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Neutrinoless Double Beta Decay with Emission of s and p Electrons

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Abstract. We have extended the light Majorana neutrino mass formalism for the neutrinoless double beta decay by including effects of the $p_{1/2}$-waves of emitted electrons and nucleon recoil. Within a standard approximation the decay rate is factorized as a sum of products of kinematical phase space factors and nuclear matrix elements. By using exact Dirac wave function with finite nuclear size and electron screening numerical computation of phase space integrals was performed. The obtained results allow to conclude that the effect of the p-wave and nucleon recoil is small, but not negligible. A more precise conclusion requires a calculation of corresponding nuclear matrix elements within an appropriate nuclear structure method.

1 Introduction

Neutrinoless double-beta decay ($0\nu\beta\beta$-decay), which involves the emission of two electrons and no neutrinos,

$$(A, Z) \rightarrow (A, Z + 2) + 2e^{-}. \tag{1}$$

is expected to occur as the total lepton number is not an exact symmetry of nature. The $0\nu\beta\beta$-decay is the most powerful tool to clarify whether the neutrino is a Dirac particle (i.e., different from its antiparticle) or a Majorana particle (i.e., identical to its own antiparticle) as the only one of all fermions [1].

The $0\nu\beta\beta$-decay has not yet been confirmed. The presently best lower bound on the $0\nu\beta\beta$-decay half-life have been achieved by GERDA ($T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) \geq 3.0\times10^{25}$ yrs) [2], EXO and KamLAND-ZEN experiments ($T^{0\nu-exp}_{1/2}(^{136}\mathrm{Xe}) \geq 3.4 \times 10^{25}$ yrs) [3]. The ultimate goal of experiments on the search for $0\nu\beta\beta$-decay is the measurement of the effective Majorana neutrino mass,

$$m_{\beta\beta} = U^2_{e1}m_1 + U^2_{e2}m_2 + U^2_{e3}m_3. \tag{2}$$

Here, U_{ei} and m_i (i=1,2,3) are elements of Pontecorvo-Maki-Nakagawa-Sakata neutrino mixing matrix and masses of neutrinos, respectively. The search for the $0\nu\beta\beta$-decay represents the new frontiers of neutrino physics, allowing in principle to fix the neutrino mass scale, the neutrino nature and possible CP violation effects.

The inverse value of the $0\nu\beta\beta$-decay half-life for a given isotope (A,Z) is commonly written as [1]

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = G_{0\nu}(Q,Z)g_A^4 \left|M^{0\nu}\right|^2 \frac{|m_{\beta\beta}|^2}{m_e^2}. \tag{3}$$

Here, $G_{0\nu}(Q,Z)$, g_A and $M^{0\nu}$ stand for the known phase-space factor, the axial-vector coupling constant and the nuclear matrix element of the process, respectively.

The goal of this paper is to derive more accurate expression for the $0\nu\beta\beta$-decay rate by considering also emission of the p-wave electrons and the nucleon recoil. To our knowledge, their impact on the calculation of the $0\nu\beta\beta$-decay half-life has not been studied yet.

2 Decay Rate for the Neutrinoless Double-Beta Decay

2.1 The hadronic currents in the non-relativistic approximation

We shall consider the $0\nu\beta\beta$-decay, assuming that the weak β-decay Hamiltonian has the standard form,

$$H^\beta = \frac{G_\beta}{\sqrt{2}} \left[(\bar{e}\gamma_\rho(1-\gamma_5)\nu_e)\ J^{\rho\dagger} + h.c.\right]. \tag{4}$$

Here, $G_\beta = G_F \cos\theta_C$, where G_F and θ_C are Fermi constant and Cabbibo angle, respectively. ν_e is the Majorana neutrino field, J^ρ is the standard hadronic current in the V-A theory:

$$\begin{aligned}\langle p(P_n')|\, J^{\mu\dagger}\, |n(P_n)\rangle = \bar{u}_p(P_n')\Big[& g_V\gamma^\mu + ig_M\frac{\sigma^{\mu\nu}}{2m_N}(P_n'-P_n)_\nu \\ & - g_A\gamma^\mu\gamma_5 - g_p\gamma_5(P_n'-P_n)^\mu \ \Big]u_n(P_n'),\end{aligned} \tag{5}$$

where the $u_p(P_n')$ and $u_n(P_n)$ are the spinors describing the proton and neutron with corresponding four-momenta $P_n'^\mu = (E_n', \mathbf{p}_n')$ and $P_n^\mu = (E_n, \mathbf{p}_n)$, respectively. m_N is the nucleon mass, $q = P_n' - P_n$ is the momentum transfer and $q_V \equiv q_V(q^2)$, $q_M \equiv q_M(q^2)$, $q_A \equiv q_A(q^2)$ and $q_P \equiv q_P(q^2)$ are the vector, weak-magnetism, axial-vector and induced pseudoscalar form-factors, respectively.

Within the non-relativistic impulse approximation, the hadronic current for the nuclear β decay can be written as (See [4, 5])

$$
\begin{aligned}
J^\rho(\mathbf{x}) = \sum_n \tau_n^+ \delta(\mathbf{x}-\mathbf{r}_n)\Big\{ & \ (g_V - g_A C_n) g^{\rho 0} \\
& + \Big(g_A \sigma_n^k - g_V D_n^k - g_P (p_n^k - p_n^{'k}) \frac{\vec{\sigma}_n \cdot (\mathbf{p}_n - \mathbf{p}_n^{'})}{2m_N}\Big) g^{\rho k} \Big\}\,, \qquad (6)
\end{aligned}
$$

where $\vec{\sigma}_n$ is the Pauli matrix, τ_n^+ is the isospin raising operator and $\mathbf{r}_n$ is the position operator. All these operators act on the n-th neutron. The nucleon recoil operators C_n and $\mathbf{D}_n$ are defined as

$$
C_n = \frac{\vec{\sigma} \cdot (\mathbf{p}_n + \mathbf{p}_n^{'})}{2m_N} - \frac{g_P}{g_A}(E_n - E_n^{'}) \frac{\vec{\sigma} \cdot (\mathbf{p}_n - \mathbf{p}_n^{'})}{2m_N}, \qquad (7)
$$

$$
\mathbf{D}_n = \frac{(\mathbf{p}_n + \mathbf{p}_n^{'})}{2m_N} - i\Big(1 + \frac{g_M}{g_V}\Big) \frac{\vec{\sigma} \times (\mathbf{p}_n - \mathbf{p}_n^{'})}{2m_N}. \qquad (8)
$$

2.2 Distorted electron wave function

In the $0\nu\beta\beta$ decay, the emitted electrons are attracted by the Coulomb force of a final nucleus. This interaction is substantial in the case of single- and double-beta decays of medium and heavy nuclei with large Z. The electron wave functions are distorted in the presence of the Coulomb field enhancing the overlap of wave functions of electrons and nucleus.

We approximate electron wave function with a sum of $s_{1/2}$ and $p_{1/2}$ waves in the partial wave expansion. Then, we have (For details see [4].),

$$
\begin{aligned}
\psi(\mathbf{r}, p, s) &\simeq \psi_{s_{1/2}}(\mathbf{r}, p, s) + \psi_{p_{1/2}}(\mathbf{r}, p, s) \\
&= \begin{pmatrix} g_{-1}(\varepsilon, r)\chi_s \\ f_1(\varepsilon, r)(\vec{\sigma}\cdot\hat{\mathbf{p}})\chi_s \end{pmatrix} + \begin{pmatrix} i g_1(\varepsilon, r)(\vec{\sigma}\cdot\hat{\mathbf{r}})(\vec{\sigma}\cdot\hat{\mathbf{p}})\chi_s \\ -i f_{-1}(\varepsilon, r)(\vec{\sigma}\cdot\hat{\mathbf{r}})\chi_s \end{pmatrix}. \qquad (9)
\end{aligned}
$$

Given the atomic potentials, $g_{\pm 1}(\varepsilon, r)$ and $f_{\pm 1}(\varepsilon, r)$, are solutions of the radial Dirac equations.

2.3 S-Matrix and Half-Life for the $0\nu\beta\beta$-Decay

The $0\nu\beta\beta$-decay is a process of second order in the perturbation theory of weak interactions. The matrix element of this process takes the form:

$$
\begin{aligned}
\langle f | \, S^{(2)} \, | i \rangle = & \frac{(-i)^2}{2} \Big(\frac{G_\beta}{\sqrt{2}}\Big)^2 \frac{1}{(2\pi)^3 \sqrt{4\varepsilon_1 \varepsilon_2}} \\
& \times \sum_i U_{ei}^2 \int dx dy \int \frac{d^4 q}{(2\pi)^4} e^{-iq(x-y)} \langle A_f | \, T[J^{\rho\dagger}(x) J^{\sigma\dagger}(y)] \, | A_I \rangle \\
& \times \bar{\psi}(p_1, \mathbf{x}) e^{i\varepsilon_1 x^0} \gamma_\rho (1-\gamma_5) \frac{i(\not{q} + m_i)}{q^2 - m_i^2} C (1-\gamma_5) \gamma_\sigma^T \bar{\psi}^T(p_2, \mathbf{y}) e^{i\varepsilon_2 y^0} \\
& - (\varepsilon_1 \leftrightarrow \varepsilon_2). \qquad (10)
\end{aligned}
$$

By neglecting the neutrino masses m_i and replacing the electron energies with half of the energy release in the process ($\varepsilon_1 \approx \varepsilon_2 \approx (M_i - M_f)/2$, where M_i and M_f are masses of the initial and final nuclei, respectively) in the energy denominators for T-matrix, we find

$$\begin{aligned}
&\langle f|\, T^{(2)}\, |i\rangle = \\
&\quad C\frac{R}{2}\int d\mathbf{x}d\mathbf{y}\int\frac{d\mathbf{q}}{2\pi^2|\mathbf{q}|}e^{i\mathbf{q}(\mathbf{x}-\mathbf{y})}\frac{\sum_n \langle A_f|\, J^\rho(\mathbf{x})\,|n\rangle\,\langle n|\, J^\sigma(\mathbf{y})\,|A_i\rangle}{|\mathbf{q}| + E_n - (M_f + M_i)/2} \\
&\quad \times [\bar{\psi}(p_1,\mathbf{x})\gamma_\rho(1-\gamma_5)\gamma_\sigma\psi^c(p_2,\mathbf{y}) + \bar{\psi}(p_1,\mathbf{y})\gamma_\sigma(1-\gamma_5)\gamma_\rho\psi^c(p_2,\mathbf{x})]. \qquad (11)
\end{aligned}$$

Here, $C = -\frac{G_\beta^2}{2}\frac{1}{\sqrt{4\varepsilon_1\varepsilon_2}}\frac{2}{4\pi R}\frac{1}{(2\pi)^3}m_{\beta\beta}$ and $|n\rangle$ is the n-th state of the intermediate nucleus.

We go beyond the standard approach by considering the electron wave function $\psi(p,\mathbf{r})$ (see Eq.(9)), which includes in addition to the $s_{1/2}$-wave also the $p_{1/2}$-wave of emitted electrons. Contributions of higher partial waves like $p_{3/2}$, $d_{3/2}, \ldots$ waves to the $0\nu\beta\beta$ decay rate are assumed to be negligible. The nuclear current $J^\rho(\mathbf{x})$ is considered within the approximation given by Eq. (6). Then, the $0\nu\beta\beta$-decay rate can be written as

$$\begin{aligned}
\Gamma^{0\nu} = \frac{1}{2}\frac{G_\beta^4 m_e^5}{16\pi^5}\frac{|m_{\beta\beta}|^2}{R^2}\Big(&|M_1|^2 G_{11} + |M_2|^2 G_{22} + |M_3|^2 G_{33} \\
&+ 2Re\{M_1 M_2^*\}G_{12} + 2Re\{M_1 M_3^*\}G_{13} + 2Re\{M_2 M_3^*\}G_{23}\Big). \qquad (12)
\end{aligned}$$

A factorization of phase-space factors G_{ij} and nuclear matrix elements was achieved by the approximation in which electron wave functions $g_{\pm1}(\varepsilon, r)$ and $f_{\pm1}(\varepsilon, r)$ are replaced by their values at the nuclear radius R. Nuclear matrix elements M_n $(n = 1, 2, 3)$ are given in the next subsection (2.4) and phase-space integrals are given by

$$\begin{aligned}
G_{ij} = \frac{1}{m_e^5}\int_{m_e}^{M_i - M_f - m_e} &\sqrt{(M_i - M_f - \varepsilon_1)^2 - m_e^2}(M_i - M_f - \varepsilon_1) \\
&\times \varepsilon_1\sqrt{\varepsilon_1^2 - m_e^2}\, g_{ij}(\varepsilon_1, M_i - M_f - \varepsilon_1)d\varepsilon_1, \qquad (13)
\end{aligned}$$

where factors $g_{ij}(\varepsilon_1, \varepsilon_2)$ are expressed in the Appendix as a superposition of different products of radial wave functions of electron evaluated at the nuclear radius $r = R$. We note that if the contribution of the $p_{1/2}$ wave of electrons to the $0\nu\beta\beta$-decay rate is neglected one ends up with Eq. (3). The relation between $G_{0\nu}^{(0)}$ introduced in (3) and the phase-factor G_{11} in Eq. (13) is $G_{0\nu}^{(0)} = \dfrac{G_\beta^4 m_e^7}{32\pi^5 R^2 \ln 2}G_{11}$.

2.4 Nuclear matrix elements

Nuclear matrix elements entering the $0\nu\beta\beta$-decay rate (Eq. 12) are derived by assuming the closure approximation for intermediate nuclear states. Within this approximation the energies of intermediate states relative to the initial ground state $(E_n - (E_i + E_f)/2)$ are replaced by an average value $\bar{E}_n - (E_i + E_f)/2 \sim$ 10 MeV, and the sum over intermediate states is taken by closure, $\sum_n |n\rangle\langle n| = 1$. The nuclear matrix elements are given by

$$\begin{aligned} M_1 &= M_F + M_{GT} + M_T \\ M_2 &= M'_F + M'_{GT} + M'_T + M_V + M_{AP} + M_{AA} \ , \end{aligned} \tag{14}$$

where

$$\begin{aligned} M_{F,GT,T} &= \sum_{r,s} \langle 0|\, h_{F,GT,T}(r_-) \mathcal{O}_{\mathcal{F},\mathcal{GT},\mathcal{T}} \,|0\rangle \\ M'_{F,GT,T} &= \sum_{r,s} \langle 0|\, h_{F,GT,T}(r_-) \mathcal{O}_{\mathcal{F},\mathcal{GT},\mathcal{T}} \left(1 - \frac{|\mathbf{r}_-|^2}{2R^2}\right) |0\rangle \\ M_V &= i \sum_{r,s} \langle 0|\, \{h_{AV}(r_-) + h_{VP}(r_-)\} \,\frac{(\mathbf{r}_- \times \mathbf{r}_+)}{R^2} \cdot \vec{\sigma}_r \,|0\rangle \\ M_{AA} &= \sum_{r,s} \langle 0|\, h_{AA}(r_-) \tau_r^+ \tau_s^+ (\vec{\sigma}_r \times \vec{\sigma}_s) \cdot \frac{\mathbf{r}_- \times \mathbf{r}_+}{R^2} \,|0\rangle \\ M_{AP} &= \sum_{r,s} \langle 0|\, \frac{h_{AP}(r_-)}{R^2} \tau_r^+ \tau_s^+ (\vec{\sigma}_r \cdot \mathbf{r}_-)(\vec{\sigma}_s \cdot \mathbf{r}_+) \,|0\rangle \ , \end{aligned} \tag{15}$$

and the nucleon recoil matrix element can be written as

$$M_3 = \sum_{r,s} \langle 0|\, h_R(r_-) \left(\mathcal{O}_{\mathcal{T}} - 2\mathcal{O}_{\mathcal{GT}}\right) |0\rangle \ . \tag{16}$$

The neutrino potentials take the form

$$\begin{aligned} h_F(r) &= \frac{2R}{\pi} \int dq\, g_V^2(q^2) \frac{q j_0(qr)}{q+\Delta} \\ h_{GT}(r) &= \frac{2R}{\pi} \int dq \Big(- g_A^2(q^2) + \frac{g_A(q^2) g_P(q^2)}{m_N} \frac{q^2}{3} - \frac{g_P^2(q^2)}{4m_N^2} \frac{q^4}{3} \Big) \frac{q j_0(qr)}{q+\Delta} \\ h_T(r) &= \frac{2R}{\pi} \int dq \Big(- \frac{g_A(q^2) g_P(q^2)}{m_N} \frac{q^2}{3} + \frac{g_P^2(q^2)}{4m_N^2} \frac{q^4}{3} \Big) \frac{q j_2(qr)}{q+\Delta} \\ h_{AV}(r) &= -\frac{2R}{\pi} \int dq\, 2 g_A(q^2) g_V(q^2) \frac{q j_0(qr)}{q+\Delta} \\ h_{VP}(r) &= \frac{2R}{\pi} \int dq\, \frac{g_V(q^2) g_P(q^2)}{m_N} \frac{q^3}{3} \frac{(j_0(qr) + j_2(qr))}{q+\Delta} \\ h_{AA}(r) &= \frac{2R}{\pi} \int dq\, g_A^2(q^2) \frac{q j_0(qr)}{q+\Delta} \end{aligned}$$

$$h_{AP}(r) = -\frac{2R}{\pi}\int dq\, \frac{g_A(q^2)g_P(q^2)}{m_N}\frac{q^3}{3}\frac{(2j_0(qr)+3j_2(qr))}{q+\Delta}$$
$$h_R(r) = -\frac{2}{3\pi}\int dq\, \frac{g_A(q^2)g_V(q^2)+g_M(q^2)g_A(q^2)}{2m_N}\frac{q^2 r j_1(qr)}{q+\Delta}\ .$$

Here, $\mathbf{r}_+ = (\mathbf{r}_r+\mathbf{r}_s)/2$, $\mathbf{r}_- = (\mathbf{r}_r-\mathbf{r}_s)$, where $\mathbf{r}_{r,s}$ is the coordinate of decaying nucleon. $\Delta = \bar{E}_n - (E_i+E_f)/2$ and $j_i(qr)$ (i=1,2,3) are the spherical Bessel functions. The double Fermi, double Gamow–Teller and tensor operators are given by

$$\begin{aligned}\mathcal{O}_F &= \tau_r^+\tau_s^+\\ \mathcal{O}_{GT} &= \tau_r^+\tau_s^+(\vec{\sigma}_r\cdot\vec{\sigma}_s)\\ \mathcal{O}_T &= 3\tau_r^+\tau_s^+(\vec{\sigma}_r\cdot\hat{\mathbf{r}}_-)(\vec{\sigma}_s\cdot\hat{\mathbf{r}}_-) - \tau_r^+\tau_s^+(\vec{\sigma}_r\cdot\vec{\sigma}_s)\ .\end{aligned} \tag{17}$$

It was assumed that $\mathbf{p}_r+\mathbf{p}'_r \simeq 0$, $E_r - E'_r \simeq 0$ and $\mathbf{p}_r - \mathbf{p}'_r \simeq q$, where q is the exchange momentum of neutrino. The form factors $g_V(q^2)$, $g_A(q^2)$, $g_M(q^2)$ and $g_V(q^2)$ are defined in [6].

2.5 Values of the phase space integrals

The phase space factors G_{ij} $(ij = 11, 12, 13, 22, 23, 33)$ (see Eq. (13)) were numerically calculated for $^{76}_{32}$Ge, $^{82}_{34}$Se, $^{130}_{52}$Te and $^{136}_{54}$Xe. In the calculation we have used exact Dirac wave functions with finite nuclear size and screening. The homogeneous electric charge distribution inside a nucleus was assumed. The results of this calculation are shown in the Table 1. We see clearly that the values of G_{ij} increase with Z of a nucleus. From the phase space factors associated with the $p_{1/2}$ wave of electrons the largest one is G_{13}. Its origin is an interference of the emission of the $s_{1/2}$ and $p_{1/2}$ electrons due to the nucleon recoil.

Table 1. Phase space factors G_{ij} obtained using screened exact finite-size Coulomb wave functions of electron in the $s_{1/2}$ and $p_{1/2}$ states.

G_{ij}	$^{76}_{32}$Ge	$^{82}_{34}$Se	$^{130}_{52}$Te	$^{136}_{54}$Xe
G_{11}	1467.	6634.	12634.	13352.
G_{22}	0.22	1.50	12.62	15.35
G_{12}	-18.28	-99.73	-399.0	-452.3
G_{33}	73.34	400.4	1599.58	1813.
G_{13}	- 328.0	- 1629.	-4494.62	-4919.
G_{23}	4.09	24.52	142.07	166.8

3 Conclusion

In this paper, we have reported about the derivation of an improved expression for the $0\nu\beta\beta$-decay rate, which accounts for effects of the $p_{1/2}$-waves of emitted electrons and of the nucleon recoil. In order to estimate their impact on the $0\nu\beta\beta$-decay half-life the phase space factor were numerically evaluated. For that purpose wave functions of an electron in the $s_{1/2}$ and $p_{1/2}$ states were exactly calculated at the nuclear radius. It was found that these effects can play an important role, if the nuclear matrix element associated with nucleon recoil is not significantly suppressed in comparison with the dominant nuclear matrix elements. The evaluation of nuclear matrix elements within given nuclear structure method is outside of the scope of this contribution.

Appendix: The g_{ij} Factors of Phase Space Integrals G_{ij}

The explicit form of g_{ij} factors, expressed with the electron wave functions $g_{\pm 1}$ and $f_{\pm 1}$ evaluated at the nuclear surface, are given by

$$
\begin{aligned}
g_{11} =& \left(f_1^2(\varepsilon_1) + g_{-1}^2(\varepsilon_1)\right)\left(f_1^2(\varepsilon_2) + g_{-1}^2(\varepsilon_2)\right) \\
g_{22} =& \left(f_{-1}^2(\varepsilon_1) + g_1^2(\varepsilon_1)\right)\left(f_{-1}^2(\varepsilon_2) + g_1^2(\varepsilon_2)\right) \\
g_{12} =& \left(f_{-1}(\varepsilon_1)g_{-1}(\varepsilon_1) - f_1(\varepsilon_1)g_1(\varepsilon_1)\right)\left(f_1(\varepsilon_2)g_1(\varepsilon_2) - f_{-1}(\varepsilon_2)g_{-1}(\varepsilon_2)\right) \\
g_{33} =& \left(f_{-1}^2(\varepsilon_2) + g_1^2(\varepsilon_2)\right)\left(f_1^2(\varepsilon_1) + g_{-1}^2(\varepsilon_1)\right) \\
&+ \left(f_{-1}^2(\varepsilon_1) + g_1^2(\varepsilon_1)\right)\left(f_1^2(\varepsilon_2) + g_{-1}^2(\varepsilon_2)\right) - 2g_{12} \\
g_{13} =& \left(f_{-1}(\varepsilon_2)g_{-1}(\varepsilon_2) - f_1(\varepsilon_2)g_1(\varepsilon_2)\right)\left(f_1^2(\varepsilon_1) + g_{-1}^2(\varepsilon_1)\right) \\
&+ \left(f_{-1}(\varepsilon_1)g_{-1}(\varepsilon_1) - f_1(\varepsilon_1)g_1(\varepsilon_1)\right)\left(f_1^2(\varepsilon_2) + g_{-1}^2(\varepsilon_2)\right) \\
g_{23} =& \left(f_1(\varepsilon_2)g_1(\varepsilon_2) - f_{-1}(\varepsilon_2)g_{-1}(\varepsilon_2)\right)\left(f_{-1}^2(\varepsilon_1) + g_1^2(\varepsilon_1)\right) \\
&+ \left(f_1(\varepsilon_1)g_1(\varepsilon_1) - f_{-1}(\varepsilon_1)g_{-1}(\varepsilon_1)\right)\left(f_{-1}^2(\varepsilon_2) + g_1^2(\varepsilon_2)\right)
\end{aligned}
$$

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