# Halo Structure of <sup>11</sup>Li and Its Effect on the (p,t)Reaction

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**Abstract.** The reaction  ${}^{11}\text{Li}(p,t){}^9\text{Li}(gs)$  is investigated in terms of a very simplistic distorted-wave Born approximation. The halo neutrons involved in the reaction may be treated as a di-neutron cluster, and it is shown that this appears to be a good approximation. The dominant contribution to the transfer reaction comes from the  $(1s_{1/2})^2$  structure component of  ${}^{11}\text{Li}$ , and the cross section angular distribution seems to be relatively insensitive to the fact that  ${}^{11}\text{Li}$  has an anomalously large radius due to its Borromean halo properties. It is perhaps significant that a simple treatment of the reaction is in much better agreement with the experimental angular distribution than a more sophisticated calculation.

# 1 Introduction

The exotic nucleus <sup>11</sup>Li with its two-neutron halo structure and so-called Borromean property is an object of considerable interest [1]. Although the two valence neutrons are very weakly bound with a two-neutron separation energy of 380 keV, the nucleus <sup>11</sup>Li has a relatively long half-life of about 8.8 ms. However, removal of one neutron causes the remaining <sup>10</sup>Li system to decay almost immediately, within  $10^{-21}$  s. Thus the three <sup>9</sup>Li-n-n components which are postulated to form <sup>11</sup>Li are symbolically reminiscent of the well-known crest of the ancient Borromeo family, which consists of three rings interlinked in such a way that removal of any single ring unties the remaining two, hence the Borromean designation.

The radius of <sup>11</sup>Li is anomalously large at about of 3.5 fm. Its ground state structure can be expressed as [2]

$$|^{11}Li(gs); 3/2^-\rangle = |\tilde{0}\rangle_{\nu} \otimes |1p_{3/2}(\pi)\rangle,$$
 (1)

with proton  $\pi$  and neutron  $\nu$  degrees of freedom and where  $|\tilde{0}\rangle_{\nu}$  is the halo wave function coupled to  $1^-$  and  $2^+$  vibrations of the <sup>9</sup>Li core, expressed as

$$|\tilde{0}\rangle_{\nu} = |0\rangle + \alpha |(p_{1/2}, s_{1/2})_{1^{-}} \otimes 1^{-}; 0\rangle + \beta |(s_{1/2}, d_{5/2})_{2^{+}} \otimes 2^{+}; 0\rangle, \quad (2)$$

where

$$|0\rangle = 0.45|s_{1/2}^2(0)\rangle + 0.55|p_{1/2}^2(0)\rangle + 0.04|d_{5/2}^2(0)\rangle.$$
(3)

#### A.A. Cowley

Clearly a two-neutron transfer reaction, such as  ${}^{11}\text{Li}(p,t){}^{9}\text{Li}$ , promises to reveal interesting properties of the system, and this has been explored by Tanihata *et al.* [3]. The short lifetime of  ${}^{11}\text{Li}$  requires an experiment in inverse kinematics, which in the normal convention is written as  ${}^{1}\text{H}({}^{11}\text{Li},t){}^{9}\text{Li}$ , where  ${}^{11}\text{Li}$  is the projectile and the target  ${}^{1}\text{H}$ . Because the target needs to be very thick to compensate for low beam currents, it is required to act simultaneously as an active detector system [3]. We indicate the reaction as  ${}^{11}\text{Li}(p,t){}^{9}\text{Li}$  to emphasize its two-neutron transfer character. This notation has no special significance or implication beyond the normal convention.

Tanihata *et al.* [3] measured the <sup>11</sup>Li(p,t)<sup>9</sup>Li reaction at a laboratory energy of 3A MeV. As a result of the low Coulomb barrier of only ~500 keV, the reaction is expected to be reasonably direct even at such a low incident energy. The reaction to the ground state of <sup>9</sup>Li( $J^{\pi} = 3/2^{-}$ ), as well as the excited state at  $E_x = 2.69$  MeV with  $J^{\pi} = 1/2^{-}$ , were explored. A direct transfer to the excited state confirms the presence of a 1<sup>+</sup> and/or 2<sup>+</sup> halo component of the ground state of <sup>11</sup>Li, as given by Eq. 2.

In this paper a very brief description of existing sophisticated theoretical analyses of the  ${}^{11}\text{Li}(p,t){}^{9}\text{Li}(gs)$  reaction which exist at an incident energy of 3A MeV will be presented. This type of analysis is also relevant at a slightly higher incident energy value of 4.4A MeV discussed mostly in this paper. Those results will be compared with a very simplistic treatment of the reaction mechanism in which only the most crucial properties of the nuclear system is included initially. Some further refinements are subsequently introduced to improve agreement with the data. This approach has the virtue that the sensitivity of the theory to the known structure of the nuclear species involved in the reaction may be investigated properly. It will be shown that certain properties of  ${}^{11}\text{Li}$  put a prominent stamp on the features of the cross section angular distribution, whereas on the other hand, some other characteristics surprisingly seem to offer very little sensitivity.

#### 2 Advanced Microscopic Formulation

A complete microscopic calculation of the angular distribution of the reaction  ${}^{11}\text{Li}(p,t){}^{9}\text{Li}(\text{gs})$ , in which all the known properties of the target and intermediate nuclei participating in the two-nucleon pickup, as well as important details of the reaction mechanism, is presented in Ref. [3]. In the distorted-wave Born approximation (DWBA) calculations several alternative three-body models are compared which have  $(1s_{1/2})^2$  components of differing strength. Direct two-nucleon pickup, as well as sequential transfer through intermediate  $1/2^-$  and  $1/2^+$  neutron states in  ${}^{10}\text{Li}$ , are considered. In spite of the relative sophistication of the calculations, the reproduction of the experimental angular distribution is disappointing, both with respect to absolute magnitude and shape.

Thompson [4] also performed preliminary calculations at a slightly higher incident energy of 4.4A MeV for the  ${}^{11}\text{Li}(p,t){}^{9}\text{Li}$  reaction to the ground state.





Figure 1. Preliminary results of Thompson [4] for the reaction  ${}^{11}\text{Li}(p,t){}^{9}\text{Li}(gs)$  at an incident energy of 4.4A MeV. The curves represent results for  $(s_{1/2})^2$ -contributions of 3% (P0), 31% (P2) and 45% (P3) in the  ${}^{11}\text{Li}$  wave function. The line indicated as  $(p_{1/2})^2$  is a simple model based on P0, but with no n - n potential to correlate the neutrons. Further details of the calculation are available in Ref. [3]. (*Figure reproduced with permission from I. J. Thompson*).

These are shown in Figure 1. In the sequential part of the mechanism the intermediate deuteron is taken to be in its ground state only, and s and p waves are used in the so-called weak-binding approximation for <sup>10</sup>Li. Core excitation is ignored. Distorted waves are calculated with standard global optical model sets, as were also used in Ref. [3]. As shown in Figure 1, the theoretical agreement with the experimental data is far from satisfactory. Furthermore, it is unlikely that improvement of the treatment of the intermediate deuteron or inclusion of core excitation will resolve the problem of the observed shape disagreement.

Potel *et al.* [2] manage to describe the  ${}^{11}\text{Li}(p,t){}^9\text{Li}(gs)$  reaction at the lower energy of 3A MeV fairly well, but for our present interest their treatment raises a number of other issues which need to be clarified, as will be explained later. Nevertheless, it is wise to keep both the results of Thompson [4] and Potel *et al.* [2] in mind when assessing the significance of the conclusions from the new study presented in the next Section.

# 3 Simplistic Di-Neutron Transfer Approach

In this paper a simplistic approach to the reaction mechanism is explored. This method is inspired by the fact that it is known to give excellent shape agreement with proton-induced two-nucleon transfer angular distributions over a large range of incident energies (See for example Refs. [5–9]). In keeping with the aim

#### A.A. Cowley

of initially finding the simplest theoretical description which gives reasonable agreement with the experimental angular distribution of  ${}^{11}\text{Li}(p,t){}^{9}\text{Li}(gs)$  reaction, a DWBA calculation is performed in zero range with the code DWUCK4 [10]. The transferred neutron pair is treated as a di-neutron cluster, with quantum numbers derived from those of the structures associated with the individual neutrons, given by

$$2N + L = \sum_{i=1}^{2} (2n_i + \ell_i), \tag{4}$$

where N and L are the principal and orbital angular momentum of the cluster and  $n_i$  and  $\ell_i$  are those of its components. This treatment, based on conservation of oscillator quanta, has been found to be useful in general for two-nucleon transfer reactions [11–13]. The form factor of the di-neutron is obtained by the usual procedure of adjusting the well depth of a Woods–Saxon potential with geometrical parameters  $r_0 = 1.15$  fm and a = 0.76 fm [11–13] to obtain the correct binding energy and wave function characteristics. This procedure, with those specific geometrical parameters, is expected to result in macroscopic form factors which have almost the same shape as their microscopic counterparts [11]. Distorted waves were generated by using the same global optical potential parameters as listed explicitly by Tanihata *et al.* [3].

Results for  ${}^{11}\text{Li}(p,t){}^{9}\text{Li}(gs)$  are presented in Figure 2. As is evident, the the-



Figure 2. Comparison between macroscopic and microscopic calculations for  ${}^{11}\text{Li}(p,t)^9\text{Li}(\text{gs})$  at an incident energy of 4.4A MeV as described in the text. In the microscopic case a pure  $(s_{1/2})^2$  configuration is assumed, whereas for the macroscopic calculation a di-neutron cluster and an angular momentum transfer of L = 0 are introduced. The normalizations to the experimental data are arbitrary.

# Halo Structure of $^{11}Li$ and Its Effect on the (p,t) Reaction

oretical prediction, with the cross section normalized to the experimental data, is in very good shape agreement with expectation. In Figure 2 this macroscopic prediction is also compared with a microscopic calculation in which only the  $(1s_{1/2})^2$  pickup component was taken into account. Standard single particle bound state geometrical parameters of  $r_0 = 1.25$  fm and a = 0.65 fm are used in the microscopic treatment. The neglect of the *p* and *d* wave contributions of the ground state of <sup>11</sup>Li is considered to be justified because of the dominance of the *s* state contribution towards the cross section magnitude, as already suggested by the results shown in Figure 1.

In Figure 3 the possible influence of a *p*-state contribution is studied further. Microscopic calculations for  $(1s_{1/2})^2$  and  $(0p_{1/2})^2$ , arbitrarily normalized as shown to reproduce a correct magnitude for a combination of these two extreme approximations, are compared with the experimental angular distribution. Somewhat better agreement with the experimental shape is achieved than with only  $(1s_{1/2})^2$ , but in view of the many approximations in the theoretical treatment the observed improvement is probably not significant.

It is noteworthy that in all the calculations shown in Figures 2 and 3 the nucleus <sup>11</sup>Li is treated as a normal species, in other words the halo structure, with its implied anomalously large nuclear radius, is totally ignored. In spite of this flaw, the reproduction of the experimental angular distribution is remarkably better in all the calculations than with the vastly more sophisticated theoretical treatment presented in Figure 1.



Figure 3. Microscopic calculations for <sup>11</sup>Li(p,t)<sup>9</sup>Li(gs) at an incident energy of 4.4A MeV. Contributions are from a component with a pure  $(s_{1/2})^2$  configuration (with angular momentum transfer L = 0), plus a yield from a pure  $(p_{1/2})^2$  structure with L = 2 transfer. The normalizations have been chosen independently and arbitrarily in order that the incoherent sum of the two contributions should best reproduce the experimental data.





Figure 4. Macroscopic calculations for  ${}^{11}\text{Li}(p,t){}^{9}\text{Li}(gs)$  at an incident energy of 4.4A MeV with various values of the bound state radius of the di-neutron cluster. The lowest indicated value of the radius corresponds to a number appropriate for nuclei in general, whereas the largest value agrees roughly with the size of the halo nucleus  ${}^{11}\text{Li}$ . The diffuseness parameter of the bound state is fixed at the standard value listed in the text. The well depth is adjusted for each case to reproduce simultaneously the separation energy of the di-neutron and properties of wave function as prescribed by the adopted quantum numbers of the cluster.

The sensitivity of the cross section to the nuclear size is investigated in Figure 4 by varying the bound state radius of the di-neutron to reproduce this effect roughly. No significant difference is observed as the value is changed from normal to an upper value which is more consistent with that of a halo nucleus. The conclusion is that the shape of the angular distribution of the transfer reaction is relatively unaffected by the large radius of <sup>11</sup>Li. This is very surprising, and it needs to be studied further.

In these investigations presented here no importance was attached to absolute cross sections, and all theoretical results were simply normalized to the experimental quantity. This approach is followed because of the known problem in transfer reactions which suffer from large momentum mismatch. For example, in spite of the low incident energy in the present case of <sup>11</sup>Li(p,t)<sup>9</sup>Li, the reaction still experiences a momentum mismatch of more than 200 MeV/c. This puts an enormous strain on the accurate prediction of absolute cross sections. Transfer reactions such as (p,t) and (p,<sup>3</sup>He) have long been suspected of underpredicting cross sections in DWBA because of the presence of a sequential reaction together with a simultaneous, one-step direct mechanism [14]. For the reaction studied most extensively, namely to the unnatural parity transfer reaction  $^{208}$ Pb(p,t)<sup>206</sup>Pb(3<sup>+</sup>) which is forbidden in zero-range DWBA, the conflicting

results of Igarashi *et al.* [15] and Nagarajan *et al.* [16] can be described at best as controversial [7]. Charlton [17] confirms the opinion that the introduction of a sequential mechanism, especially in an allowed transition, may not be much more than a parametrization to give slightly better agreement with experimental data. That is to say, when it is not good enough already without the need to include such a mechanism explicitly.

As was already mentioned earlier, Potel *et al.* [2] do reproduce the cross section distributions of  ${}^{11}\text{Li}(p,t){}^9\text{Li}$  at an incident energy of 3A MeV for the ground state as well as the excited state. To achieve this, a coupled channel approach is followed and all the structure and reaction mechanism information, as required for the sophisticated theory as discussed in Sec. 2, is employed. However, in this work the orthogonal term [18] which comes from sequential transfer appears to be carefully chosen [19] to preserve the shape of the simultaneous transfer to the ground state. To a great extent the inclusion of core polarization serves to reduce the theoretical cross section to a final value which is in agreement with the experimental quantity.

# 4 Summary and Conclusions

Proton-induced two-neutron transfer on <sup>11</sup>Li was investigated. A simplistic DWBA calculation was shown to give a reasonably good reproduction of a cross section angular distribution for the reaction  ${}^{11}\text{Li}(p,t){}^{9}\text{Li}(gs)$  at an incident energy of 4.4A MeV. Similar agreement is obtained irrespective of whether dineutron transfer is assumed, or a more realistic microscopic two-particle mechanism is considered. Results from this present study were compared with those from more sophisticated analyses at the same or lower incident energy, and found to be mostly superior. The response to the transfer reaction is similar to a normal nuclear species. In view of the peculiar halo valence structure of  ${}^{11}\text{Li}$ , it is very surprising that the theoretical transfer results appear to be fairly insensitive to its large nuclear size.

The simplicity of the theoretical treatment, combined with its ability to reproduce the main characteristics of experimentally observed angular distributions, is encouraging. Clearly this needs to be investigated further.

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## A.A. Cowley

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