Rotation-Vibration Excited States of Odd Ta Isotopes Built on $g_{7/2}$ Orbital

M.J. Ermamatov¹, H. Yépez-Martínez², P.C. Srivastava³

¹Institute of Nuclear Physics, Ulughbek, Tashkent 100214, Uzbekistan
²Universidad Autónoma de la Ciudad de México, Prolongación San Isidro 151, Col. San Lorenzo Tezonco, Del. Iztapalapa, 09790 México D.F., Mexico
³Department of Physics, Indian Institute of Technology, Roorkee-247 667, India

Abstract. The ground state band of the proton-odd Ta isotopes, built on $g_{7/2}$ orbital, is investigated within the framework of a recently-developed extended Bohr Hamiltonian model. Energy levels of ground state band with their admixture with other possible bands built on $g_{7/2}$ orbital and $B(E2)$ values inside ground state band are calculated and compared with available experimental data.

1 Introduction

The Bohr Hamiltonian [1] has long been used in the study of important nuclear structure properties [2–7], further development has been done in the recent works [8–22]. The collective-single-particle structure of deformed odd nuclei has been studied in our earlier works [23–25] using the same mass parameter for all the vibration and rotation modes. The idea of different mass parameters for different modes of motion in a nucleus, originating from Ref. [19], has been used in a simple model where quantum numbers of the projection of angular momentum of a nucleus, $K$ and that of an external nucleon, $Ω$, are good quantum numbers in Refs. [26, 27]. In Ref. [28], the Coriolis interaction, to which a number of earlier interesting works are devoted [29, 30], and its effects on spectra and reduced $E2$ transition probabilities, has been studied in the case that the projection of the angular momentum to the third axis connected with a nucleus and that of the external nucleon are not conserved.

It is well known that in the Nilsson model, single-particle energies are calculated by solving the Schrödinger equation for a particle moving in a deformed potential. The model and its features are discussed in detail, for example, in Ref. [31]. The corresponding interacting boson fermion model is discussed in Ref. [32]. In particular it has been shown that the Nilsson model corresponds to the classical limit of the interacting boson-fermion model with a pure quadrupole boson-fermion interaction. The relationship between the Nilsson model with interacting boson model is discussed in detail in Refs. [30, 32, 33].
The Nilsson model can be applied to determine the ordering of levels in odd nuclei, placing particles in each $K$-level \[32\]. In this work we utilize the model which we developed in Ref. [28] to describe the ground state band of the $^{175,177,179,181}$Ta isotopes built on the $g_{7/2}$ orbital. For the $^{175,177,179,181}$Ta isotopes, the ground state spin $L_0$ equals to the angular momentum $j$ of last proton. This model is good for this type of nuclei, as is mentioned also in [6, 7].

2 Model

We write the Schrödinger equation in the following form

$$\left( H_v + H_{\text{rot}} + H_p + H_{\text{int}} \right) \Psi = E \Psi,$$

where the vibrational component of this, the Bohr Hamiltonian, for the case of an odd-mass nucleus is

$$\begin{align*}
H_v &= \frac{-\hbar^2}{2} \left\{ \frac{1}{B_\beta} \frac{\partial^2}{\partial \beta^2} + \frac{2}{B_\gamma} \frac{\partial}{\partial \beta} + \frac{2}{B_\gamma} \frac{\partial}{\partial \beta} + \frac{1}{B_\gamma} \frac{\partial}{\partial \gamma} \left( \frac{\partial}{\partial \gamma} \right) \\
&\quad - \frac{1}{B_\gamma} \frac{1}{4 \beta^2} \left( \frac{1}{\gamma^2} + \frac{1}{3} \right) (\hat{L}_3 - j_3)^2 \right\} + V(\beta, \gamma),
\end{align*}$$

and the operator of rotational energy is

$$H_{\text{rot}} = \frac{\hbar^2}{6 B_{\text{rot}} \beta^2} \left[ L^2 + j^2 - L_3^2 - j_3^2 - 2(L_1 j_1 + L_2 j_2) \right],$$

$H_p$ takes into account the central-symmetrical part, and the interaction operator is

$$H_{\text{int}} = -\beta \langle T \rangle (3 j_3^2 - j^2),$$

where $L$ is the total angular momentum of the nucleus, $L_1$, $L_2$, and $L_3$ are its projections on the principal axes of the nucleus, and $j$, $j_1$, $j_2$, and $j_3$ are the total angular momentum operators of a single nucleon external to the core, and those of its projections. In Refs. [6] and [7], $T(r)$ is a function of the distance between the single nucleon and the center of the core nucleus. It appears in the Hamiltonian of Eq. (2.2) in Ref. [6] and Eq. (2) of Ref. [7]. $\langle T \rangle$, which is introduced in those references and used here, is the average of the $T(r)$ in the states of the extra nucleon, assuming zero nuclear surface oscillation.

The same potential of Eq. (6) in Refs. [26–28] is considered. The eigenvalues of the Hamiltonian in Eq. (1) are determined by the following expression:

$$E_{n_\beta j_\beta L_\beta m_\gamma} = [2n_\beta + q^*_{n_\beta} (L, |m|) + 3/2] \sqrt{2g_\beta},$$

where

$$q_{n_\beta}^* (L, |m|) = \sqrt{\Lambda - \Lambda_0 + 2g_\beta + 1/4 - 1/2},$$

46
Rotation-vibration excited states of Ta isotopes built on $g_{7/2}$ orbital

and

$$\Lambda - \Lambda_0 = \frac{2}{3} \frac{B_{\beta}}{B_{\gamma}} \left( 2n_\gamma + |m| + \frac{m^2}{3} \right) + \varepsilon_{|m|L\tau} - \varepsilon_{0L_01},$$

(7)

where $\Lambda$ is the eigenvalue of the $\gamma$-vibrational part of the Hamiltonian plus the third term of the rotational section of the Hamiltonian, $\Lambda_0$ is that of ground state, $L_0$ is lowest state for each Nilsson band, $\tau$ distinguishes between different states of the same $L$, and $n_\beta$ and $n_\gamma$ are the quantum numbers of $\beta$ and $\gamma$ rotations, respectively. Values of $m$ are connected with $K$ and $\Omega$ through the condition $K - \Omega = 2m$ [6], where $m$ is of integer value, $g_{\beta} = \frac{1}{2} B_{\beta}^2 V_0/\hbar^2$ and $g = \frac{1}{2} \frac{\hbar^2}{\sqrt{B_{\beta}^2 C}}$.

The following determinant is calculated in order to determine eigenvalues and eigenfunction of the rotational part of the Hamiltonian:

$$||\langle L \gamma K \gamma | \hat{X} | L \gamma K \gamma' \gamma' \rangle - \varepsilon_{|m|L\tau} \delta_{KK'} \delta_{mm'}|| = 0,$$

(8)

where

$$\hat{X} = \frac{1}{3} \frac{B_{\beta}}{B_{\text{rot}}} \left[ L(L+1) + j(j+1) - L_3^2 - j_3^2 - 2(L_1 j_1 + L_2 j_2) \right]$$

$$- \frac{\hbar^2}{3\xi} \left[ 3j_3^2 - j(j+1) \right],$$

(9)

and $\xi = \frac{\hbar^2}{6B_{\beta} \beta_0^2 \langle T \rangle}$. Since $K$ and $\Omega$ are not good quantum numbers, not only diagonal elements of the Hamiltonian, but also non-diagonal elements, contribute to the energies and $E2$ transition probabilities. Diagonal elements are as follows:

$$\langle L \gamma K \gamma | \hat{X} | L \gamma K \gamma \rangle =$$

$$\frac{1}{3} \frac{B_{\beta}}{B_{\text{rot}}} \left[ L(L+1) + j(j+1) - K^2 - (K - 2m)^2 \right]$$

$$- (-1)^{j+1} (L + 1/2)(j + 1/2) \delta_{K1/2} \delta_{mn}$$

$$- \frac{\hbar^2}{3\xi} [3(K - 2m)^2 - j(j+1)].$$

(10)

Non-diagonal elements are

$$\langle L \gamma K \gamma | \hat{X} | L \gamma K \pm 1 \gamma \rangle =$$

$$\frac{1}{3} \frac{B_{\beta}}{B_{\text{rot}}} \sqrt{(L \mp K)(L \pm K + 1)(j + K \pm 2m)(j \pm K \mp 2m + 1)}.$$

(11)

We denote $E_{00L_0\tau} - E_{00L_{g.s.}01} = E(L)$ as rotational-single-particle energies of the bands built on Nilsson orbitals. Here $L_{g.s.}$ is ground state spin of the nucleus.
The corresponding wave function is expanded as

\[ \Psi = \beta^{-1} F(\beta) \sum_{mK} A^{\alpha}_{mK} \chi_K|\alpha|LM^jKm), \]

where

\[ |LM^jKm) = \sqrt{\frac{2L + 1}{16\pi^2}} \left[ D_{MK}^j(\theta_i) \varphi_{K-2m}(x_i) \right. \]

\[ \left. + (\text{if})^{L-K} D_{MK}^j(\theta_i) \varphi_{K+2m}(x_i) \right] \quad i = 1, 2, \text{or} 3 \]

\[ \chi_{n,|m|}(\gamma) = N_{\gamma}\left(\frac{1}{g} \right)^{\frac{|m|}{2}} \text{F}_1 \left(-n_{\gamma},|m| + 1, \frac{\gamma^2}{g}\right) e^{-\gamma^2} \]

and

\[ F_{n,\alpha L,|m|}(\beta) = N_{\beta}\beta^{n_{\alpha}(L,|m|)+1} \exp \left(-\frac{\beta^2}{2|g|}\right) L_{\alpha\beta}^{n_{\alpha}(L,|m|)+1/2} \left(\frac{\beta^2}{|g|}\right). \]

Here \( N_{\beta} \) and \( N_{\gamma} \) are normalization coefficients for \( \beta \) and \( \gamma \) wave functions, respectively, \( \text{F}_1 \left(-n_{\gamma},|m| + 1, \frac{\gamma^2}{g}\right) \) is a confluent hypergeometric function, \( g_{\beta} = \frac{B_{\beta}^{\alpha}(\beta) g_{\alpha}(\beta)}{k_{\beta}}, \quad g = \frac{k_{\gamma}}{\sqrt{B_{\gamma}}}, \quad b = \frac{\beta_{\alpha}(L,|m|)+1/2}{\sqrt{2g_{\beta}}}, \quad L_{\alpha\beta}^{n_{\alpha}(L,|m|)+1/2} \) are Laguerre polynomials, \( D(\theta_i) \) is Wigner function, \( \varphi(x_i) \) is the wave function of the single-particle states, and \( A^{\alpha}_{mK} \) is determined to be eigenvectors with matrix elements \( \langle L^jK^m|X|L^jKm \rangle \).

Then, if we denote deformability with respect to \( \beta \) vibration as

\[ S_{n,\alpha L,|m|,\tau;n',\alpha' L',|m'|,\tau'} = \int_0^\infty F_{n,\alpha L,|m|,\tau;n',\alpha' L',|m'|,\tau'} \frac{\beta}{\beta_0} F_{n',\alpha' L',|m'|,\tau'} d\beta \]

for the ground state intraband transitions the following expression is obtained:

\[ B(E2; L_{g.s.} + 2 \to L_{g.s.}) = \frac{5Q_0^2}{16\pi} \left| \sum_{K K'} A^{01}_{LJK} A^{01}_{L'JK'} (L2K0|L'K') \right|^2 \times S_{00L01;00L01}^2, \]

where

\[ S_{00L01;00L01} = \frac{2}{\sqrt{2g_{\beta}} \sqrt{\Gamma(q_0^1(L,0) + 3/2)}} \frac{1}{\sqrt{\Gamma(q_0^1(L',0) + 3/2)} \sqrt{\Gamma(q_0^1(L,0) + 4)}} \]

\[ \times \Gamma\left[ q_0^1(L',0) + q_0^1(L,0) + \frac{3}{2} \right]/2. \]
3 Results and Discussions

In the case $m = 0$, the angular momentum vector of the prolate core is perpendicular to the axis of symmetry, and thus it cannot contribute to the value of $K$. Then, the value of $K$ is determined by the projection of the angular momentum of the last proton. The $7/2^+ [404]$ ground state of the Ta isotopes we have determined from the Nilsson model. Since this state corresponds to $g_{7/2}$ spherical orbit, $K = 1/2, 3/2, 5/2$ and $7/2$ values are possible.

The ground state band of the Ta isotopes are calculated by diagonalizing (8) with all possible $K$ taking into account the $K \pm 1$ mixture of non-diagonal elements.

Table 1. The values of the parameters used in calculations

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\xi$</th>
<th>$g_{7/2}$</th>
<th>$B_{\gamma}/B_{rot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{175}$Ta</td>
<td>0.01</td>
<td>187</td>
<td>3.2</td>
</tr>
<tr>
<td>$^{177}$Ta</td>
<td>0.01</td>
<td>219</td>
<td>3.2</td>
</tr>
<tr>
<td>$^{179}$Ta</td>
<td>0.01</td>
<td>227</td>
<td>3.2</td>
</tr>
<tr>
<td>$^{181}$Ta</td>
<td>0.01</td>
<td>587</td>
<td>3.2</td>
</tr>
</tbody>
</table>

The values of parameters used in the calculation are given in Table 1. The ground state band energies do not depend on parameters $g$ and $B_{\gamma}/B_{rot}$, therefore they depend only on three parameters given in the Table 1. The parameter $\xi$ which connects single-particle and $\beta$ vibrations and $B_{\gamma}/B_{rot}$ are chosen the same for all isotopes and $g_{7/2}$ is different for each isotope.

Comparison of the calculated values of the ground state excited energies with experimental data relative to $E(9/2^+)$ energy are given in Tables 2 and 3. The experimental and intraband reduced $B(E2)$ transition probabilities with respect

Table 2. The calculated and experimental values of the $E(L)/E(9/2^+)$ for $^{175}$Ta and $^{177}$Ta. The experimental values are taken from Ref. [34]

<table>
<thead>
<tr>
<th>$L$</th>
<th>$^{175}$Ta</th>
<th>$^{177}$Ta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calc</td>
<td>Exp</td>
</tr>
<tr>
<td>$11/2^+$</td>
<td>2.19</td>
<td>2.19</td>
</tr>
<tr>
<td>$13/2^+$</td>
<td>3.55</td>
<td>3.55</td>
</tr>
<tr>
<td>$15/2^+$</td>
<td>5.08</td>
<td>5.07</td>
</tr>
<tr>
<td>$17/2^+$</td>
<td>6.74</td>
<td>6.72</td>
</tr>
<tr>
<td>$19/2^+$</td>
<td>8.53</td>
<td>8.49</td>
</tr>
<tr>
<td>$21/2^+$</td>
<td>10.44</td>
<td>10.34</td>
</tr>
<tr>
<td>$23/2^+$</td>
<td>12.44</td>
<td>12.27</td>
</tr>
<tr>
<td>$25/2^+$</td>
<td>14.54</td>
<td>14.27</td>
</tr>
<tr>
<td>$27/2^+$</td>
<td>16.72</td>
<td>16.32</td>
</tr>
<tr>
<td>$29/2^+$</td>
<td>18.97</td>
<td>18.46</td>
</tr>
</tbody>
</table>
Table 3. The calculated and experimental values of the $E(L)/E(9/2^+)$ for $^{179}\text{Ta}$ and $^{181}\text{Ta}$. The experimental values are taken from Ref. [34].

<table>
<thead>
<tr>
<th>$L$</th>
<th>$^{179}\text{Ta}$</th>
<th>$^{181}\text{Ta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calc</td>
<td>Exp</td>
</tr>
<tr>
<td>11/2^+</td>
<td>2.20</td>
<td>2.20</td>
</tr>
<tr>
<td>13/2^+</td>
<td>3.57</td>
<td>3.60</td>
</tr>
<tr>
<td>15/2^+</td>
<td>5.12</td>
<td>5.17</td>
</tr>
<tr>
<td>17/2^+</td>
<td>6.81</td>
<td>6.91</td>
</tr>
<tr>
<td>19/2^+</td>
<td>8.65</td>
<td>8.80</td>
</tr>
<tr>
<td>21/2^+</td>
<td>10.61</td>
<td>10.81</td>
</tr>
<tr>
<td>23/2^+</td>
<td>12.69</td>
<td>12.93</td>
</tr>
<tr>
<td>25/2^+</td>
<td>14.87</td>
<td>15.14</td>
</tr>
<tr>
<td>27/2^+</td>
<td>17.14</td>
<td>17.41</td>
</tr>
<tr>
<td>29/2^+</td>
<td>19.50</td>
<td>19.74</td>
</tr>
</tbody>
</table>

to $B(E2;11/2 \rightarrow 7/2)$ for $^{181}\text{Ta}$ we have listed in Table 4. The ground state and the first excited states energies are not listed in the Tables since they are always 0 an 1, respectively. As is seen from the Tables lower excited states energies are almost the same for all isotopes. The higher excited states energies increase with the increasing of the number of neutrons in both calculations and experiment. The increase of excited states energies is faster in the calculation. As is seen from Table 4 the values of the calculated intraband $B(E2)$ transition probabilities agrees with the experimental data within the uncertainties of the experimental measurements.

The other $K$ bands originating from $g_{7/2}$ orbital are located much higher than $K = 7/2$ ground state band in the calculation. This may be the reason why they do not exist in the experiment. In the experiment there exist other bands which are not built in $g_{7/2}$ orbital. This suggest us to investigate the case when angular momentum of last proton is not conserved where the bands originating from other orbitals and their contribution to the ground state band energies as well as interband $B(E2)$ transition probabilities could be investigated.

Table 4. The calculated and experimental values of the $B(E2; L + 2 \rightarrow L)$ in units of $B(E2; 11/2^+ \rightarrow 7/2^+)$ for $^{181}\text{Ta}$. The experimental values are taken from Ref. [34].

<table>
<thead>
<tr>
<th>$B(E2; L + 2 \rightarrow L)$</th>
<th>$^{181}\text{Ta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calc</td>
</tr>
<tr>
<td>$B(E2; 11/2^+ \rightarrow 7/2^+)$</td>
<td></td>
</tr>
<tr>
<td>13/2^+ → 9/2^+</td>
<td>1.74</td>
</tr>
<tr>
<td>15/2^+ → 11/2^+</td>
<td>2.23</td>
</tr>
<tr>
<td>17/2^+ → 13/2^+</td>
<td>2.58</td>
</tr>
<tr>
<td>19/2^+ → 15/2^+</td>
<td>2.83</td>
</tr>
<tr>
<td>21/2^+ → 17/2^+</td>
<td>3.01</td>
</tr>
</tbody>
</table>
4 Conclusion

The ground state band energies of Ta isotopes have been studied using the different mass parameters for each allowed collective mode. Calculated energy levels and reduced $B(E2)$ transition probabilities are compared with existing experimental data. It is shown that both deformation of the core and the interaction of the last odd proton with the core make a significant impact on the spectra of these nuclei. Also this investigation suggests us to complicate the model with inclusion of triaxility and the case when angular momentum of last nucleon is not conserved which allows to consider more possible mixture of the states with different $K$ observed in the experiment.

Acknowledgements

The authors are grateful to Pieter Van Isacker for fruitful discussions. M.J.E. thanks to Nikolay Minkov for the discussion of the future perspectives on this topic and acknowledges support from grant F2-FA-F177 of Uzbekistan Academy of Sciences.

References