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Rotation-Vibration Excited States of Odd Ta Isotopes Built on $g_{7/2}$ Orbital

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Abstract. The ground state band of the proton-odd Ta isotopes, built on $g_{7/2}$ orbital, is investigated within the framework of a recently-developed extended Bohr Hamiltonian model. Energy levels of ground state band with their admixture with other possible bands built on $g_{7/2}$ orbital and $B(E2)$ values inside ground state band are calculated and compared with available experimental data.

1 Introduction

The Bohr Hamiltonian [1] has long been used in the study of important nuclear structure properties [2–7], further development has been done in the recent works [8–22]. The collective-single-particle structure of deformed odd nuclei has been studied in our earlier works [23–25] using the same mass parameter for all the vibration and rotation modes. The idea of different mass parameters for different modes of motion in a nucleus, originating from Ref. [19], has been used in a simple model where quantum numbers of the projection of angular momentum of a nucleus, K and that of a external nucleon, Ω, are good quantum numbers in Refs. [26, 27]. In Ref. [28], the Coriolis interaction, to which a number of earlier interesting works are devoted [29, 30], and its effects on spectra and reduced $E2$ transition probabilities, has been studied in the case that the projection of the angular momentum to the third axis connected with a nucleus and that of the external nucleon are not conserved.

It is well known that in the Nilsson model, single-particle energies are calculated by solving the Schrödinger equation for a particle moving in a deformed potential. The model and its features are discussed in detail, for example, in Ref. [31]. The corresponding interacting boson fermion model is discussed in Ref. [32]. In particular it has been shown that the Nilsson model corresponds to the classical limit of the interacting boson-fermion model with a pure quadrupole boson-fermion interaction. The relationship between the Nilsson model with interacting boson model is discussed in detail in Refs. [30, 32, 33].

The Nilsson model can be applied to determine the ordering of levels in odd nuclei, placing particles in each K-level [32]. In this work we utilize the model which we developed in Ref. [28] to describe the ground state band of the ^{175,177,179,181}Ta isotopes built on the $g_{7/2}$ orbital. For the ^{175,177,179,181}Ta isotopes, the ground state spin L_0 equals to the angular momentum j of last proton. This model is good for this type of nuclei, as is mentioned also in [6, 7].

2 Model

We write the Schrödinger equation in the following form

$$(H_{\rm v} + H_{\rm rot} + H_{\rm p} + H_{\rm int})\Psi = E\Psi, \tag{1}$$

where the vibrational component of this, the Bohr Hamiltonian, for the case of an odd-mass nucleus is

$$H_{\rm v} = -\frac{\hbar^2}{2}\left\{\frac{1}{B_\beta}\frac{\partial^2}{\partial\beta^2} + \frac{2}{B_\gamma}\frac{1}{\beta}\frac{\partial}{\partial\beta} + \frac{2}{B_\beta}\frac{1}{\beta}\frac{\partial}{\partial\beta} + \frac{1}{B_\gamma}\frac{1}{\gamma}\frac{\partial}{\partial\gamma}\left(\gamma\frac{\partial}{\partial\gamma}\right)\right.$$
$$\left. - \frac{1}{B_\gamma}\frac{1}{4\beta^2}\left(\frac{1}{\gamma^2}+\frac{1}{3}\right)(\hat{L}_3 - j_3)^2\right\} + V(\beta,\gamma), \tag{2}$$

and the operator of rotational energy is

$$H_{\rm rot} = \frac{\hbar^2}{6B_{\rm rot}\beta^2}\left[L^2 + j^2 - L_3^2 - j_3^2 - 2(L_1 j_1 + L_2 j_2)\right], \tag{3}$$

$H_{\rm p}$ takes into account the central-symmetrical part, and the interaction operator is

$$H_{\rm int} = -\beta\,\langle T\rangle\,(3j_3^2 - j^2), \tag{4}$$

where L is the total angular momentum of the nucleus, L_1, L_2, and L_3 are its projections on the principal axes of the nucleus, and j, j_1, j_2 and j_3 are the total angular momentum operators of a single nucleon external to the core, and those of its projections. In Refs. [6] and [7], $T(r)$ is a function of the distance between the single nucleon and the center of the core nucleus. It appears in the Hamiltonian of Eq. (2.2) in Ref. [6] and Eq. (2) of Ref. [7]. $\langle T\rangle$, which is introduced in those references and used here, is the average of the $T(r)$ in the states of the extra nucleon, assuming zero nuclear surface oscillation.

The same potential of Eq. (6) in Refs. [26–28] is considered. The eigenvalues of the Hamiltonian in Eq. (1) are determined by the following expression:

$$E_{n_\beta n_\gamma L|m|\tau} = [2n_\beta + q^\tau_{n_\gamma}(L,|m|) + 3/2]\sqrt{2g_\beta}, \tag{5}$$

where

$$q^\tau_{n_\gamma}(L,|m|) = \sqrt{\Lambda - \Lambda_0 + 2g_\beta + 1/4} - 1/2, \tag{6}$$

and

$$\Lambda - \Lambda_0 = \frac{2}{\mathrm{g}} \frac{B_\beta}{B_\gamma} \left(2n_\gamma + |m| + \frac{m^2}{3} \right) + \varepsilon_{|m|L\tau} - \varepsilon_{0L_0 1}, \tag{7}$$

where Λ is the eigenvalue of the γ-vibrational part of the Hamiltonian plus the third term of the rotational section of the Hamiltonian, Λ_0 is that of ground state, L_0 is lowest state for each Nilsson band, τ distinguishes between different states of the same L, and n_β and n_γ are the quantum numbers of β and γ rotations, respectively. Values of m are connected with K and Ω through the condition $K - \Omega = 2m$ [6], where m is of integer value, $g_\beta = B_\beta \beta_0^2 V_0/\hbar^2$ and $g = \dfrac{1}{\beta_0^2} \dfrac{\hbar^2}{\sqrt{B_\gamma C_\gamma}}$.

The following determinant is calculated in order to determine eigenvalues and eigenfunction of the rotational part of the Hamiltonian:

$$||\langle LjKm|\hat{X}|LjK'm'\rangle - \varepsilon_{|m|L\tau}\delta_{KK'}\delta_{mm'}|| = 0, \tag{8}$$

where

$$\hat{X} = \frac{1}{3}\frac{B_\beta}{B_{\text{rot}}} \left[L(L+1) + j(j+1) - L_3^2 - j_3^2 - 2(L_1 j_1 + L_2 j_2) \right] \\ - \frac{1}{3\xi} \left[3j_3^2 - j(j+1) \right], \tag{9}$$

and $\xi = \hbar^2/(6B_\beta\beta_0^3\langle T\rangle)$. Since K and Ω are not good quantum numbers, not only diagonal elements of the Hamiltonian, but also non-diagonal elements, contribute to the energies and $E2$ transition probabilities. Diagonal elements are as follows:

$$\langle LjKm|\hat{X}|LjKm\rangle = \\ \frac{1}{3}\frac{B_\beta}{B_{\text{rot}}}[L(L+1) + j(j+1) - K^2 - (K-2m)^2 \\ - (-1)^{I-j}(L+1/2)(j+1/2)\delta_{K1/2}\delta_{m0}] \\ - \frac{1}{3\xi}[3(K-2m)^2 - j(j+1). \tag{10}$$

Non-diagonal elements are

$$\langle LjKm|\hat{X}|LjK \pm 1m\rangle = \\ \frac{1}{3}\frac{B_\beta}{B_{\text{rot}}}\sqrt{(L \mp K)(L \pm K + 1)(j \mp K \pm 2m)(j \pm K \mp 2m + 1)}. \tag{11}$$

We denote $E_{00L0\tau} - E_{00L_{\text{g.s.}}01} = E(L)$ as rotational-single-particle energies of the bands built on Nilsson orbitals. Here $L_{\text{g.s.}}$ is ground state spin of the nucleus.

The corresponding wave function is expanded as

$$\Psi = \beta^{-1-\frac{B_\beta}{B_\gamma}} F(\beta) \sum_{mK} A_{LK}^{m\tau} \chi_{K|m|}(\gamma) |LMjKm\rangle, \tag{12}$$

where

$$\begin{aligned} |LMjKm\rangle = \sqrt{\frac{2L+1}{16\pi^2}} \Big[& D_{MK}^L(\theta_i) \varphi_{K-2m}^j(x_i) \\ & + (-1)^{L-j} D_{M-K}^L(\theta_i) \varphi_{-K+2m}^j(x_i) \Big] \quad i = 1, 2, \text{or } 3 \end{aligned} \tag{13}$$

$$\chi_{n_\gamma|m|}(\gamma) = N_\gamma \Big(\frac{\gamma^2}{\mathrm{g}}\Big)^{\frac{|m|}{2}} {}_1F_1\Big(-n_\gamma, |m|+1, \frac{\gamma^2}{\mathrm{g}}\Big) e^{-\frac{\gamma^2}{2\mathrm{g}}} \tag{14}$$

and

$$F_{n_\beta n_\gamma L|m|\tau}(\beta) = N_\beta \beta^{q_{n_\gamma}^\tau(L,|m|)+1} \exp\Big(-\frac{\beta^2}{2b^2}\Big) L_{n_\beta}^{q_{n_\gamma}^\tau(L,|m|)+1/2}\Big(\frac{\beta^2}{b^2}\Big). \tag{15}$$

Here N_β and N_γ are normalization coefficients for β and γ wave functions, respectively, ${}_1F_1\left(-n_\gamma, |m|+1, \frac{\gamma^2}{\mathrm{g}}\right)$ is a confluent hyper-geometric function, $g_\beta = \frac{B_\beta V_0 \beta_0^2}{\hbar^2}$, $\mathrm{g} = \frac{1}{\beta_0^2}\frac{\hbar^2}{\sqrt{B_\gamma C_\gamma}}$, $b = \frac{\beta_0}{\sqrt[4]{2g_\beta}}$, $L_{n_\beta}^{q_{n_\gamma}^\tau(L,|m|)+1/2}$ are Laguerre polynomials, $D(\theta_i)$ is Wigner function, $\varphi(x_i)$ is the wave function of the single-particle states, and $A_{LK}^{m\tau}$ is determined to be eigenvectors with matrix elements $\langle LjK'm'|\hat{X}|LjKm\rangle$.

Then, if we denote deformability with respect to β vibration as

$$S_{n_\beta n_\gamma L|m|\tau; n'_\beta n'_\gamma L'|m'|\tau'} = \int_0^\infty F_{n_\beta n_\gamma L|m|\tau} \frac{\beta}{\beta_0} F_{n'_\beta n'_\gamma L'|m'|\tau'} d\beta \tag{16}$$

for the ground state intraband transitions the following expression is obtained:

$$\begin{aligned} B(E2; L_{\mathrm{g.s.}}+2 \to L_{\mathrm{g.s.}}) = \frac{5Q_0^2}{16\pi} & \left| \sum_{KK'} A_{LK}^{01} A_{L'K'}^{01} (L2K0|L'K') \right|^2 \\ & \times S_{00L01;00L'01}^2, \end{aligned} \tag{17}$$

where

$$\begin{aligned} S_{00L01;00L'01} = \frac{2}{\sqrt[4]{2g_\beta}} & \frac{1}{\sqrt{\Gamma(q_0^1(L,0)+3/2)}} \frac{1}{\sqrt{\Gamma(q_0^1(L',0)+3/2)}} \\ & \times \Gamma\left[\frac{q_0^1(L',0)+q_0^1(L,0)+4}{2}\right]. \end{aligned} \tag{18}$$

3 Results and Discussions

In the case $m = 0$, the angular momentum vector of the prolate core is perpendicular to the axis of symmetry, and thus it cannot contribute to the value of K. Then, the value of K is determined by the projection of the angular momentum of the last proton. The $7/2^+[404]$ ground state of the Ta isotopes we have determined from the Nilsson model. Since this state corresponds to $g_{7/2}$ spherical orbit, $K = 1/2$, $3/2$, $5/2$ and $7/2$ values are possible.

The ground state band of the Ta isotopes are calculated by diagonalizing (8) with all possible K taking into account the $K \pm 1$ mixture of non-diagonal elements.

Table 1. The values of the parameters used in calculations

Nucleus	ξ	g_β	$B_\beta/B_{\rm rot}$
^{175}Ta	0.01	187	3.2
^{177}Ta	0.01	219	3.2
^{179}Ta	0.01	227	3.2
^{181}Ta	0.01	587	3.2

The values of parameters used in the calculation are given in Table 1. The ground state band energies do not depend on parameters g and $B_\gamma/B_{\rm rot}$, therefore they depend only on three parameters given in the Table 1. The parameter ξ which connects single-particle and β vibrations and $B_\beta/B_{\rm rot}$ are chosen the same for all isotopes and g_β is different for each isotope.

Comparison of the calculated values of the ground state excited energies with experimental data relative to $E(9/2)$ energy are given in Tables 2 and 3. The experimental and intraband reduced $B(E2)$ transition probabilities with respect

Table 2. The calculated and experimental values of the $E(L)/E(9/2^+)$ for ^{175}Ta and ^{177}Ta. The experimental values are taken from Ref. [34]

L	^{175}Ta		^{177}Ta	
	Calc	Exp	Calc	Exp
$11/2^+$	2.19	2.19	2.19	2.20
$13/2^+$	3.55	3.55	3.57	3.59
$15/2^+$	5.08	5.07	5.11	5.15
$17/2^+$	6.74	6.72	6.80	6.88
$19/2^+$	8.53	8.49	8.63	8.75
$21/2^+$	10.44	10.34	10.58	10.75
$23/2^+$	12.44	12.27	12.65	12.86
$25/2^+$	14.54	14.27	14.81	15.07
$27/2^+$	16.72	16.32	17.07	17.35
$29/2^+$	18.97	18.46	19.40	19.70

Table 3. The calculated and experimental values of the $E(L)/E(9/2^+)$ for ^{179}Ta and ^{181}Ta. The experimental values are taken from Ref. [34]

L	^{179}Ta		^{181}Ta	
	Calc	Exp	Calc	Exp
$11/2^+$	2.20	2.20	2.21	2.21
$13/2^+$	3.57	3.60	3.63	3.63
$15/2^+$	5.12	5.17	5.24	5.26
$17/2^+$	6.81	6.91	7.05	7.08
$19/2^+$	8.65	8.80	9.04	9.09
$21/2^+$	10.61	10.81	11.20	11.29
$23/2^+$	12.69	12.93	13.54	13.67
$25/2^+$	14.87	15.14	16.03	16.21
$27/2^+$	17.14	17.41	18.66	18.93
$29/2^+$	19.50	19.74	21.45	21.78

to $B(E2; 11/2 \rightarrow 7/2)$ for ^{181}Ta we have listed in Table 4. The ground state and the first excites states energies are not listed in the Tables since they are always 0 an 1, respectively. As is seen from the Tables lower excited state energies are almost the same for all isotopes. The higher excited states energies increase with the increasing of the number of neutrons in both calculations and experiment. The increase of excited states energies is faster in the calculation. As is seen from Table 4 the values of the calculated intraband $B(E2)$ transition probabilities agrees with the experimental data within the uncertainties of the experimental measurements.

The other K bands originating from $g_{7/2}$ orbital are located much higher than $K = 7/2$ ground state band in the calculation. This may be the reason why they do not exist in the experiment. In the experiment there exist other bands which are not built in $g_{7/2}$ orbital. This suggest us to investigate the case when angular momentum of last proton is not conserved where the bands originating from other orbitals and their contribution to the ground state band energies as well as interband $B(E2)$ transition probabilities could be investigated.

Table 4. The calculated and experimental values of the $B(E2; L+2 \rightarrow L)$ in units of $B(E2; 11/2^+ \rightarrow 7/2^+)$ for ^{181}Ta. The experimental values are taken from Ref. [34]

$\frac{B(E2; L+2 \rightarrow L)}{B(E2; 11/2^+ \rightarrow 7/2^+)}$	^{181}Ta	
	Calc	Exp
$13/2^+ \rightarrow 9/2^+$	1.74	1.98(50)
$15/2^+ \rightarrow 11/2^+$	2.23	2.59(67)
$17/2^+ \rightarrow 13/2^+$	2.58	2.48(66)
$19/2^+ \rightarrow 15/2^+$	2.83	2.88(117)
$21/2^+ \rightarrow 17/2^+$	3.01	3.22(83)

4 Conclusion

The ground state band energies of Ta isotopes have been studied using the different mass parameters for each allowed collective mode. Calculated energy levels and reduced $B(E2)$ transition probabilities are compared with existing experimental data. It is shown that both deformation of the core and the interaction of the last odd proton with the core make a significant impact on the spectra of these nuclei. Also this investigation suggests us to complicate the model with inclusion of triaxility and the case when angular momentum of last nucleon is not conserved which allows to consider more possible mixture of the states with different K observed in the experiment.

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References

[1] A. Bohr, *Mat. Fys. Medd. K. Dan. Vidensk. Selsk.* **26** (1952) 1.

[2] J.M. Eisenberg and W. Greiner, "*Nuclear Models: Collective and Single-Particle Phenomena*" (North Holland, Amsterdam, 1970).

[3] A.S. Davydov and A.F Filipov, *Nucl. Phys.* **8** (1986) 237.

[4] A.S. Davydov and A.A. Chaban, *Nucl. Phys.* **20** (1996) 499.

[5] A. Faessler and W. Greiner, *Z. Phys.* **168** (1962) 425.

[6] A.S. Davydov and R.A. Sardaryan, *Nucl. Phys.* **37** (1962) 106.

[7] V.V. Pashkevich and R.A. Sardaryan, *Nucl. Phys.* **65** (1965) 401.

[8] F. Iachello, *Phys. Rev. Lett.* **87** (2001) 052502.

[9] R.F. Casten and N.V. Zamfir, *Phys. Rev. Lett.* **87** (2001) 052503.

[10] D. Bonatsos, D. Lenis, N. Minkov, P.P. Raychev and P.A. Terziev, *Phys. Rev. C* **69** (2004) 014302.

[11] D. Bonatsos, P.E. Georgoudis, D. Lenis, N. Minkov and C. Quesne, *Phys. Rev. C* **83** (2011) 044321.

[12] I. Yigitoglu and D. Bonatsos, P.E. Georgoudis, D. Lenis, N. Minkov and C. Quesne, *Phys. Rev. C* **83** (2011) 014303.

[13] D. Bonatsos *et al.*, *Phys. Rev. C* **76** (2007) 064312.

[14] L. Fortunato, *Phys. Rev. C* **70** (2004) 011302.

[15] R. Rowe, *Nucl. Phys.* **735** (2004) 372.

[16] M.A. Caprio, *Phys. Rev. C* **83** (2011) 064309.

[17] R.V. Jolos and P. von Brentano, *Phys. Rev. C* **74** (2006) 064307.

[18] R.V. Jolos and P. von Brentano, *Phys. Rev. C* **76** (2007) 024309.

[19] R.V. Jolos and P. von Brentano, *Phys. Rev. C* **78** (2008) 064309.

[20] R.V. Jolos and P. von Brentano, *Phys. Rev. C* **79** (2009) 044310.

[21] Bonatsos *et al.*., *Phys. Rev. C* **88** (2013) 034316.

[22] R. Jolos, N. Shirikova and A. Sushkov, *Phys. At. Nucl.* **76** (2013) 557.

[23] Sh. Sharipov and M.J. Ermamatov, *Int. J. Mod. Phys. E* **12** (2003) 41.

[24] Sh. Sharipov, M.J. Ermamatov and J.K. Bayimbetova, *Int. J. Mod. Phys. E* **12** (2006) 951.

[25] Sh. Sharipov, M.J. Ermamatov and J.K. Bayimbetova, *Int. J. Mod. Phys. E* **71** (2008) 215.

[26] M.J. Ermamatov and P.R. Fraser, *Phys. Rev. C* **84** (2011) 044321.

[27] M.J. Ermamatov, P.C. Srivastava, P.R. Fraser and P. Stránský, *Euro. Phys. J. A* **48** (2012) 1.

[28] M.J. Ermamatov *et al.*, *Phys. Rev. C* **84** (2012) 034307.

[29] A. Frank *et al.*, *Phys. Lett. B* **84** (1986) 233.

[30] A. Arima, A. Gelberg and O. Scholten, *Phys. Lett. B* **185** (1987) 259.

[31] R.F. Casten, "*Nuclear Nuclear Structure from a Simple Perspective*" (Oxford University Press, 1990).

[32] F. Iachello and P. Van Isacker, "*The Interacting Boson Fermion Model*" (Cambridge University Press, Cambridge, 1991).

[33] A. Frank, P. Van Isacker, *Phys. Lett. B* **197** (1987) 474.

[34] http://www.nndc.bnl.gov/ensdf