Systematics of Nuclear Level Densities

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Abstract. A Python code was developed to search the Nuclear Data Base for experimental known nuclear levels with the purpose of looking at the nuclear level densities. The code downloads the evaluated nuclear levels of the nucleus/nuclei of interest from the data base, and produces a graph for each nucleus. In the graph, there are given the number of states per MeV, parity ratio, and spin cutoff per each energy bin of 0.5 MeV. The purpose of the code is to be used as a preliminary selection of nuclei in a certain region of interest for calculating nuclear level densities. A comparison with the calculated values using a systematic parametrization developed by Ohio University group is also implemented.

1 Introduction

Nuclear level densities are important in nuclear physics and astrophysics. They are an important input into compound nuclear reactions. Predicting the energy distribution of all of the excited levels of the nucleus is still very challenging using the available theoretical models, due to the complexity of the nuclear interaction and dimension of the size of the basis. Nucleosynthesis studies in astrophysics, medical applications of nuclear physics, and fission and fusion reactor design need nuclear level densities.

There are several challenges in obtaining nuclear level densities from both experimental and theoretical sides. From an experimental view, it is very hard to obtain complete spectroscopy for a nucleus. Then, there are many kinds of nuclei, and each kind presents specific experimental approaches. Level densities in unstable nuclei are challenging to measure. From the theoretical perspective, calculating all energy levels requires building a model in a huge basis. The problems increase exponentially by increasing the dimension of the space, then calculating all the terms in the interaction, and extracting the energy levels.

Assuming non-interacting nucleons and an energy independent density of single particle states g, Bethe [1] showed that the density of states $\rho(u)_s$ at an excitation energy u, can be written as:

$$\rho(u)_s = \frac{\sqrt{\pi}}{12} \frac{\exp(2\sqrt{au})}{a^{1/4}u^{5/4}}, \qquad (1)$$

where $a = (\pi^2/6)g$ is the level density parameter proportional to the density of the single particle states at the Fermi level. Since the parameter g is expected to

be proportional to A, the parameter a is also proportional to the nucleon number of the nucleus. From this expression, the density of levels can be inferred. For a spherical nucleus, it is assumed that the states have a Gaussian distribution in J_z, which leads to a factor multiplying Eq. (1) of

$$\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left[\frac{-J_z^2}{2\sigma^2}\right], \tag{2}$$

where $\sigma^2 = \langle J_z^2 \rangle$. The level density is obtained by differentiation and becomes

$$\rho(u, J) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\frac{(J + 1/2)}{\sigma^2}\exp\left[\frac{-(J + 1/2)^2}{2\sigma^2}\right]\rho(u), \tag{3}$$

where J_z is one of the $2J + 1$ projections of a level of spin J. If the state distribution is Gaussian in J_z, then

$$\rho_L(u) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\rho(u), \tag{4}$$

where σ is the spin cutoff parameter,

$$\sigma = \langle J_z^2 \rangle^{1/2} = \sqrt{\frac{1}{3}\langle J(J + 1) \rangle} . \tag{5}$$

In this work, we extract the energy levels with known angular momentum and parity from the Evaluated Nuclear Structure Data File (ENSDF [2]) Retrieval maintained by the Brookhaven National Laboratory. The code uploads the files, reads the number of energy levels in each 0.5 MeV energy interval, and saves them in a table. The code has the options to plot either the number of level density per MeV or the number of states (for a spherical nucleus, there are $2J + 1$ states for each level) per MeV.

The next quantity needed in evaluating the nuclear level density, is the cutoff parameter σ, calculated for each energy bin:

$$\sigma(u) = \left(\frac{\Sigma_J J(J + 1)\rho(u, J)}{\Sigma_J 3\rho(u, J)}\right)^{1/2} . \tag{6}$$

The ratio of levels of positive parity p (=+1) versus total number of levels per energy bin is also calculated and plotted

$$\Pi(u) = \frac{\rho^+(u)}{\rho^+(u) + \rho^-(u)}. \tag{7}$$

2 Program Description and Results

The program consists of several Python [4] files. The main code called NucUtil.py is calling several units. Here is the list of modules:

Table 1. Here are the modules used to generate the graphs of number of states per MeV, spin cutoff and parity ratio.

chem.py	a dictionary of the elements with N and Z
download.py	downloads the file from the ENSDF website
ENSDFparse.py	selects the levels from the raw data
fileops.py	
graphN.py	generate the plot
NucUtil.py	main part
calc.py	calculates based on theoretical models

The modules download.py and fileops.py use Hyper Text Transfer Protocol (HTTP) requests and responses, as well as HTML, its input and form elements, and its compatibility with XML-based parsers. The main code NuclUtil.py can be run without arguments for detail explanations and step by step procedure, or with arguments. The code can be used to generate the graphs for a nucleus at a time, or for a series of nuclei.

When using the code, there is the option to check if the file for the nucleus is already downloaded and processed. If it is, it checks if the graphs already exist. If the data is not downloaded previously, then the code will download it first. Also there is the option to download again, even if the file exists. The code checks if there are changes in the file, since the last time it was downloaded. If changes are found, the file will be processed and a new graph will be generated, if not, a message will be writen "No changes since last download."

As an example, an excerpt from the downloaded file for ^{30}Al, from the ENSDF library is given in Figure 1.

```
30AL  XD30SI(T,3HE)
30AL  XE30SI(7LI,7BE)
30AL  PN                                                          6
30AL  L 0              3+            3.62 S    6
30ALX L XREF=ABCDE
30AL cL J$From absence of |b{+-} feeding (from J|p=0+) in {+30}Mg |b{+-} decay
30AL2cL (2008Hi05) and comparison of measured {+30}Si(t,{+3}He) cross sections
30AL3cL with calculation (1987Pe06).
30AL2 L MOMM1=3.010 7
30AL cL MOMM1$Using |b{+-}Nuclear Magnetic Resonance method
```

Figure 1. Excerpt from the Evaluated Nuclear Structure Data File for ^{30}Al.

After the extraction of the energy levels, the information is saved in the file 30Al.in, that looks like in Table 2.

For the nucleus ^{30}Al, the parity was not assigned for a lot of levels. Also, for the level at 4570.8 keV, the most probable values for angular momentum are 5 or 6. In other nuclei, we can find a lot of measured energy levels with neither parity nor angular momentum assigned.

Based on the mass range, for example $25 \leq A \leq 30$, all data file for nuclei with $25 \leq A \leq 30$, are downloaded. Then each file is processed. All the levels

Table 2. Here are first few energy levels for ^{30}Al. The first column is the energy in keV, the second one the uncertainty in energy, the third one is angular momentum, and the fourth one is the parity of the level.

0	0	3	+	3.62	S	
243.90	8	2	+	8	NS	LT
687.52	12	1	+	0.7	PS	
1000	30	0				
1118.9	3	3	+	83	FS	
1243.99	10	4	+	118	FS	
1799.6	4	0				

found are counted and added in energy bins of 0.5 MeV. In the same time, all the levels of positive and negative parity are sorted and counted. For each energy bin, the average J^2 value is calculated, too.

After downloading the data from ENSDF library, the program makes a file with all energy levels found with or without assigned angular momentum, and with or without assigned parity. After that, the code will start processing the information and count the energy levels in bins of 0.5 MeV. The program have the option to stop counting levels based on various scenaria, given in the input file.

Table 3. Here are all the energy levels for ^{30}P. The first column is the energy in keV, the second one is sigma, the third one is the number of states per MeV, the fourth one is the parity ratio for the energy bin, and the last one is the calculated density of states

Energy [keV]	σ	nr. of states exp.	parity ratio	nr. of states model
250	1.0	6	1.0	3.72
750	0.5	8	1.0	3.48
1250	3.33	10	1.0	4.52
1500	7.0	14	1.0	6.17
2500	5.167	48	1.0	11.63
3250	1.0	12	1.0	15.80
3500	3.78	30	1.0	21.29
4250	7.47	90	0.67	28.46
4500	3.28	58	0.51	37.72
5250	7.16	48	0.34	49.63
5500	2.46	90	0.78	64.81
6250	3.96	90	0.73	84.07
6500	2.15	74	0.64	108.37
7250	3.63	154	0.54	138.89
7500	4.73	242	0.73	177.01
8250	3.13	244	0.46	224.45

Scenario 1

If this option is selected, the code stops counting levels when the first level with unassigned parity is found. A summary file is printed out in the form of a table that includes the nucleus, total number of levels, and last energy considered. This option is valuable just to have an idea how much experimental data there is for a certain nucleus.

Scenario 2

If this option is selected, the code stops counting levels when the first level with unassigned angular momentum is found. This option is also just used for a preliminary calculation.

Scenario 3

In this option, the code stops counting levels, when for three energy bins in a row, there are still energy levels found, but neither angular momentum nor parity assignment are known for these levels. This option is used for plotting the experimental data. There is more processing done for this option. Since there are energy levels of unknown parity here and there, the code counts all energy levels in the total number, but does not take this level into consideration when the parity ratio is calculated for this energy bin. So the unknown parity of this level, does not change the parity ratio for this energy bin. If an energy level of unknown angular momentum is found, again this level is not used in the spin cutoff parameter for this energy bin, or we may say that we assigned an angular momentum zero for that level. In this case we are going to further improve the code.

Based on the third scenario, the data can be analyzed for any nucleus.

Scenario 4

Read all data until the end of file is reached. This option will just read all energy levels with or without assigned angular momentum and parity. The generated output file is used to make a decision about where to stop generating the graphs, since the uncertainty in angular momentum and parity is increasing. We continue our efforts to improve this option.

Nuclear level density $\rho(u)$, as a function of energy, is calculated and compared with the one found from level counting at low energy. The calculated density uses the parameters found by Al-Quraishi [3], by analysing a wide range of nuclei with $20 \leq A \leq 110$. For this work we choose the following parametrization $a = \alpha A + \beta A^{2/3}$, with $\alpha = 0.0481$, and $\beta = 0.2037$. Other parametrizations can be easily implemented.
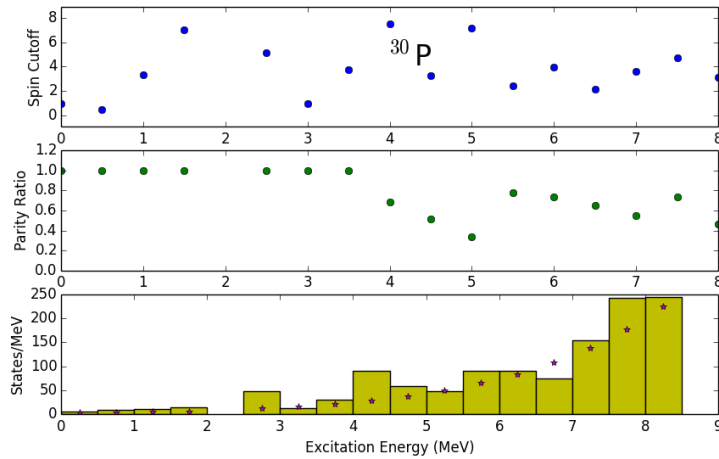
Figure 2. Spin cutoff, parity ratio, and number of states per MeV for ^{30}P.

3 Conclusions

This code started from the need to have a quick evaluation and visualization of the experimental data for various regions of nuclei. We use Python to create this code to take advantage of the easy manipulation of data and plotting libraries. The code can be used for fast analyzing of a large number of nuclei, like all isotopes of Aluminum, or all nuclei with mass number A=30, or all nuclei with $25 \leq A \leq 30$. The code is fast in generating the plots for the number of states per MeV, spin cutoff and parity ratio.

Using the parametrization $a = \alpha A + \beta A^{2/3}$, with $\alpha = 0.0481$ [3], and $\beta = 0.2037$, the calculated and experimental number of states per Mev are plotted. As an example we present the plots obtained for ^{30}P in Figure 2. Since other parametrizations can be easily implemented, we are planning to apply the code to a wide range of nuclei, and test also the N-Z dependence.

References

[1] H.A. Bethe, *Phys. Rev.* **50** (1936) 332.

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[3] S.I. Al-Quraishi, S.M. Grimes, T.N. Massey, and D.A. Resler, *Phys. Rev. C* **67** (2003) 015803.

[4] Python [online], URL: `http://www.python.org`[downloaded 16 January 2014].