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Differential Cross Section of Elastic and Inelastic p^{15}N Scattering

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Abstract. The calculation of differential cross sections for elastic proton scattering at energies 0.6 and 1.0 GeV and inelastic proton scattering (at the level $J = 5/2^+$) at energies 0.415, 0.8 and 1.0 GeV from ^{15}N nucleus at E = 0.4, 0.8 and 1.0 GeV was made within the Glauber's diffraction theory. It is shown that in the approximation of double scattering using the shell wave function ^{15}N the amplitude of ^{15}N- process can be calculated analytically. The calculation of differential cross section in the optical limit, allows us taking into account the collisions with nucleons on different shells.

1 Introduction

^{15}N nucleus is stable, the number of its neutrons is just by one more than the number of the protons. Though its abundance in nature is just 0.36%, it plays an important role in CNO-cycle, being a basis of the nucleosynthesis of ^{12}C, ^{16}O and ^{4}He. Therefore, in literature, the processes involving ^{15}N are mainly reviewed at low (astrophysical) energies [1].

The various characteristics of the ^{15}N(p,γ)^{16}O, ^{15}N(p,α)^{12}C reactions are calculated, which form the branch point of the CNO-cycle [2, 3]. Thus, definition of the reaction correlation parameters is necessary in order to simulate the process of energy production in stars, and nucleosynthesis of carbon, nitrogen and oxygen isotopes in the process of hydrogen burning in stars.

Here we consider the elastic and inelastic scattering of protons from ^{15}N nucleus at higher energies (from 0.4 to 1.0 GeV). Therefore we use the Glauber diffraction theory [4], which describes the proton scattering at the intermediate energies in the most appropriate way. The Glauber theory is attractive because it allows us to separate the structural (depending on the wave function (WF) of the target nucleus) and dynamic (depending on the operator of multiple scattering) components of the scattering amplitude.

The input parameters of the theory are the WF of the target and the elementary nucleon-nucleon amplitude. Parameters of the elementary nucleon-nucleon amplitudes are taken from the experiments by pp and pn scattering [5–7].

Currently, when all precise calculations are made by the numerical method with the help of large computer software, we have made the analytical calculation of the matrix element with the WF of ^{15}N in the shell model and the operator, where the first and the second order of collisions are taken into account. With this approach, we were able to show the contribution to the differential cross section (DCS) by different orders of collisions and to show the contribution to the DCS from the interior region of the nucleus and the nuclear periphery.

One of the aim of the present paper to calculate the DCS of proton scattering on ^{15}N nucleus in the optical limit of the Glauber diffraction theory (when only single collisions are taken into account in the operator of multiple scattering) and to analyses this cross section sensitivity in proton scattering on the nucleons of different shells. Therefore to describe the internal structure of ^{15}N, we use the shell model [8]. It is known that the multiple scattering series converges rapidly and each next term of the series contributes to the cross section by several orders less than the previous one. The optical limit approximation properly describes the DCS at the front angles.

2 Brief Formalism

The scattering matrix element (amplitude) within the Glauber theory is written as follows [4]:

$$M_{if} = \sum_{M_j M_j'} \frac{ik}{2\pi} \int d\vec{\rho} d\vec{R}_A \exp{(i\vec{q}\vec{\rho})} \delta(\vec{R}_A) \langle \Psi_f^{J' M_j'} | \Omega | \Psi_i^{J M_j} \rangle, \tag{1}$$

where $\Psi_i^{JM_j}$ and $\Psi_f^{J'M_j'}$ are the WFs of initial and final states, $\vec{\rho}$ is the impact parameter, perpendicular to the direction of the projection; A is the number of nucleons in a target, $\vec{R}_A = \frac{1}{A}\sum_{n=1}^{A} \vec{r}_n$ is the coordinate of the nucleus mass centre, $\vec{q} = \vec{k} - \vec{k}'$ is the transferred momentum, $\vec{k}, \vec{k}'$ are the momenta of the projectile and the scattered particles in the center-of-mass system, $\langle \Psi_f^{J'M_j'} | \Omega | \Psi_i^{JM_j} \rangle$ is the matrix element of the transfer from the initial to the final states.

The ground state of ^{15}N nucleus is the level of negative parity with $J^\pi = 1/2^-, T = 1/2$ and $|(1s)^4(1p)^{11}\rangle$ configuration. The shell WF be represented in the form [8]

$$\begin{aligned} \Psi_{i,f}(\vec{r}_i) &= \Psi_{n_0,l_0,m_0}(\vec{r}_1, \ldots, \vec{r}_4) \Psi_{n_1,l_1,m_1}(\vec{r}_5, \ldots, \vec{r}_{15}) \\ &= \Psi_{000}(\vec{r}_1, \ldots, \vec{r}_4) \Psi_{11m_1}(\vec{r}_5, \ldots, \vec{r}_{15}), \end{aligned} \tag{2}$$

where $\Psi_{n,l,m}(\vec{r}_1, \vec{r}_2, \ldots) = \prod_\nu \Psi_{n,l,m}(\vec{r}_\nu)$ is the product of single-particle functions, $\vec{r}_\nu$ are the single-particle coordinates of nucleons.

The excited state of ^{15}N nucleus is the level of positive parity with $J^\pi = 5/2^+$ and $|(1s)^4(1p)^{10}(1d)^1\rangle$ configuration [8]

$$\begin{aligned} \Psi_f(\vec{r}_1, \vec{r}_2, \ldots) &= |(1s)^4(1p)^{10}(1d)^1\rangle \\ &= \sum_{m_1 m_2} \Psi_{000}(\vec{r}_1, \ldots, \vec{r}_4) \Psi_{11m_1}(\vec{r}_5, \ldots, \vec{r}_{14}) \Psi_{22m_2}(\vec{r}_{15}). \end{aligned} \tag{3}$$

Operator Ω in the Glauber theory is written in the form of a multiple-scattering series

$$\Omega = 1 - \prod_{\nu=1}^{A} 1 - \omega_\nu(\vec{\rho} - \vec{\rho}_\nu)$$
$$= \sum_{\nu=1}^{A} \omega_\nu - \sum_{\nu<\tau} \omega_\nu\omega_\tau + \sum_{\nu<\tau<\eta} \omega_\nu\omega_\tau\omega_\eta + \cdots + (-1)^{A-1}\omega_1\omega_2\omega_A, \quad (4)$$

where the first term consider the single collisions, the second term is the double collisions and so on, up to the last term which consider A-multiple collision.

Separate profile functions ω_ν are expressed through the elementary pN-amplitudes $f_{pN}(q)$:

$$\omega_\nu(\vec{\rho} - \vec{\rho}_\nu) = \frac{1}{2\pi ik} \int d\vec{q}_\nu \exp\left(-i\vec{q}_\nu(\vec{\rho} - \vec{\rho}_\nu)\right) f_{pN}(q_\nu). \quad (5)$$

The elementary amplitude itself is written in the standard manner

$$f_{pN}(q_\nu) = \frac{k\sigma_{pN}}{4\pi}(i + \varepsilon_{pN}) \exp\left(-\beta_{pN}^2 q_\nu^2/2\right), \quad (6)$$

where σ_{pN} is the total proton scattering cross section by the nucleon, ε_{pN} is the ratio of the real part of the amplitude to the imaginary one, and β_{pN} is the slope parameter at the amplitude corn are taken from [5–7].

To announce the key items of the derivation.

1. In the expression (4) we are restricted only the first two terms, owing to that fact, that each next term gives a contribution to the cross section smaller by several orders of magnitude than the previous one.

2. Substituting the series of multiple scattering (4) in the amplitude (1), and then integrating it with the impact parameter $d\vec{\rho}$ and momentum $d\vec{q}_\nu$ leads to the following result:

$$\Omega = \frac{2\pi}{ik} f_{pN}(q) \sum_{i=1}^{15} \widetilde{\omega}_i - \left(\frac{2\pi}{ik} f_{pN}\left(\frac{q}{2}\right)\right)^2 \sum_{i<j=1}^{15} \widetilde{\omega}_i\widetilde{\omega}_j, \quad (7)$$

where

$$\begin{aligned} \sum_{i=1}^{15} \widetilde{\omega}_i &= \sum_{i=1}^{15} \exp\left(i\vec{q}\vec{\rho}_i\right), \\ \sum_{i<j=1}^{15} \widetilde{\omega}_i\widetilde{\omega}_j &= \sum_{i<j=1}^{15} \exp\left(i\frac{\vec{q}}{2}(\vec{\rho}_i + \vec{\rho}_j)\right)\delta(\vec{\rho}_i - \vec{\rho}_j). \end{aligned} \quad (8)$$

3. We represent the operators of single and double collisions as the sum of operators affecting the nucleons at different shells:

$$\sum_{i=1}^{15} \widetilde{\omega}_i = \sum_{i=1}^{4} \widetilde{\omega}_i + \sum_{i=5}^{14} \widetilde{\omega}_i + \widetilde{\omega}_{15}, \quad (9)$$

$$\sum_{i<j=1}^{15} \widetilde{\omega}_i\widetilde{\omega}_j = \sum_{i<j=1}^{14} \widetilde{\omega}_i\widetilde{\omega}_j + \sum_{i=1}^{15} \widetilde{\omega}_i\widetilde{\omega}_{15}. \tag{10}$$

4. Replacing the flat vectors $\vec{\rho}_i$ (on which they depend $\widetilde{\omega}$) with three-dimentional $\vec{r}_i, \vec{r}_{15} \to \vec{r}$, expending $\widetilde{\omega} = \exp{(i\vec{q}\vec{r})}$ in the Bessel series:

$$\exp{(i\vec{q}\vec{r})} = 4\pi \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} (i)^{\lambda} \sqrt{\frac{\pi}{2qr}} J_{\lambda+1/2}(qr) Y_{\lambda\mu}\left(\Omega_r\right) Y_{\lambda\mu}\left(\Omega_q\right),$$

and representing the WF in the centrally symmetric field is factorized into radial $R_{nl}(r_\nu)$ and angular $Y_{lm}\left(\Omega_k\right)$ parts: $\Psi_{nlm}(\vec{r}_i) = R_{nl}(r_i)Y_{lm}\left(\Omega_r\right)$, then it is possible to integrate the matrix element (1) in the spherical system of coordinates.

Detailed description of the elastic p^{15}N scattering may be found in [9], here we show the result for the inelastic scattering. The matrix element of single scattering:

$$M_{if}^{(1)}(\vec{q}) = \frac{k}{k'} 2\pi f_{pN}(q) \sum_{\lambda\mu} (i)^{\lambda} \left\langle R_{22}(r) \middle| \frac{1}{\sqrt{r}} J_{\lambda+1/2}(qr) \middle| R_{11}(r) \right\rangle \times \langle Y_{2m_2}(\Omega_r)|Y_{\lambda\mu}(\Omega_r)|Y_{1m}\rangle Y_{\lambda\mu}(\Omega_q), \tag{11}$$

where

$$\left\langle R_{22}(r) \middle| \frac{1}{\sqrt{r}} J_{\lambda+1/2}(qr) \middle| R_{11}(r) \right\rangle = \sqrt{\frac{\pi}{2q}} \int_0^{\infty} R_{22}^*(r) R_{11}(r) J_{\lambda+1/2}(qr) r^{3/2} dr, \tag{12}$$

$$\sum_{\lambda\mu} \langle Y_{2m_2}(\Omega_r)|Y_{\lambda\mu}(\Omega_r)|Y_{1m}\rangle Y_{\lambda\mu}(\Omega_q) = \sum_{\lambda\mu} \sqrt{\frac{5(2\lambda+1)}{3\cdot 4\pi}} \langle \lambda 010|20\rangle \langle \lambda\mu 1m|2m_2\rangle. \tag{13}$$

The matrix element of double scattering:

$$M_{if}^{(2)}(\vec{q}/2) = M_{if}^{(2)-sd}(\vec{q}/2) + M_{if}^{(2)-pd}(\vec{q}/2), \tag{14}$$

the upper indexes indicate the nucleons of the shell where collision occurs.

$$M_{if}^{(2)-sd}(\vec{q}/2) = = C(q/2) \sum_{\lambda\mu} (i)^{\lambda} \left\langle R_{00}(r)R_{22}(r) \middle| \frac{1}{\sqrt{r}} J_{\lambda+1/2}(qr) \middle| R_{00}(r)R_{11}(r) \right\rangle \times \langle Y_{00}(\Omega_r)Y_{2m_2}(\Omega_r)|Y_{\lambda\mu}(\Omega_r)|Y_{00}(\Omega_r)Y_{1m}(\Omega_r)\rangle Y_{\lambda\mu}(\Omega_q), \tag{15}$$

where

$$C(q/2) = \frac{ik'}{(ik)^2} 6(2\pi)^2 \sqrt{\frac{\pi}{q/2}} f^2 \left(\frac{q}{2}\right), \tag{16}$$

$$\left\langle R_{00}(r)R_{22}(r) \middle| \frac{1}{\sqrt{r}} J_{\lambda+1/2}(qr) \middle| R_{00}(r)R_{11}(r) \right\rangle$$
$$= \int_0^\infty |R_{00}(r)|^2 R_{22}^*(r) R_{11}(r) J_{\lambda+1/2}(qr) r^{3/2} dr, \tag{17}$$

$$\langle Y_{00}(\Omega_r)Y_{2m_2}(\Omega_r)|Y_{\lambda\mu}(\Omega_r)|Y_{00}(\Omega_r)Y_{1m}(\Omega_r)\rangle$$
$$= \frac{1}{4\pi} \int Y_{2m_2}^*(\Omega_r) Y_{\lambda\mu}^*(\Omega_r) Y_{1m}^*(\Omega_r) d\Omega_r. \tag{18}$$

The last integral is analog of (13).

$$M_{if}^{(2)-pd}(\vec{q}/2) =$$
$$= \widetilde{C}(q/2) \sum_{\lambda\mu} (i)^\lambda \left\langle R_{11}(r)R_{22}(r) \middle| \frac{1}{\sqrt{r}} J_{\lambda+1/2}(qr) \middle| R_{11}(r)R_{11}(r) \right\rangle$$
$$\times \langle Y_{1m_1}(\Omega_r)Y_{2m_2}(\Omega_r)|Y_{\lambda\mu}(\Omega_r)|Y_{1m}(\Omega_r)Y_{1m}(\Omega_r)\rangle Y_{\lambda\mu}(\Omega_q), \tag{19}$$

$$\widetilde{C}(q/2) = \frac{15}{2} C(q/2),$$

$$\left\langle R_{11}(r)R_{22}(r) \middle| \frac{1}{\sqrt{r}} J_{\lambda+1/2}(qr) \middle| R_{11}(r)R_{11}(r) \right\rangle$$
$$= \int_0^\infty |R_{11}(r)|^2 R_{22}^*(r) R_{11}(r) J_{\lambda+1/2}(qr) r^{3/2} dr, \tag{20}$$

$$\langle Y_{1m_1}(\Omega_r)Y_{2m_2}(\Omega_r)|Y_{\lambda\mu}(\Omega_r)|Y_{1m}(\Omega_r)Y_{1m}(\Omega_r)\rangle Y_{\lambda\mu}(\Omega_q)$$
$$= \sum_{LML'M'} (-1)^{m_2} \frac{\sqrt{15(2\lambda+1)}}{(4\pi)^{3/2}} \langle\lambda 020|L'0\rangle\langle L'010|L0\rangle\langle L010|10\rangle$$
$$\times \langle\lambda\mu 2-m_2|L'M'\rangle\langle L'M'1m|LM\rangle\langle LM1m|1m_1\rangle Y_{\lambda\mu}(\Omega_q), \tag{21}$$

It is important to note that in approximation of two multiple collisions all integrals are taken analytically and, therefore there is no loss of precision inherent for numerical integration.

The DCS is determined by the squared modulus of the respective matrix elements as

$$\frac{d\sigma}{d\Omega} = \frac{1}{2J+1} \left| M_{if}^{(1)}(\vec{q}) - M_{if}^{(2)}(\vec{q}) \right|^2. \tag{22}$$

In the optical limit, the first term of the multiple scattering series (7) remains only in operator Ω, being the sum of single collisions with all nucleons

$$\Omega = \sum_{\nu=1}^{A} \omega_\nu, \tag{23}$$

Let's divide (23) into terms corresponding to the scattering over s- and -shells:

$$\sum_{\nu=1}^{15} \omega_\nu = \sum_{\nu=1}^{4} \omega_\nu + \sum_{\nu=5}^{15} \omega_\nu, \tag{24}$$

After substituting the operator (24) into (1), we obtain the matrix element as a sum of two components

$$M_{if}^{(1)}(\vec{q}) = \frac{k}{k'} f_{pN}(q) \left[M_s^{(1)}(\vec{q}) + M_p^{(1)}(\vec{q}) \right]. \tag{25}$$

where

$$M_s^{(1)}(\vec{q}) = \int \left| \prod_{\nu=1}^{4} |\Psi_{000}(\vec{r}_\nu) \right|^2 \sum_{\nu=1}^{4} \omega_\nu \prod_{\nu=1}^{4} d\vec{r}_\nu, \tag{26}$$

$$M_p^{(1)}(\vec{q}) = \sum_m \int \left| \prod_{\nu=5}^{15} |\Psi_{11m}(\vec{r}_\nu) \right|^2 \sum_{\nu=5}^{15} \omega_\nu \prod_{\nu=5}^{15} d\vec{r}_\nu, \tag{27}$$

$M_s^{(1)}(\vec{q})$ is responsible for the scattering on the 1s-shell nucleons; $M_p^{(1)}(\vec{q})$ is responsible for the scattering on the 1-shell nucleons. Those matrix elements are the overlap integrals of the WFs and the operators ω_ν acting over the coordinates of the nucleons in the corresponding shells.

The DCS is determined by the squared modulus of the respective matrix elements as

$$\frac{d\sigma^{OL}}{d\Omega} = \frac{1}{2J+1} \left| M_s^{(1)}(\vec{q}) + M_p^{(2)}(\vec{q}) \right|^2. \tag{28}$$

3 Results and Discussions

We have calculated the DCSs for elastic scattering at energies 0.6 and 1.0 GeV and inelastic scattering at the level $J = 5/2^+$ at energies 0.415, 0.8 and 1.0 GeV.

Figure 1 shows the contributions from different collision multiplicities to the DCS for elastic scattering at energies 0.6 (a) and 1.0 GeV (b).

As it can be seen from the figures, the single scattering (dash curve) gives the main contribution to the forward angles, but it decreases quickly and then the double scattering (dotted curve) begins to dominate at the larger angles. The minima at $\theta \sim 14^\circ$ in Figure 1a and at $\theta \sim 9^\circ$ in Figure 1b, caused by the destructive interference, appear at points where the partial cross sections is crossed

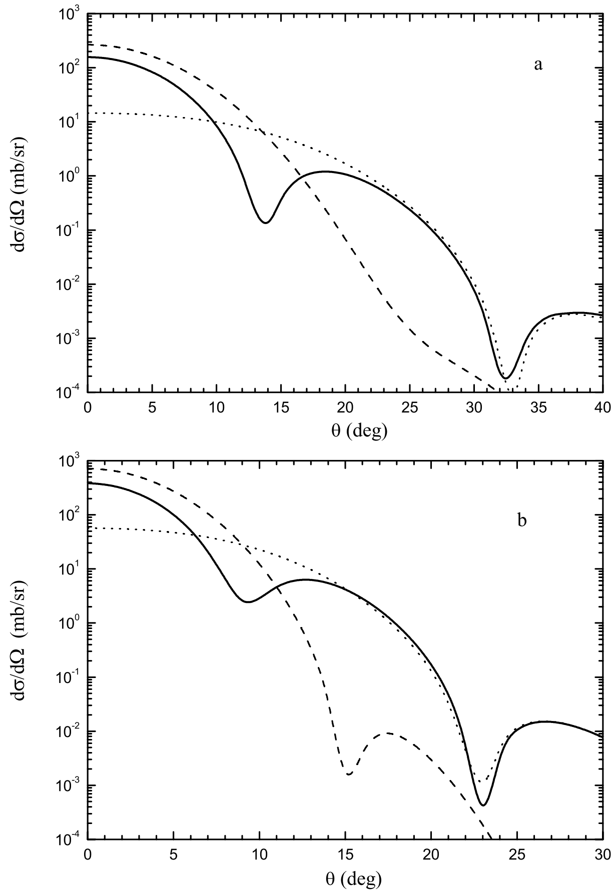

Figure 1. Contributions from the different collision multiplicities to the DCS at $E = 0.6$ (a) and 1.0 (b) GeV. Curves dash, dotted and solid are the contributions single, double collisions and their sum.

each other, because the series of multiple scattering are sign-changing. In all this cases the resulting cross section at small angles is lower than the partial cross section of single scattering. The reason of such behavior is the fact that the resulting cross section is equal to the difference single and double amplitudes (see Eq. (22)).

Additional minima in the partial cross sections at $\theta \sim 33^\circ$ in Figure 1a and at $\theta \sim 23^\circ$ in Figure 1b arise because of the use of the realistic WFs that contain the angular part. As a result, the amplitude can change sigh. Since the DCS is the square of the amplitude, it leads to a minimum in the DCS.

Figure 2 shows the contribution to the DSC from the scattering on the nucleons of 1s (dash curve) and 1p (dotted curve) shells in the optical limit (when we

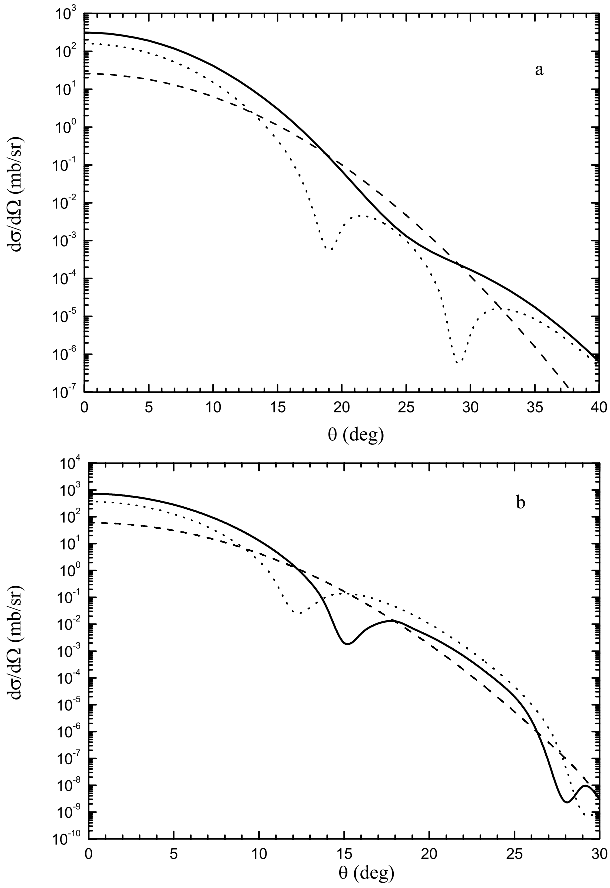

Figure 2. Contribution to the single cross-section (solid curve) from the scattering on the nucleons of 1s (dash curve) and 1p (dotted curve) shells at $E = 0.6$ GeV (a) and 1.0 GeV (b). Here, the solid curve is the same as dash curve in Figure 1.

take into account only single scattering in series of multiple scattering, it is solid curve). We can see that at small angles the main contribution to the DCS gives 1p-shell nucleons scattering. With increasing scattering angles the contribution from the 1s-shell nucleons scattering decreases slowly than the DCS from the 1p-shell nucleon scattering. At some angles they become equal. At these points (where the DCS are equal) arise the interference of amplitudes (constructive or destructive depend on sigh of amplitudes). For example there are the constructive interference at $\theta \sim 13°$ and $32°$ in Figure 2a, and the constructive interference at $\theta \sim 9°$ and destructive one at $\theta \sim 15°$ and $28°$ in Figure 2b. Whereas, in Figure 2a the deep minima observed in the DCS on 1p-shell nucleons do not affect the behavior of the resulting cross section.

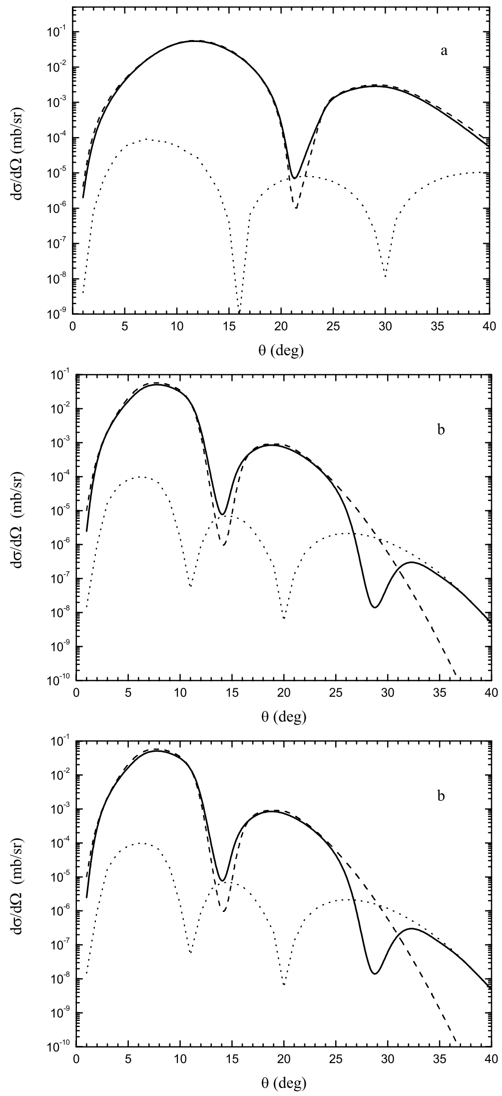

Figure 3. Contributions from different collision multiplicities to the DCS of inelastic scattering at $E = 0.415$ (a), 0.8 (b) and 1.0 GeV (c). Dash, dotted and solid curves correspond to single, double and their sum scattering.

Why the scattering on 1-shell nucleons is dominated at forward angles and the scattering on 1s-shell nucleons is dominated at larger angles? This occurs due to the fact that the momentum transfer increases with an increase of the scattering angles and proton can penetrate deeper into the nucleus and it interacts with the nucleons of internal 1s-shell.

Let's discuss inelastic scattering. Figure 3 shows the contribution to inelastic scattering DCS from single and double collisions at $E = 0.415$ (a), 0.8 (b), and 1.0 (c) GeV. The DCS at zero angle tends to zero because of the orthogonality of the initial and the final states WFs.

Figure 3a shows that single scattering dominates in the whole angular region. This can be explained the fact, that the scattering of proton (occurs at the nuclear periphery) on the outside nucleon, located on 1d-shell. Double collisions in the first maximum are by three orders of magnitude smaller than the single collisions, and in the resulting cross section they lead to slight filling of the minimum at $\theta \sim 22°$ and slight decrease the cross section at $\theta > 35°$. Another pictures are shown in Figures 3b,c. In the region of forward angles (θ up to $25°$) the single collisions give the main contribution to the resulting DCS, but they rapidly decrease at $\theta > 25°$, and the double collisions at $\theta > 30°$ become comparable with the single ones. At the large angles they begin to dominate and determining the behavior of the cross section. This happens because with increase of the energy the higher order collisions comes more significant contribution to the DCS. This is the result of the fact, that the more energetic colliding particles can penetrate deeper into the interior of the nucleus and collide with a more number of nucleons. The transfer momentum in the reaction also increases with increase of the scattering angle, so that the contributions of the collisions of higher multiplicity become rather more significant at the large angles.

As in the elastic p^{15}N scattering, the additional minima in the double partial cross sections (at $\theta \sim 16°$ and $30°$ in Figure 3a, at $\theta \sim 11°$ and $20°$ in Figure 3b, at $\theta \sim 17°$ and $22°$ in Figure 3c) arise due to the use realistic WFs that include an angular part. As a result, the amplitude can change sigh and it leads to the minimum in the DCS.

4 Conclusions

The differential cross sections of the elastic and inelastic (the level $J = 5/2^+$) p^{15}N scattering were calculated within the framework of the Glauber diffraction theory at intermediate energies from 0.415 to 1.0 GeV. It is show that the application of the Glauber diffraction theory to the p^{15}N scattering with WF presented in the form of Gaussian functions and with the operator expressed by an exponential function allows us to calculate analytically the scattering amplitude and take into account the structural components of the WFs.

It is revealed the single scatterings are dominated in the region of forward angles, while the contribution from the double scatterings only slightly decrease the DCS. At larger angles the double scatterings become dominant and determine the behavior of the cross section. Such behavior of DCS is interpreted as follows. With the increase of proton energy, they can penetrate deeper into the interior of the nucleus and can be rescattered on a more number of nucleons.

Considering the dependence of the DCS on scattering over nucleons in different shells (in the optical limit) we can see that the small number of nucleons on the 1s-shell lead to their substantially decrease of the contribution to the resulting cross section in the forward angles in comparison to the scattering on the 1p-shell nucleons. However, at large angles the partial DCS of scattering on the nucleons of 1s-shell begins to dominate and the resulting DCS is determined by the competing contribution of both partial cross sections.

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References

[1] R.J. deBoer, P.J. LeBlanc, S. Falahat, *et al.*, *Phys Rev. C* **85** (2012) 038801.

[2] A.M. Mukhamedzhanov, M.La Cognata, and V. Kroha, *Phys Rev. C* **83** (2011) 044604.

[3] P.J. LeBlanc, *et al.*, *Phys Rev. C* **82** (2010) 055804.

[4] *R. G. Glauber, in Lectures in Theoretical Physics* (Interscience, New York; London, 1959).

[5] L. Ray, *Phys Rev. C* **19** (1979)1855; *Phys Rev. C* **20** (1979) 1857.

[6] J.P. Auger, C. Lazard and R.J. Lombard, *J. Phys. G* **7** (1981) 1627; J.P. Auger, *et al.*, *J. Phys. G* **11** (1985) 751.

[7] D.K. Hasell, *Phys Rev. C* **34** (1986) 236.

[8] M.A. Zhusupov, *et al.*, *Izv. Rus. Akad. Nauk. Fisika* **32** (1968) 332-339.

[9] E.T. Ibraeva, *et al.*, *Phys.of Atom.Nucl.* **73** (2010) 1451.