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On the Modified Two- and Three-Body Approaches, the Asymptotic Normalization Coefficients and Their Astrophysical Application

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Abstract. In the present review report, the modified two-body potential approaches in which the direct radiative capture astrophysical S factor for the $\mathrm{A}(a,\gamma)\mathrm{B}$ reaction is expressed in term of the asymptotic normalization coefficient for $\mathrm{A}+a\to\mathrm{B}$ as well as the modified three-body DWBA for the peripheral charged-particle transfer reaction $\mathrm{A}(x,y)\mathrm{B}$ ($x=y+a$ and $B=A+a$), are presented. The results of the analysis of the specific experimental astrophysical S factors for some nuclear astrophysical reactions of the pp-chain and the CNO cycle are discussed.

1 Introduction

It is well known that detailed information on a cross section $\sigma_{aA}(E)$ (or a respective astrophysical factor $S_{aA}(E)$) for the radiative capture $A(a,\gamma)B$ reaction at extremely low is essential for such problems as the solar neutrino flux as well as the abundance of light elements and isotopes in Universe. However, up to now the problem of a reliable extrapolation of measured astrophysical S factors to the experimentally inaccessible energy regions (≤ 25 keV) is not entirely solved.

One of the possible solutions of this problem is based on the idea that at stellar energies the amplitude of the direct capture $A(a,\gamma)B$ reaction of astrophysical interest proceeds through the tail of the overlap integral [1], and, hence, that is completely determined by the respective asymptotic normalization coefficient (ANC) [2], which determines the probability of the $(A+a)$-configuration in nucleus B at distances greater than the radius of two-body nuclear $\mathrm{A}a$-interaction (see, e.g., [3-6] and references therein).

In this review work, we will briefly present the main methods of determination of ANC for $A+a\to B$ (see, e.g., [3-6] and references therein) and their application for the extrapolation of the astrophysical S factors to the solar energy region for some specific radiative capture reactions.

2 Methods of Determination of ANCs

2.1 ANCs from proton transfer reactions

One of such methods uses the DWBA approach for nuclear reactions of manifest peripheral character. The DWBA cross section for reaction $\mathrm{A(x,y)B}$ with particle a transferring can be written in the following form [7-9]

$$\frac{d\sigma^{\rm DWBA}}{d\Omega}(E_i,\theta)=\mathcal{R}^2\sum_{j_xj_B}C^2_{ya;\,l_xj_x}C^2_{Aa;\,l_Bj_B} \times\frac{\sigma^{\rm DWBA}_{l_xj_xl_Bj_B}(E_i,\theta;\,C^{(sp)}_{ya;\,l_xj_x},C^{(sp)}_{Aa;\,l_Bj_B})}{(C^{(sp)}_{ya;\,l_xj_x})^2(C^{(sp)}_{Aa;\,l_Bj_B})^2} \qquad (1)$$

where the single-particle differential cross section $\sigma^{\rm DWBA}_{l_xj_xl_Bj_B}$ is calculated in the "post"-approximation of DWBA. Here, $C^2_{ya;\,l_xj_x}$ and $C^2_{Aa;\,l_Bj_B}$ are the ANCs for $y+a\to x$ and $A+a\to B$, which determine the amplitude of the tail of the overlap functions for the nuclei x and B in the $(y+a)$- and $(A+a)$-configurations [2], respectively, $C^{(sp)}_{ya;\,l_xj_x}$ ($C^{(sp)}_{Aa;\,l_Bj_B}$) is the single particle ANC, which determines the amplitude of the tail of the shell model wave function of the bound $x\,(=y+a)(B(=A+a))$ state, l_x and j_x (l_B and j_B) are the orbital and total angular momentums of the particle a in the nucleus $x=(y+a)(B=(A+a))$, E_i is the relative kinetic energy of the colliding particles and θ is the scattering energy in c.m.s. In (1), the factor $\mathcal{R}^2$ is the Coulomb renormalization factor arising due to the correct taking into account of the three-body Coulomb dynamics in the transfer mechanism in the DWBA amplitude [10, 11]. If the reaction under consideration is peripheral the ratio in the r.h.s. of Eq. (1) is independent on $C^{(sp)}_{ya;\,l_xj_x}$ and $C^{(sp)}_{Aa;\,l_Bj_B}$ [7, 8]. In this case, the expression (1) can be applied for determination of the ANC for $A+a\to B$ if the ANC value for $y+a\to x$ is known and the $d\sigma^{\rm DWBA}(E_i,\theta)/d\Omega$ in the left hand site of (1) is replaced by its experimental data near the stripping peak of the angular distribution [7–9].

2.1.1 ANC for $^{14}\mathrm{N}+p\to{}^{15}\mathrm{O}$

In Table 1, the weighted ANC means for $^{14}\mathrm{N}+p\to{}^{15}\mathrm{O}$, the values of the Coulomb renormalization factor $\mathcal{R}^2$ are presented, which were found in [12] from the analysis of the experimental differential cross sections for the peripheral $^{14}\mathrm{N}(^3\mathrm{He},d)^{15}\mathrm{O}$ reaction performed in [12–14] by using the expression (1). The intervals of change of the calculated values for $\mathcal{R}^2$ pointed out in Table 1 correspond to the dependence of $\mathcal{R}^2$ on energy of the incident ^{3}He-ions.

As is seen from Table 1, the contribution of the three-body Coulomb dynamics in the proton transfer mechanism to the DWBA cross sections ($\triangle\%$) is about up to 60%. It is seen that the values of $\triangle$ increase as the residual nucleus ^{15}O

Table 1. The Coulomb renormalization factors $\mathcal{R}^2$ and the weighted means of the ANCs for $^{14}\mathrm{N}+p\rightarrow{}^{15}\mathrm{O}$ corresponding to the ground and excited (E^*;J^π) states of the ^{15}O nucleus with the quantum number (l_B, j_B).

$A(x,y)B$ reaction and the energy E_x, MeV	E^* (MeV); J^π	l_B, j_B	$\mathcal{R}^2$	$C^2_{pA;\,l_B j_B}$ fm^{-1}
$^{14}\mathrm{N}(^3\mathrm{He},d)^{15}\mathrm{O}$	0.0;1/2$^-$	(1,1/2)+	1.278	45.4±5.5 [12]
22.3 [12]; 20.0 [13];		(1,3/2)		63±14 [13]
26.3 [14]	5.183;1/2$^-$	0,1/2	1.531	0.065±0.020 [12]
				0.11±0.04 [13]
	5.241;5/2$^+$	1,5/2	1.601	0.069±0022 [12]
				0.12±0.03 [13]
	6.176;3/2$^-$	1,1/2	1.240-1.203	0.31±0.06 [12]
				0.46±0.10 [13]
		1,3/2		(1.21±0.24)x10^{-3} [12]
	6.793;3/2$^+$	0,1/2		23.0±3.0^a [12]
				21±5 [13]
		1,5/2		0.095±0.03 [12]
				0.084±0.019 [13]
	6.859;5/2$^+$	1,5/2		0.38±0.07 [12]
				0.36±0.08 [13]
	7.276;7/2$^+$	1,5/2		(2.4±0.5)x10^6 [12]
				(2.7±0.6)x10^6 [13]

^{a} obtained from the $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$ analysis performed within the MTBPA (see the text)

are formed in the weakly bound excited states being of astrophysical interest. As a comparison, the ANC values for $^{14}\mathrm{N}+p\rightarrow{}^{15}\mathrm{O}$ recommended in [13] are also presented in Table 1. Besides, as is seen from this table, the weighted ANC values recommended in [12] and [13] differ noticeably from each other. One of the main reason of this discrepancy is discussed in detail in [6].

The results of the analysis of the experimental data for the $^{14}\mathrm{N}(p,\gamma)^{15}\mathrm{O}$ radiative capture reaction using the ANC values recommended in [12] are presented below.

2.2 Coulomb breakup ^{208}Pb(^{6}Li,αd) ^{208}Pb reaction and the ANC for $\alpha+d\rightarrow{}^6$Li

The results of the ANC values for $\alpha+d\rightarrow{}^6\mathrm{Li}$ obtained within the different methods are presented in Refs. [5, 6]. One of them was obtained in [15] from the analysis of the experimental triple-differential cross section (TDCS) of the $^{208}\mathrm{Pb}(^6\mathrm{Li},\alpha d)^{208}\mathrm{Pb}$ Coulomb breakup reaction [16] performed with a correct taking into account contributions from the pure $E1$- and $E2$-multipoles as well as their interference. Note that in [16], the contribution of the $E1$-multipole was ignored assuming that this contribution is sufficiently unnoticeable.

As it follows from Refs. [5,6], the "indirect determined" ANC value $\alpha+d\rightarrow$

^{6}Li recommended in [4,15], which is equal to $C_{\alpha d;\,0\,1}$=5.24±0.51 fm^{-1}, is in an excellent agreement with the value of $C_{\alpha d:\,0\,1}$=5.24±0.26 fm^{-1} deduced in [17] by the extrapolation in energy E of the partial wave amplitudes of the elastic $d\alpha$-scattering using the phase shift analysis. Therefore, they can be considered as straightforward best "indirect determined" (empirical) value, since another the ANC value, $C_{\alpha d;\,0\,1}$=5.24±0.77 fm^{-1}, which was deduced in [18] from the analysis of the exchanged d^6Li-scattering performed within the dispersion peripheral model, has fairly large uncertainty.

The results of their application for the radiative capture $d(\alpha,\gamma)^6$Li reaction are presented below.

2.3 $\boldsymbol{R}$-matrix method for the radiative $\boldsymbol{A(a,\gamma)B}$ capture reaction and ANC for $\boldsymbol{A+a\to B}$

In the R-matrix method [19, 20], the space of interaction for the $A+a$ system is divided into two regions: the internal region ($0\le r\le r_c$), where nuclear forces are important, and external regions ($r_c<r<\infty$), where the interaction between the nuclei is governed by the Coulomb force only.

In the single-level R-matrix approximation, a contribution in the amplitude of the $A(a,\gamma)B$ reaction from the internal region is determined by a single resonance pole amplitude $M^{(R\lambda)}_{l_{B^*}s_{B^*}J_{B^*}IJ_B}(E)$, which corresponds to the mechanism $A+a\to B^*\to B+\gamma$ with spin J_{B^*} of the resonance level λ and are determined by energy $E^{(R)}_{\lambda}$, the channel radius r_c, ANC for $A+a\to B$, the partial widths for the particle a and γ-ray ($\Gamma^a_{\lambda;\,l_{B^*}s_{B^*}J_{B^*}}(E)$ and $\Gamma^{\gamma}_{\lambda;\,J_{B^*}IJ_B}(E)$). The direct amplitude $M^{(EI,M1)}_{l_is_BIJ_{B^*}J_B}(E)$ is determined only by the channel radius r_c and the ANC $C_{Aa;\,l_Bs_B}$, where s_B is the channel spin [2].

The total radiative capture capture amplitude reads as

$$M_{l_il_Bs_BIJ_{B^*}J_B}(E)=M^{(R\lambda)}_{l_Bs_BJ_{B^*}IJ_B}(E)+M^{(EI,M1)}_{l_is_BIJ_{B^*}J_B}. \tag{2}$$

Then the R-matrix radiative $A(p,\gamma)B$ cross section for transition to the state of the nucleus B with spin J_B at fixed angular momentum l_B is given by [20]

$$\sigma_{Aa;\,l_BJ_B}(E)=\sum_{J_{B^*}}\sigma_{l_BJ_{B^*}J_B}(E)=\frac{\pi}{k^2}\frac{2J_{B^*}+1}{(2J_a+1)(2J_A+1)}\times\sum_{J_{B^*}Is_Bl_i}\mid M_{l_il_Bs_BIJ_{B^*}J_B}(E)\mid^2. \tag{3}$$

The astrophysical S factor $S_{aA}(E)$ for the direct radiative capture $A(a,\gamma)B$ reaction is defined by the relation

$$S_{aA}(E)=Ee^{2\pi\eta}\sigma_{aA}(E), \tag{4}$$

where η is the Coulomb parameter corresponding to Aa-scattering.

2.3.1 The ^{14}N$(p,\gamma)^{15}$O reaction

The experimental astrophysical S factors of this reaction were determined by directly measuring at energies $E \gtrsim 200$ KeV in [21] and from $E \geq 70.1$ keV in [22]. The analysis of these experimental data performed in Refs. [14, 23] within the modified R-matrix approach showed that the contribution of the subthreshold (6.793 MeV; $J^{\pi} = 3/2^{+}$) state of ^{15}O to direct radiative capture is dominant. Nevertheless, the $S_{1\,14}(0)$ values for the ^{14}N$(p,\gamma)^{15}$O reaction populating the ground and excited bound states of the ^{15}O nucleus, which have been recommended in [14] and [23], differ noticeably from each other.

To find out the true reason of this discrepancy, in Ref. [12], the new reanalysis of the experimental astrophysical S factors $S_{1\,14}(E)$ is done within the modified R-matrix approach. The resulting improved ANC values for ^{14}N$+p \to {}^{15}$O recommended in [12] (see the last column of Table 1), which have really been compiled from all the results of Refs. [12–14], were used in [12].

Here, the results of the analysis performed in [12] within the single-level R-matrix for new experimental data of the astrophysical S factor of the reaction ^{14}N$(p,\gamma)^{15}$O are presented. The analysis of the available experimental data [21, 22, 24, 25] near the first resonance energy region was performed using the "indirect determined" ANC values for ^{14}N $+ p \to {}^{15}$O from Table 1. The results for the $S_{1\,14}(0)$ obtained in [12] and their comparison with those recommended in other works are presented in Table 2. From here, one can see that the $S_{1\,14}(0)$ values obtained by different authors agree well within errors with each other, with exception of the result of 3.04 ± 0.49 keV b [13]. Besides, the $S_{1\,14}(0)$ value of $S_{1\,14}(0) = 1.26 \pm 0.17$ keV b [12], which corresponds to the transition to the subthreshold excited ($E^{*} = 6.793$ MeV) state of ^{15}O, within its uncertainty is in an agreement with that of $S_{1\,14}(0) = 1.47$ keV b [26]. Whereas, as it is mentioned above, the summarized $S_{1\,14}(0)$ recommended in [12] differs strongly from that deduced in [13]. Apparently, this is associated with the observed discrepancy between the ANC values used in [12] and [13] for calculation of the $S_{1\,14}(0)$ (see Table 1).

Table 2. Values of the astrophysical S factors $S_{1\,14}(0)$ (keV b) for the ^{14}N$(p,\gamma)^{15}$O reaction.

E_x (keV)	[12]	[24]	[25]	[14]	[23]	[13]
0	0.25±0.05	0.25±0.06	0.49±0.08	0.15±0.07	$0.08^{+0.13}_{-0.05}$	1.67±0.40
5183	0.033±0.025	0.010±0.003				0.0023±0.0008
5241	0.066±0.025	0.070±0.003				0.0110±0.0026
6173	0.117±0.037	0.08± 0.03	0.04±0.01	0.13±0.02	$0.06^{+0.01}_{-0.02}$	0.138±0.033
6793	1.26±0.17	1.20±0.05	1.15±0.05	1.40±0.20	1.63±0.17	1.17±0.28
6859	0.042±0.007*			0.03±0.04		0.0349±0.0084
7276	0.019±0.004*					0.0186±0.0045
summarized	1.79±0.31	1.61±0.08	1.68±0.09	1.70±0.22	1.77±0.20	3.04±0.49

* The uncertainty in $S^{exp}(E)$ was not included to the total error.

2.3.2 The $d(\alpha,\gamma)^6$Li reaction

Direct measurements of the experimental astrophysical S factor $S_{42}^{exp}(E)$ cover the energy region $E \gtrsim 700$ keV [27, 28] presented in Figure 1 by the open triangle and star symbols. In [16], the $S_{42}^{exp}(E)$ were determined in the energy region 100$\lesssim E \lesssim$ 600 keV from the analysis of the experimental TDCS $^{208}\mathrm{Pb}(^6\mathrm{Li},\alpha d)^{208}\mathrm{Pb}$ Coulomb breakup (see, the open cycle symbols in Figure 1) with taking into account only the contribution of the $E2$-multipole. In [15], the aforementioned ANC value for $\alpha + d \to {}^6$Li, which was deduced in [4, 15] from the same experimental TDCS $^{208}\mathrm{Pb}(^6\mathrm{Li},\alpha d)^{208}\mathrm{Pb}$ analysis, were utilized for obtaining the new information about the correct $S_{42}^{exp}(E)$-values with taking into account the $E1$- and $E2$-multipoles in the energy region $E \leq 250$ keV(see the filled triangle symbols in Figure 1).

As is seen from Figure 1, the rather large uncertainties (from 26% to 50%) for the $S_{42}^{exp}(E)$ occur for the energy region $E \leq 190$ keV. This is connected with the uncertainties for the $C^2_{\alpha d;\,0\,1}$ values [4], which are conditioned by the large uncertainties in the measured TDCS [16]. Nevertheless, the $S_{42}^{exp}(E)$ data of [16], deduced from the same experimental TDCS but without taking into

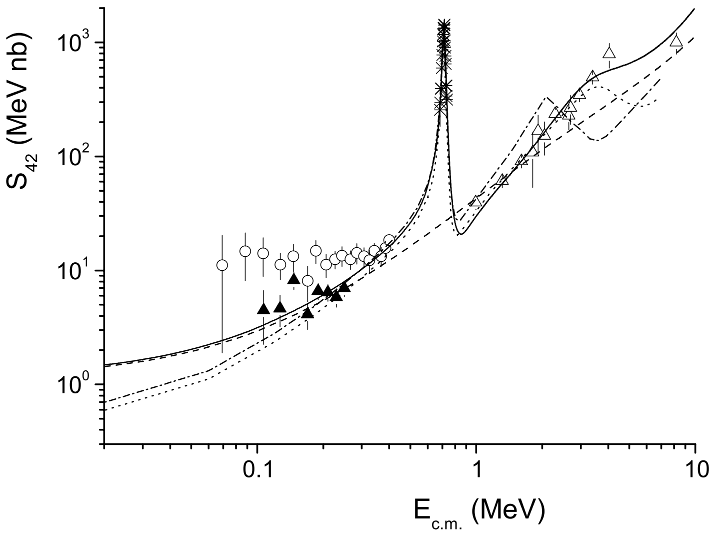

Figure 1. The $d(\alpha,\gamma)^6$Li astrophysical S factor. The experimental data are taken from [27](open triangle symbols), [28](star symbols), [16](open cycle symbols) and [15](blacked triangle symbols). The solid (dashed) lines are the result obtained in [4] (only for the direct radiative captures). The dotted (dash-dotted) line is the results taken from [29] for the MN (V2) form of the NN potential.

account the contribution of the $E1$-multipole, are in reality overestimated in the energy region $E \leq 250$ keV. Besides, the resulting $S_{42}(0)$ recommended in [15, 30, 31], which is equal to 1.30 MeV nb, differs also considerably on $S_{42}(0) = 9.10 \pm 1.80$ MeV nb recommended [16]. It should be noted that the new data for $S_{42}^{exp}(E)$ obtained in [15] are in a good agreement with the theoretical predictions [30,31], which can also be obtained using the ANC value for $\alpha + d \to {}^6\mathrm{Li}$ recommended in [17]. Therefore, the $S_{42}^{exp}(E)$ values at $E \leq$ 250 keV obtained in [4,15] could so far be considered as the "best values" of the experimental $S_{42}^{exp}(E)$.

Nevertheless, we recommend new decisive direct measurements of the astrophysical S factors for the direct $d(\alpha,\gamma)^6\mathrm{Li}$ reaction at $E \leq 700$ keV or the inverse ${}^6\mathrm{Li}(\gamma,d)\alpha$ reaction at near threshold energies E_γ. It would allow one to find out the true reason of the aforementioned drop of $S_{42}^{exp}(E)$ at $E \lesssim 400$ keV recommended in Refs. [15] and [16].

2.4 The direct radiative capture $\boldsymbol{A(a,\gamma)B}$ reaction and the ANC for A+ $\boldsymbol{a} \to$ B

In work [32], the modified two-body potential approach (MTBPA) was proposed for an analysis of the precisely measured astrophysical S factors for the direct radiative capture $A(a,\gamma)B$ reaction. According to [32], the astrophysical S factor has the form

$$S_{aA}(E) = (\sum_{j_B} C^2_{Aa;\,l_B j_B})\mathcal{R}_{l_B}(E, C^{(sp)}_{Aa;\,l_B j_B}). \tag{5}$$

where $\mathcal{R}_{l_B}(E, C^{(sp)}_{Aa;\,l_B j_B}) = \tilde{S}_{l_B j_B}(E)/(C^{(sp)}_{Aa;\,l_B j_B})^2$, $\tilde{S}_{l_B j_B}(E) = \sum_\lambda \tilde{S}_{l_B j_B \lambda}$ is the single-particle astrophysical S factor and λ is the multipolarity of the electromagnetic transition. If the reaction under consideration is peripheral, the $\mathcal{R}_{l_B}(E, C^{(sp)}_{Aa;\,l_B j_B})$ function does not depend practically on the model parameter $C^{(sp)}_{Aa;\,l_B j_B}$ [32]. Therefore, the expression (5) can be used for finding the ANC $C^2_{Aa;\,l_B j_B}$ if the $S_{aA}(E)$ in the left hand side of Eq. (5) is replaced by the experimental $S_{aA}^{exp}(E)$.

2.4.1 ANCs for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$ and ${}^7\mathrm{Be} + p \to {}^8\mathrm{B}$ and the ${}^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and ${}^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reactions

The direct radiative capture ${}^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and ${}^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reactions are one of the main links in the *pp* chain of solar hydrogen burning (see, the review work [33] and the references therein). Their rates at a stellar temperature $T_6 \sim$ 15 K ($E_G = 3.68 T_6^{2/3}$ keV [34] $\sim$ 22 keV for the ${}^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction and $E_G = 2.94 T_6^{2/3}$ keV [34] $\sim$ 18 keV for the ${}^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction) determine how much the ${}^7\mathrm{Be}$ and ${}^8\mathrm{B}$ branches of the *pp* chain contribute to solar hydrogen burning. In the standard solar model, the predicted flux of solar neutrinos is given by the relation [35]

$$\phi_\nu \sim S_{11}^{-2.5}(0)S_{33}^{-0.3}(0)S_{43}^{0.8}(0)[1+3.5S_{17}(0)\tau_{e7}] \qquad (6)$$

and it depends noticeably on the flux of the $^7\mathrm{Be}$ and $^8\mathrm{B}$ neutrinos, where τ_{e7} is the $^7\mathrm{Be}$ electron capture ($^7\mathrm{Be}+e^- \to {}^7\mathrm{Li}+\nu_e$) lifetime. As is seen from Eq.(6), the solar neutrino flux ϕ_ν is mainly determined by the accuracy of the astrophysical S factors of the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reactions at experimentally inaccessible solar energy E=0. Therefore, in (6), the extrapolated astrophysical S factors $S_{43}(0)$ and $S_{17}(0)$ must be known to $\le \pm$ 5% [35] so that their uncertainties not be dominant for obtaining the precise values of the predicted solar neutrino fluxes [33, 35].

At present, although the precisely measured experimental S factors for the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reactions at extremely low energies are available (see, Ref. [33] and the references therein), some ambiguities associated with the prediction for $S_{43}(0)$ and $S_{17}(0)$ [33].

Therefore, in works [36] and [37], to determine the ANC values for $\alpha+{}^3\mathrm{He}\to{}^7\mathrm{Be}\,(E^*=0.0\,\mathrm{MeV};\,J^\pi=3/2^-)$, $\alpha+{}^3\mathrm{He}\to{}^7\mathrm{Be}\,(E^*=0.429\,\mathrm{MeV};\,J^\pi=1/2^-)$ and $p+{}^7\mathrm{Be}\to{}^8\mathrm{B}$ the new analysis has been performed within the MTBPA for the aforementioned precisely measured experimental astrophysical S factors for the direct capture $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reactions. The scrupulous analysis, performed in [36] and [37], has quantitatively shown that these reactions are strongly peripheral. Therefore, the expression (5) can be used for determination of the aforementioned ANCs since the calculated function $\mathcal{R}_{l_B}(E,C^{(sp)}_{Aa;\,l_Bj_B})$ does not depend from the free parameter $C^{(sp)}_{Aa;\,l_Bj_B}$.

In [36] and [37], the new estimations have been obtained for the ANC values for $\alpha+{}^3\mathrm{He}\to{}^7\mathrm{Be}$(g.s.), $\alpha+{}^3\mathrm{He}\to{}^7\mathrm{Be}$ (0.429 MeV) and $p+{}^7\mathrm{Be}\to{}^8\mathrm{B}$, respectively, which have then been used for extrapolating the $S_{43}(E)$ and $S_{17}(E)$ to solar energies, including $E=0$. The weighed means for square of the ANCs recommended in [36, 37] for $\alpha+{}^3\mathrm{He}\to{}^7\mathrm{Be}$ (g.s.), $\alpha+{}^3\mathrm{He}\to{}^7\mathrm{Be}$ (0.429 MeV) and $p+{}^7\mathrm{Be}\to{}^8\mathrm{B}$ are equal to 23.3 ± 1.7, 15.9 ± 1.1 [36] and 0.628 ± 0.017 fm^{-1} [37], respectively. These weighed ANC means allowed to estimate the $S_{43}(0)$ and $S_{17}(0)$ values, which are equal to 0.613 ± 0.045 kev b [36] and 23.4 ± 0.6 eV b [37], respectively. One notes that these results for $S_{43}(0)$ and $S_{17}(0)$ are more (about of $\pm9.5\%$ and $\pm30\%$) than those recommended in [33] and [38], which were determined by the model dependent ways [36, 37, 39].

Therefore, since the astrophysical S factors $S_{43}(0)$ and $S_{17}(0)$ must be known to $\pm$ 3–5% [33,40] in order that their uncertainties not be the dominant error in prediction of the solar neutrino flux [35,41], the $S_{43}(0)$ and $S_{17}(0)$-values recommended in [36, 37], which have the uncertainties about of $\pm7\%$ and $\pm3\%$, respectively, could be used as the main input data for a correct estimation of the solar neutrino flux.

Nevertheless, we recommend the new decisive precise measurement of $S_{43}(E)$ for the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction at $E\le600$ keV (or for the inverse $^7\mathrm{Be}(\gamma,\alpha)^3\mathrm{He}$

reaction at $E_\gamma \leq 2.5$ MeV) as the absolute values of the available experimental data for $S_{43}^{\rm exp}(E)$ measured by different authors within the aforementioned energy region differ (about of 10%) from each other (e.g., see Refs. [33, 36] and references therein). It would make it possible to reduce the uncertainty of the ANC for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}$(g.s.) and, consequently, the uncertainty for $S_{43}(0)$ up to $\pm$ 3–5%.

3 Conclusion

Now we have at our disposal the different methods for determination of asymptotic normalization coefficients on the basis of the analysis of experimental data on nuclear processes such as the nuclear-astrophysical radiative capture reactions and the peripheral one-particle transfer reactions. The MTBPA and the R-matrix method are a good tool of both obtaining the valuable information about ANCs being astrophysical interest and the reliable extrapolation of the astrophysical S factor $S_{aA}(E)$ at stellar energies E with uncertainty not exceeding the experimental one. Besides, the MTBPA can be used as a tool of test of the reliability of the modified DWBA calculations widely used for determination of the ANC-values of astrophysical interest.

This indicates on the need for further systematic accumulation of information about the ANC values for different nuclei being astrophysical interest, which requires to have experimental data with fairly high precision ($\lesssim 5\%$) on the cross sections of Sub-barrier peripheral transfer reactions as well as of radiative capture and photodisintegration reactions. Obtaining such data are planned within the acting multilateral (Italy-Kazakhstan-Russia-Poland-Uzbekistan-USA) collaboration.

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