasi parity-doublets in odd-mass nuclei [6,7].

As mentioned below Eq. (20) the index n also labels the different q.p. states to which the different doublets are coupled. This imposes the condition that the signs of the average parity in the ground and excited q.p. states obtained in DSM correspond to the parities of the respective experimentally observed states. Another important condition is that the third angular momentum projections $K_b = K^{(n)}$ ($n = 0, 1, 2, ...$) obtained in DSM for these states also coincide with the experimentally estimated ones or with those assumed in the collective part of the model. These conditions together with the requirement to obtain appropriate values for the Coriolis decoupling and K-mixing factors entering $\overset{\circ}{X}(I, K)$ in (34) impose a number of limitations on the possible quadrupole and octupole deformation values β_2 and β_3 which can be used in the DSM+BCS calculation. The choice of these deformation parameters is crucial for the present approach. One possibility is to obtain the solutions of the DSM+BCS problem on a net in the (β_2, β_3)-space and get the deformations which provide the appropriate K-values for the ground state and the excited bandhead states and give, at the same time, the best description of the spectrum including the Coriolis decoupling and K-mixing effects. A more sophisticated possibility is to determine $\beta_2 = \beta_2^0$ and $\beta_3 = \beta_3^0$ from the collective model before using them in DSM. For this reason one can try to obtain their average ("dynamical") values from the CQOM solution for the even-even core. This is not so straightforward because of the pronounced softness of the shape in the assumed regime of coherent quadrupole-octupole motion. Beyond the coherent limit of the model, one can get the values which provide a minimum in the (β_2, β_3) potential (3) after numerically solving the two-dimensional quadrupole-octupole problem [15]. In both cases the use of the so obtained "collective" estimations for the quadrupole-octupole deformations in the DSM+BCS part of the scheme is a subtle task which requires a careful determination of the collective model parameters, especially the parameters of the potential (3), in conjunction with the characteristics of the odd nucleon motion (K-values, average parity, Coriolis interaction). In the preliminary work [10] the β_2 and β_3 deformations for the DSM procedure are essentially determined by examining the evolution of the s.p. characteristics

in the quadrupole-octupole space. This seems to be a reasonable starting point in the prospective application of the formalism.

4.2 Transition Probabilities

By using the most general Coriolis mixed wave functions (30) one can calculate B(Eλ) transition probabilities ($\lambda = 1, 2, 3$) within the yrast and non-yrast quasi-doublet bands as well as between the different bands. The relevant formalism was originally developed in [3, 4] and further extended to the non-yrast states of even-even nuclei [5]. In this work we use the formalism of [5] and below develop it in the case of the Coriolis mixed quasi-doublet states (30).

The electric multipole operators are

$$\hat{M}_{\mu}(E\lambda)=\sqrt{\frac{2\lambda+1}{4\pi(4-3\delta_{\lambda,1})}}\hat{Q}_{\lambda 0}\sum_{\nu}D^{\lambda}_{\mu\nu}, \quad \lambda=1,2,3;\ \mu=0,\pm1,...,\pm\lambda. \tag{35}$$

The explicit form of the operators $\hat{Q}_{\lambda 0}$ is given by Eqs. (31)–(33) in [5]. The matrix elements for the $E1$, $E2$ and $E3$ transitions between the unperturbed states (20) are obtained in the form

$$\begin{aligned}\langle|\hat{M}_{\mu}(E\lambda)|\rangle &= \langle\Psi^{\pi_f,\pi^{b_f}}_{n_f k_f I_f M_f K_f}(\eta,\phi)|\hat{M}_{\mu}(E\lambda)|\Psi^{\pi_i,\pi^{b_i}}_{n_i k_i I_i M_i K_i}(\eta,\phi)\rangle \\ &= \widetilde{R}_{\lambda,\mu}C^{I_f K_f}_{I_i K_i \lambda(K_f-K_i)}\frac{P_{K_f K_i}\langle\mathcal{F}^{(\pi^{b_f})}_{K_f}|\mathcal{F}^{(\pi^{b_i})}_{K_i}\rangle}{N_{K_f}N_{K_i}} \\ &\quad\times\left[1-\pi_f\pi^{b_f}\pi_i\pi^{b_i}(-1)^{\lambda+K_f+K_i}\right], \end{aligned}\tag{36}$$

where

$$\widetilde{R}_{\lambda,\mu}=\frac{1}{2}\sqrt{\frac{2\lambda+1}{4\pi(4-3\delta_{\lambda,1})}}\sqrt{\frac{2I_i+1}{2I_f+1}}C^{I_f M_f}_{I_i M_i \lambda\mu}R^{\pi_f,\pi^{b_f};\pi_i,\pi^{b_i}}_{\lambda}, \tag{37}$$

with $C^{I_f M_f}_{I_i M_i \lambda\mu}$ being the Clebsch-Cordan coefficient and

$$R^{\pi_f,\pi^{b_f};\pi_i,\pi^{b_i}}_{\lambda}=\int_{\eta}\int_{\phi}\Phi^{\pi_f\cdot\pi^{b_f}}_{n_f k_f I_f}(\eta,\phi)Q_{\lambda 0}(\eta,\phi)\Phi^{\pi_i\cdot\pi^{b_i}}_{n_i k_i I_i}(\eta,\phi)\eta d\eta d\phi. \tag{38}$$

The two-dimensional integral (38) is factorized into radial (η) and angular (ϕ) integrals the explicit analytic forms of which are obtained in [5].

In the case of good parity ($\beta_3 = 0$) the overlap integral in (36) gives $\delta_{i,f}$ and the transition matrix element between the unperturbed states becomes (see also the text above Eq. (23))

$$\begin{aligned}\langle|\hat{M}_{\mu}(E\lambda)|\rangle &= \widetilde{R}_{\lambda,\mu}C^{I_f K_f}_{I_i K_i \lambda(K_f-K_i)}\delta_{K_i,K_f}\delta_{i,f} \\ &\quad\times\left[1-\pi_f\pi^{b_f}\pi_i\pi^{b_i}(-1)^{\lambda+K_f+K_i}\right]. \end{aligned}\tag{39}$$

The matrix element (39) allows transitions between states only belonging to the same doublet built on the same s.p./q.p. state. This is the situation considered in the previous applications of the CQOM approach to odd-mass nuclei [4, 7] (see Eq. (13) in [7] and the related text). This restriction of the model scheme is removed below by taking into account the Coriolis K-mixing effect.

The matrix elements of the electric transition operators between the Coriolis perturbed states (30) with initial quantum numbers $n = n_i$, $k = k_i$, $I = I_i$, $K = K_i$ and final quantum number $n = n_f$, $k = k_f$, $I = I_f$, $K = K_f$ are given by

$$\begin{aligned}
\langle \widetilde{\Psi}^{\pi_f,\pi^{b_f}}_{n_f k_f I_f M_f K_f} | \hat{M}_\mu(E\lambda) | \widetilde{\Psi}^{\pi_i,\pi^{b_i}}_{n_i k_i I_i M_i K_i} \rangle &= \frac{1}{\widetilde{N}_{K_f}\widetilde{N}_{K_i}} \widetilde{R}_{\lambda,\mu} \delta_{\pi^{b_f}\pi^{b_i}} \\
&\times \left[1 + \pi_f \pi_i (-1)^\lambda\right] \Bigg[\delta_{K_f K_i} C^{I_f K_f}_{I_i K_i \lambda 0} \frac{P_{K_f K_i} \langle \mathcal{F}^{(\pi^{b_f})}_{K_f} | \mathcal{F}^{(\pi^{b_i})}_{K_i} \rangle}{N_{K_f} N_{K_i}} \\
&+ C^{I_f K_f}_{I_i K_f \lambda 0} \sum_{\substack{\nu \neq i \\ (K_\nu = K_i \pm 1, \frac{1}{2})}} \delta_{K_f K_\nu} C^{I}_{K_\nu K_i} \frac{P_{K_f K_\nu} \langle \mathcal{F}^{(\pi^{b_f})}_{K_f} | \mathcal{F}^{(\pi^{b_i})}_{K_\nu} \rangle}{N_{K_f} N_{K_\nu}} \\
&+ C^{I_f K_i}_{I_i K_i \lambda 0} \sum_{\substack{\nu \neq f \\ (K_\nu = K_f \pm 1, \frac{1}{2})}} \delta_{K_\nu K_i} C^{I}_{K_\nu K_f} \frac{P_{K_\nu K_i} \langle \mathcal{F}^{(\pi^{b_f})}_{K_\nu} | \mathcal{F}^{(\pi^{b_i})}_{K_i} \rangle}{N_{K\nu} N_{K_i}} \\
&+ \sum_{\substack{\nu'' \neq f \\ (K_{\nu''} = K_f \pm 1, \frac{1}{2})}} \sum_{\substack{\nu' \neq i \\ (K_{\nu'} = K_i \pm 1, \frac{1}{2})}} \delta_{K_{\nu''} K_{\nu'}} C^{I_f K_{\nu''}}_{I_i K_{\nu'} \lambda K_{\nu''} - K_{\nu'}} \\
&\times C^{I}_{K_{\nu''} K_f} C^{I}_{K_{\nu'} K_i} \frac{P_{K_{\nu''} K_{\nu'}} \langle \mathcal{F}^{(\pi^{b_f})}_{K_{\nu''}} | \mathcal{F}^{(\pi^{b_i})}_{K_{\nu'}} \rangle}{N_{K_{\nu''}} N_{K_{\nu'}}} \Bigg]. \quad (40)
\end{aligned}$$

The B(Eλ) transition probabilities between initial (i) and final (f) model states (30) belonging to the same ($n_i = n_f$) or different ($n_i \neq n_f$) quasidoublets are determined by

$$\begin{aligned}
&B(E\lambda; \pi^{b_i} n_i k_i I_i K_i \to \pi^{b_f} n_f k_f I_f K_f) = \\
&\frac{1}{2I_i + 1} \sum_{M_i M_f \mu} \left| \left\langle \widetilde{\Psi}^{\pi_i,\pi^{b_i}}_{n_i k_i I_i M_i K_i}(\eta,\phi) \middle| \mathcal{M}_\mu(E\lambda) \middle| \widetilde{\Psi}^{\pi_i}_{n_i k_i I_i M_i 0}(\eta,\phi) \right\rangle \right|^2. \quad (41)
\end{aligned}$$

After carrying out the integration over the rotation part in (41) one obtains the $B(E\lambda)$ transition probabilities in the form

$$B(E\lambda;\pi^{b_i}n_ik_iI_iK_i\to\pi^{b_f}n_fk_fI_fK_f)=\frac{1}{4}\frac{2\lambda+1}{4\pi(4-3\delta_{\lambda,1})}$$
$$\times R^2_\lambda(\pi^{b_i}n_ik_iI_i\to\pi^{b_f}n_fk_fI_f)\delta_{\pi^{b_f}\pi^{b_i}}\left[1+\pi_f\pi_i(-1)^\lambda\right]^2$$
$$\times\frac{1}{\widetilde{N}^2_{K_f}\widetilde{N}^2_{K_i}}\Bigg[\delta_{K_fK_i}C^{I_fK_f}_{I_iK_i\lambda0}\frac{P_{K_fK_i}\langle\mathcal{F}^{(\pi^{b_f})}_{K_f}|\mathcal{F}^{(\pi^{b_i})}_{K_i}\rangle}{N_{K_f}N_{K_i}}$$
$$+C^{I_fK_f}_{I_iK_f\lambda0}\sum_{\substack{\nu\neq i\\(K_\nu=K_i\pm1,\frac{1}{2})}}\delta_{K_fK_\nu}C^{I}_{K_\nu K_i}\frac{P_{K_fK_\nu}\langle\mathcal{F}^{(\pi^{b_f})}_{K_f}|\mathcal{F}^{(\pi^{b_i})}_{K_\nu}\rangle}{N_{K_f}N_{K_\nu}}$$
$$+C^{I_fK_i}_{I_iK_i\lambda0}\sum_{\substack{\nu\neq f\\(K_\nu=K_f\pm1,\frac{1}{2})}}\delta_{K_\nu K_i}C^{I}_{K_\nu K_f}\frac{P_{K_\nu K_i}\langle\mathcal{F}^{(\pi^{b_f})}_{K_\nu}|\mathcal{F}^{(\pi^{b_i})}_{K_i}\rangle}{N_{K\nu}N_{K_i}}$$
$$+\sum_{\substack{\nu''\neq f\\(K_{\nu''}=K_f\pm1,\frac{1}{2})}}\sum_{\substack{\nu'\neq i\\(K_{\nu'}=K_i\pm1,\frac{1}{2})}}\delta_{K_{\nu''}K_{\nu'}}C^{I_fK_{\nu''}}_{I_iK_{\nu'}\lambda K_{\nu''}-K_{\nu'}}$$
$$\times C^{I}_{K_{\nu''}K_f}C^{I}_{K_{\nu'}K_i}\frac{P_{K_{\nu''}K_{\nu'}}\langle\mathcal{F}^{(\pi^{b_f})}_{K_{\nu''}}|\mathcal{F}^{(\pi^{b_i})}_{K_{\nu'}}\rangle}{N_{K_{\nu''}}N_{K_{\nu'}}}\Bigg]^2. \quad (42)$$

In the good parity case $\beta_3=0$ one has

$$B(E\lambda;\pi^{b_i}n_ik_iI_iK_i\to\pi^{b_f}n_fk_fI_fK_f)=\frac{1}{4}\frac{2\lambda+1}{4\pi(4-3\delta_{\lambda,1})}$$
$$\times R^2_\lambda(\pi^{b_i}n_ik_iI_i\to\pi^{b_f}n_fk_fI_f)\delta_{\pi^{b_f}\pi^{b_i}}\left[1+\pi_f\pi_i(-1)^\lambda\right]^2$$
$$\times\frac{1}{\widetilde{N}^2_{K_f}\widetilde{N}^2_{K_i}}\Bigg[\delta_{K_fK_i}\delta_{b_fb_i}C^{I_fK_f}_{I_iK_i\lambda0}+C^{I_fK_f}_{I_iK_f\lambda0}C^{I}_{K_fK_i}+C^{I_fK_i}_{I_iK_i\lambda0}C^{I}_{K_iK_f}$$
$$+\sum_{\substack{\nu\neq f,i\\(K_\nu=K_f\pm1,K_i\pm1,\frac{1}{2})}}C^{I_fK_\nu}_{I_iK_\nu\lambda0}C^{I}_{K_\nu K_f}C^{I}_{K_\nu K_i}\Bigg]^2. \quad (43)$$

It is seen that the term on the third line of Eq. (42) and the first term in the third line of (43) correspond to the unperturbed parts of the wave functions. However, the terms on the fourth and fifth lines of (42) and the second and third terms on the third line of (43) provide a first order K mixing effect connecting model states whose doublet bandheads differ by $\Delta K=|K_f-K_i|=1$. Further, the last terms in (42) and (43) involve a second order mixing effect connecting states with $\Delta K=2$. Therefore, the transition probability expression (42) provides a possibility for the description of various kinds of transition probabilities within and between different quasiparity doublets in the spectra of odd-mass

nuclei. Thus, the formalism developed in the present work provides a complete collective (CQOM) plus microscopic (DSM+BCS) model framework for description of quadrupole-octupole spectra in odd-mass nuclei with a consistent and fully microscopic treatment of the Coriolis interaction.

5 Summary

In summary, a quadrupole-octupole core plus particle model scheme is proposed for description of yrast and non-yrast quasi parity-doublet spectra in odd-mass nuclei. The even-even core is considered in the CQOM model, while the odd particle is described within the reflection-asymmetric DSM with pairing. A Coriolis mixed particle-core coupling wave function providing a good parity of the total system is constructed. Within this scheme the yrast and the non-yrast quasi-doublet states appear as collective quadrupole-octupole rotation-vibration modes coupled to the ground and some higher quasi-particle states. The approach allows a fully microscopic calculation of the Coriolis decoupling/K-mixing effects in the quasi parity-doublet spectra and the attendant intra and inter-doublet transition probabilities. The test calculations and the systematic application of the approach in different regions of odd-mass nuclei is the subject of further work.

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