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Superscaling Analysis and Neutrino-Induced Charged-Current Pion Production at MiniBooNE Kinematics

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Abstract. Superscaling approximation (SuSA) predictions to neutrino-induced charged-current pion production in the Δ-resonance region are explored under MiniBooNE experimental conditions. The results obtained within SuSA for the flux-averaged double-differential cross sections of the pion production for the $\nu_\mu + \mathrm{CH}_2$ reaction as a function of the muon kinetic energy and of the scattering angle, the cross sections averaged over the angle, the total cross section for the pion production, as well as $\mathrm{CC}1\pi^+$ to CCQE cross section ratio are compared with the corresponding MiniBooNE experimental data. The SuSA charged-current π^+ predictions are in good agreement with data on neutrino flux average cross-sections, but a somewhat different dependence on the neutrino energy (for charged-current π^+ and π^0 production) is predicted than the one resulting from the experimental analysis.

The studies of the neutrino oscillations are of particular importance being related to the information on the limits of the Standard Model. In most neutrino experiments, the interactions of the neutrinos occur with nucleons bound in nuclei. The influence of nucleon-nucleon interactions on the response of nuclei to neutrino probes must then be considered, ideally in a model independent way. Model predictions for these reactions involve many different effects such as nuclear correlations, interactions in the final state, possible modification of the nucleon properties inside the nuclear medium, that presently cannot be computed in an unambiguous and precise way. This is particularly true for the channels where neutrino interactions take place by means of excitation of a nucleon resonance and ulterior production of mesons. The data set of neutrino-induced charged-current (CC) charged and neutral pion production cross sections on mineral oil recently released by the MiniBooNE collaboration [1] provides an unprecedented opportunity to carry out a systematic study of double differential cross section of the processes:

$$\nu(k) + p(p) \to \mu^-(k') \quad + \quad \Delta^{++}(p') \\ \hookrightarrow p(p'') + \pi^+(p_\pi). \tag{1}$$

or

$$\nu(k) + n(p) \to \mu^-(k') \quad + \quad \Delta^{+}(p') \\ \hookrightarrow n(p'') + \pi^+(p_\pi) \tag{2}$$

or

$$\hookrightarrow p(p'') + \pi^0(p_\pi). \tag{3}$$

averaged over the neutrino flux (k and p being the corresponding four-momentum). The momentum transfer will be denoted by $q = k - k'$.

The extensive analyses of scaling [2–4] and superscaling [5–10] phenomena observed in electron-nucleus scattering lead to the use of the scaling function directly extracted from (e, e') data to predict neutrino (antineutrino)-nucleus cross sections [11], (not relying on a particular nuclear structure model). Within SuSA a "*superscaling function*" $f(\psi)$ is built by factoring-out the single-nucleon content off the double-differential cross section and plotting the remaining nuclear response versus a scaling variable $\psi(q, \omega)$. Approximate scaling of the first kind, *i.e.,* no explicit dependence of $f(\psi)$ on the momentum transfer q, can be seen at transfer energies below the quasielastic (QE) peak. Scaling of second kind,*i.e.,* no dependence of $f(\psi)$ on the mass number, turns out to be excellent in the same region. When scaling of both first and second types occur, one says that superscaling takes place.

The analyses of the world data on inclusive electron-nucleus scattering [7] confirmed the observation of superscaling and thus justified the extraction of a universal nuclear response to be also used for weak interacting probes. However, while there is a number of theoretical models that exhibit superscaling, such as for instance the relativistic Fermi gas (RFG) [5, 6], the nuclear response they predict departs from the one derived from the experimental data. The point is that the scaling function in the RFG model is $f^{\rm QE}_{\rm RFG}(\psi) = 0$ for $\psi \leq -1$, whereas the experimental scaling function extracted from (e, e') data extends to large negative values of the scaling variable ψ up to $\psi \approx -2$ where effects beyond the mean-field approximation are important. This showed the necessity to consider more complex dynamical pictures of finite nuclear systems -beyond the RFG- in order to describe the nuclear response at intermediate energies.

The SuSA model is based on the phenomenological superscaling function extracted from the world data on quasielastic electron scattering [12]. The model has been extended to the Δ-resonance region [11] and to neutral current scattering [13]. It has been already employed to describe the non pionic

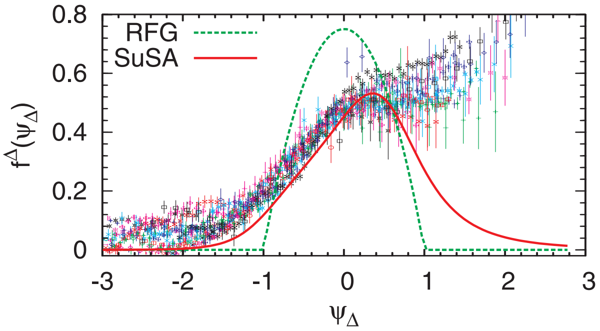

Figure 1. The SuSA scaling function in the Δ-region $f^{\Delta}(\psi_{\Delta})$ (solid line) extracted from the world data on electron scattering [11]. The dotted line shows the scaling functions $f^{\Delta}(\psi_{\Delta})$ in the RFG model.

(QE) cross-section of the MiniBooNE ν- and $\overline{\nu}$-nucleus cross-section [14–16] and in Ref. [17] it has been applied to neutrino (antineutrino) CCQE on ^{12}C for energy range up to 100 GeV with a comparison with the MiniBooNE and NOMAD [18] data. Here and in [19] we extend the analysis to CC pion production cross-section measured at MiniBooNE. As a reference, we also show results obtained within the RFG where the scaling function in the Δ-domain is simply given as

$$f^{\Delta}_{RFG}(\psi_{\Delta}) = \frac{3}{4}(1-{\psi_{\Delta}}^2)\theta(1-{\psi_{\Delta}}^2), \tag{4}$$

with ψ_{Δ} the dimensionless scaling variable extracted from the RFG analysis that incorporates the typical momentum scale for the selected nucleus [8, 11].

In Figure 1 we compare the Δ-region SuSA [11] and RFG scaling functions, which we use in our study. We present the results of applying the SuSA and RFG Δ-scaling function to neutrino-induced CC charged pion production. We follow the formalism given in [11]. The charged current neutrino cross section in the target laboratory frame is given in the form

$$\frac{d^2\sigma}{d\Omega dk'} = \frac{(G\cos\theta_c k')^2}{2\pi^2}\left(1-\frac{|Q^2|}{4\epsilon\epsilon'}\right)\mathcal{F}^2 \tag{5}$$

where Ω, k' and ϵ' are the scattering angle, momentum and energy of the outgoing muon, G is the Fermi constant and θ_c is the Cabibbo angle. The function $\mathcal{F}^2$ depends on the nuclear structure through the R responses and can be written as [11, 20]:

$$\mathcal{F}^2 = \widehat{V}_{\rm CC}R_{\rm CC} + 2\widehat{V}_{\rm CL}R_{\rm CL} + \widehat{V}_{\rm LL}R_{\rm LL} + \widehat{V}_{\rm T}R_{\rm T} + 2\widehat{V}_{\rm T'}R_{\rm T'}$$

that is, as a generalized Rosenbluth decomposition having charge-charge (CC), charge-longitudinal (CL), longitudinal-longitudinal (LL) and two types of transverse (T,T′) responses (R's) with the corresponding leptonic kinematical factors

(V's). The nuclear response functions in Δ-region are expressed in terms of the nuclear tensor $W^{\mu\nu}$ in the corresponding region. The basic expressions used to calculate the single-nucleon cross sections are given in [11]. These involve the leptonic and hadronic tensors as well as the response and structure functions for single nucleons. A convenient parametrization of the single-nucleon $W^+n \to \Delta^+$ vertex is given in terms of eight form-factors: four vector ($C^V_{3,4,5,6}$) and four axial ($C^A_{3,4,5,6}$) ones. Vector form factors have been determined from the analysis of photo and electro-production data, mostly on a deuteron target. Among the axial form factors, the most important contribution comes from C^A_5. The factor C^A_6, whose contribution to the differential cross section vanishes for massless leptons, can be related to C^A_5 by PCAC. Since there are no other theoretical constraints for $C^A_{3,4,5}(q^2)$, they have to be fitted to data. We use two different parameterizations: the one given in [21] where deuteron effects were evaluated, denoted as "PR1", and the one from [22], called "PR2".

With these ingredients, we evaluate the cross section for CC Δ^{++} and Δ^+ production on proton and neutron, respectively. Once produced, the Δ decays into πN pairs. For the amplitudes $\mathcal{A}$ of pion production the following isospin decomposition applies:

$$\begin{aligned}
\mathcal{A}(\nu_l\, p \to l^- p\, \pi^+) &= \mathcal{A}_3, \\
\mathcal{A}(\nu_l\, n \to l^- n\, \pi^+) &= \frac{1}{3}\mathcal{A}_3 + \frac{2\sqrt{2}}{3}\mathcal{A}_1, \\
\mathcal{A}(\nu_l\, n \to l^- p\, \pi^0) &= -\frac{\sqrt{2}}{3}\mathcal{A}_3 + \frac{2}{3}\mathcal{A}_1,
\end{aligned}$$

with $\mathcal{A}_3$ being the amplitude for the isospin $3/2$ state of the πN system, predominantly Δ, and $\mathcal{A}_1$ the amplitude for the isospin $1/2$ state that is not considered here.

First we present the double-differential cross section for π^+ production from Δ resonance region of neutrino-induced CC ν_μ–CH_2 reaction averaged over the neutrino flux $\Phi(\epsilon_\nu)$, namely

$$\frac{d^2\sigma}{dT_\mu d\cos\theta} = \frac{1}{\Phi_{\text{tot}}}\int \left[\frac{d^2\sigma}{dT_\mu d\cos\theta}\right]_{\epsilon_\nu} \Phi(\epsilon_\nu) d\epsilon_\nu, \qquad (6)$$

where T_μ and θ are correspondingly the kinetic energy and scattering angle of the outgoing muon, ϵ_ν is the neutrino energy and Φ_{tot} is the total integrated ν_μ flux factor for the MiniBooNE experiment ($\Phi_{\text{tot}} = 5.19\times 10^{-10}$ [ν_μ/cm^2/POT]). The double-differential cross section averaged over the neutrino energy flux as a function of the muon kinetic energy T_μ is presented in Figure 2. Each panel corresponds to a bin of $\cos\theta$. The "PR2" parametrization has been considered. Results with the PR1 parameterization are about 5% higher. We compare the predictions of SuSA and RFG with the MiniBooNE data [1]. The nuclear target has been considered as carbon and hydrogen in the mineral oil target. Figure 2

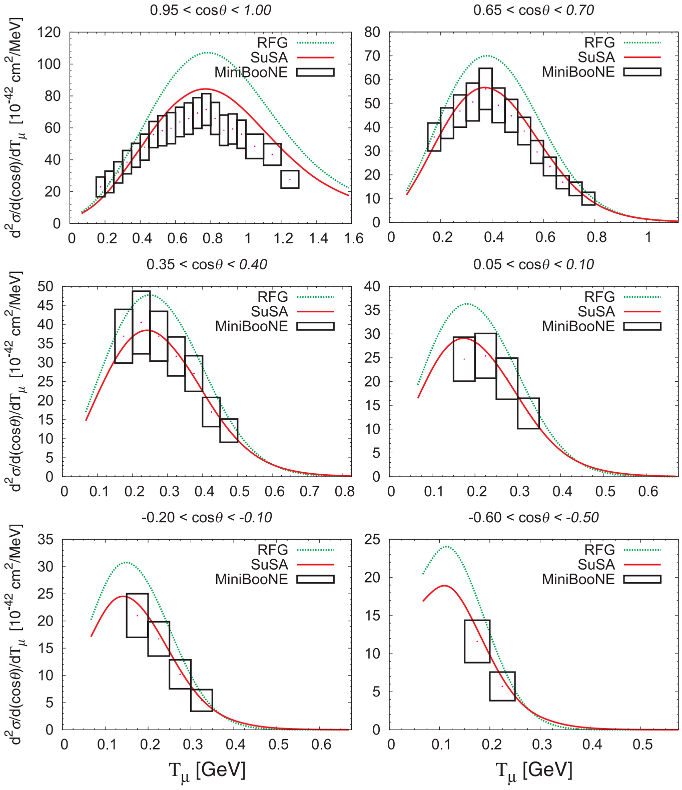

Figure 2. The double-differential cross section averaged over the neutrino energy flux as a function of the muon kinetic energy T_μ obtained by SuSA (solid line) and RFG (dotted line) Δ-region scaling functions. In each subfigure the results have been averaged over the corresponding angular bin of $\cos\theta$. "PR2" parametrization [22] is used. The results are compared with the MiniBooNE data [1].

shows a good agreement between data and the SuSA predictions for the flux-averaged double-differential cross sections. This applies to both parameterizations of the vector and axial form factors. As expected, RFG results have similar shape as SuSA ones, but they overestimate the data to a large extent.

In Figure 3 are shown the results for π^+ and π^0 production obtained by integrating the flux-averaged double-differential cross sections over angle:

$$\left\langle \frac{d\sigma}{dT_\mu} \right\rangle = \frac{1}{\Phi_{\text{tot}}} \int \Phi(\epsilon_\nu) \int \left(\frac{d^2\sigma}{dT_\mu d\cos\theta} \right)_{\epsilon_\nu} d(\cos\theta) d\epsilon_\nu. \tag{7}$$

The total cross section for π^+ and π^0 production as a function of the neutrino energy along with the MiniBooNE data are displayed in Figure 4. Poorer agreement with data than for the flux-averaged cross sections presented in Figures 2

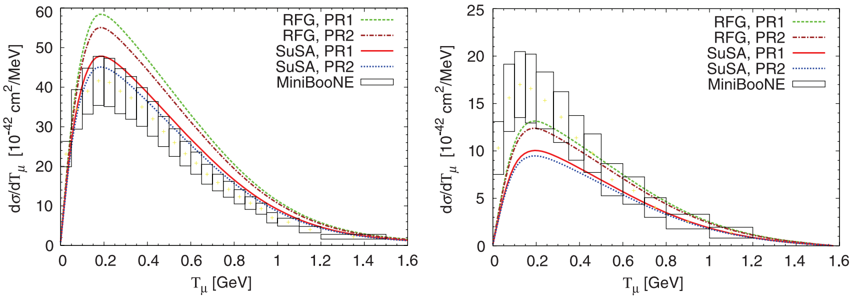

Figure 3. The $d\sigma/dT_\mu$ results for π^+ (left panel) and π^0 (right panel) production obtained by integrating the flux-averaged double-differential cross sections over $\cos\theta$ [Eqs. (7)] are compared with the MiniBooNE data [1]. For vector and axial form-factors two parameterizations, "PR1" [21] and "PR2" [22], are used.

and 3 is clearly observed in the case of π^+ production. The data seems to follow a more linear dependence with the energy up to 2 GeV than the theory. However, before drawing definite conclusions, one has to consider that the unfolding procedure used to extract the data of Figure 4 is model dependent, while the direct comparison with the data of Figures 2 and 3 is more significant. In the case of π^0 production the RFG results are closer to the experimental MiniBooNE data while the SuSA results clearly underpredict the data.

Figure 5 shows the ratio of CC1π^+ (CC single-pion production) to CCQE (CC quasielastic scattering) cross sections from SuSA, SuSA+MEC ($2p$–$2h$ meson–exchange current) [14], and RFG approaches in comparison with the MiniBooNE data corrected for final state interactions. All these ratios have been rescaled to an isoscalar target [23]. The results are obtained on the basis of total cross sections for CC1π^+ (given in Figure 4) and CCQE [14]. A similar conclusion as the one in the previous figure could be drawn here. It seems that there

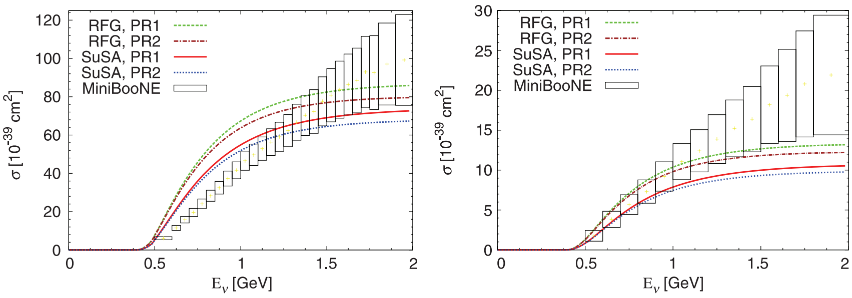

Figure 4. The total cross section for π^+ (left panel) and π^0 (right panel) production are compared with the MiniBooNE data [1]. For vector and axial form-factors two parameterizations, "PR1" [21] and "PR2" [22], are used.

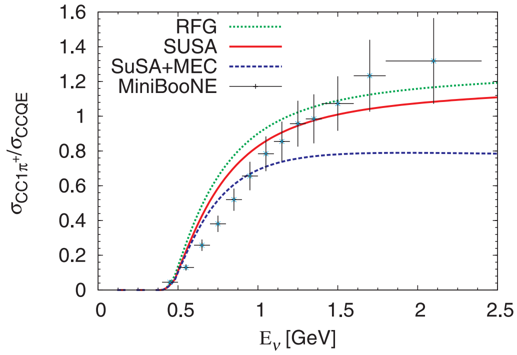

Figure 5. The results for $CC1\pi^+$ to CCQE cross section ratio are compared with MiniBooNE data (corrected for final state interactions and rescaled for an isoscalar target) [23].

is too much π^+ production strength below 1.2 GeV, and too little beyond that, compared to data.

Summarizing, in this work we present results for the cross sections of neutrino–induced CC π^+ and π^0 production obtained with the SuSA and RFG (shown as reference) models. The SuSA approach provides nuclear-model-independent neutrino-nucleus cross-section predictions, based on the observed nuclear response to electron projectile and the universal character of the scaling function. Notice that SuSA predictions incorporate effects of final state interaction (FSI), the properties of the Δ resonance in the nuclear medium, *etc.* The role of the FSI on the one-pion production has been considered for instance within the GIBUU transport model [24], where it was shown that in order to reproduce the data, the total π^+ cross section obtained with FSI included has to be multiplied by a factor of 1.5. SuSA predictions are in good agreement with the MiniBooNE experimental data for charged pionic cross-section in the case of the flux averaged data, while some disagreement remains in the comparison to unfolded neutrino energy data. Notice that the accordance between SuSA and data here is better than the one for the non-pionic case, where the model was found to underpredict the data unless meson exchange currents were explicitly included [14]. SuSA predictions underpredict the data in the case of neutral pion production, this result is in agreement with other theoretical calculations [25–27]. We conclude that the SuSA approach for the Δ-region (extracted from electron scattering experiments) and its extension to neutrino processes is very useful in predicting highly-model-independent cross sections for neutrino-induced CC π^+ production.

Acknowledgements

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