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# Influence of Cranking Inertia on Binary Nuclear Processes

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**Abstract.** The sub-barrier fusion reactions, though they occur with a lower cross section, have the advantage of a final state closer to the ground state. Consequently one hopes for a longer lifetime of the nucleus. Superheavy nuclei are stable only due to the shell and pairing corrections. The macroscopic energy produces no barrier. We calculated here the pairing corrections within a specialized binary model, the deformed two center shell model. The pairing corrections have been obtained by solving the BCS system, in order to obtain the Fermi level for paired nucleons and the energy gap which appears when protons and neutrons are under pairing interaction.

Compared to the shell correction, the pairing energy is in antiphase and smaller. When we have paired nucleons, they have an occupation and non-occupation probability different from 1. These probabilities depend on the energy gap and the new Fermi level. These quantities determine the inertia tensor, which is necessary in the dynamics of the process via the action integral.

The binary character appears in the use of the deformed two center Hamiltonian. The deformation parameters are: the ratios of the semiaxes, the small semiaxis of the projectile and the distance between centers.

The total penetrability is calculated within the WKB approximation and the final transmission factor for the sub-barrier cross-section is obtained. Calculations have been performed for superheavy nuclei. We obtained for example the most favorable reaction $^{160}$Yb+$^{132}$Sn $\rightarrow$ $^{292}$120 (highest penetrability).

## 1 Introduction

In binary nuclear phenomena, like fusion and fission, the use of a two-center model is necessary. When the total deformation energy is calculated along the distance between centers for binary configurations, some valleys appear for different mass asymmetries. These valleys can be obtained as the result of multidimensional minimization of the action integral within the space of deformation. In order to take as many as possible deformation parameters into account, one has to calculate all the terms in the total deformation energy with the appropriate binary model able to describe the stages of the fission or fusion process. Such models have been developed along the years [1, 2]. The importance of the deformed valleys in the potential energy surfaces is that they provide the most

favored fission channels for the decay of superheavy nuclei, or synthesis paths for the fusion process. For the dynamics study one has to introduce the influence of the mass tensor. We use the results from the pairing calculations for the occupation probabilities. In this way the mass tensor components contain the binary character of the process, since the pairing parameters are calculated with the two-center shell model levels. The mass tensor components are calculated with the cranking model, specialized for binary configurations. Finally the penetrabilities are calculated within the WKB approximation, and favoured synthesis or decay channels are obtained.

## 2 The Deformed Binary Potential

The axial symmteric binary configurations are used for the total deformation energy calculation. A typical shape is displayed in Figure 1, where $b_1, a_1$ and $b_2, a_2$ are the small and large semi-axes of the daughter (target) and emitted fragment (projectile) respectively, $z_s$ is the position of the separation plane and $R$ is the distance between centers. All these geometrical parameters form the space of deformation, and further on one shall work with $\chi_d = b_1/a_1$, $\chi_e = b_2/a_2$, $b_2$ and $R$ as degrees of freedom.

The microscopic part starts with the binary Hamiltonian written for a single particle system:

$$H = -\frac{\hbar^2}{2m_0}\nabla^2 + V(\rho, z) + V_{ls} + V_{l^2}\,, \tag{1}$$

where the potentials are deformation dependent and $m_0$ is the nucleon (proton and neutron) mass. The same equation is valid for protons and neutrons. The

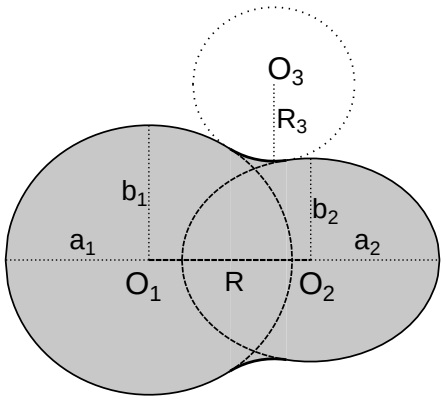


Figure 1. Binary configuration for fusion and fission description in the axial symmetric hypothesis

deformed two-center oscillator potential for the two fission fragment regions reads

$$V_{\text{DTCSM}}(\rho,z)=\begin{cases}\frac{1}{2}m_0\omega_{\rho_1}^2\rho^2+\frac{1}{2}m_0\omega_{z1}^2(z+z_1)^2\\ \begin{cases}V_{g1}(\rho,z)=2V_0-\left[\frac{1}{2}m_0\omega_g^2(\rho-\rho_3)^2+\frac{1}{2}m_0\omega_g^2(z-z_3)^2\right]\\ V_{g2}(\rho,z)=V_0\end{cases}\\ \frac{1}{2}m_0\omega_{\rho_2}^2\rho^2+\frac{1}{2}m_0\omega_{z_2}^2(z-z_2)^2\end{cases} \tag{2}$$

In the case of fusion like configurations the neck terms $V_{g1}$ and $V_{g2}$ are missing. It is the fusion part we will treat further on. Angular momentum dependent potentials, $V_{\Omega s}$ and $V_{\Omega^2}$ are constructed to comply to the $V(\rho,z)$-dependence and hermiticity of the operators, so that

$$V_{so}=\begin{cases}-\left\{\dfrac{\hbar}{m_0\omega_{0T}}\kappa_T(\rho,z),(\nabla V^{(r)}\times\mathbf{p})\mathbf{s}\right\} & ,v_T-\text{region}\\ -\left\{\dfrac{\hbar}{m_0\omega_{0\text{P}}}\kappa_P(\rho,z),(\nabla V^{(r)}\times\mathbf{p})\mathbf{s}\right\} & ,v_P-\text{region}\end{cases} \tag{3}$$

and similarly for the $l^2$ term. The matrix diagonalization of $H$ generates the level scheme of the fission configuration, for spheroidally deformed nuclei, at any given distance $R$ between centers and intermediary independent $b_2$, $\chi_d$ and $\chi_e$. The level scheme sequence from the compound nucleus up to complete separation is input data for the Strutinsky method [3], and calculations are performed separately for protons and neutrons.

The shell correction energy is obtained as the difference between the simple sum of level energies and the smoothed part of the same scheme:

$$E_{\text{sh}}=\sum_i E_i-\tilde{U} \tag{4}$$

where the summation is performed for all occupied levels. The main part of the calculation consists in obtaining the smoothed term $\tilde{U}$. A smoothed level distribution density $\tilde{g}(\epsilon)$ is defined by averaging the actual distribution over a finite interval $\gamma$ (here equal to 1.2 in $\hbar\omega$ units). If the level energies in units of $\hbar\omega$ are denoted with $\epsilon_i$, one can write the integral which replaces the discrete sum and one obtains the smoothed distribution:

$$\tilde{g}(\epsilon)=\frac{1}{\gamma}\int_{-\infty}^{\infty}\zeta\Big(\frac{\epsilon-\epsilon'}{\gamma}\Big)g(\epsilon')d\epsilon'=\frac{1}{\gamma}\sum_{i=1}^{\infty}\zeta\Big(\frac{\epsilon-\epsilon_i}{\gamma}\Big)\,. \tag{5}$$

This work utilizes a smoothing function $\zeta$ of the form:

$$\zeta(x)=\frac{1}{\sqrt{\pi}}\exp\left(-x^2\right)f_m(x)\,, \tag{6}$$

where $x = (\epsilon - \epsilon')/\gamma$ and the smoothing function $f$ is taken as a polynomial sum:

$$f_m(x) = \sum_{k=0}^{m} a_{2k} H_{2k}(x)\,. \tag{7}$$

$H_n(x)$ are the Hermite polynomials, and the maximum degree $m$ (here 3) is taken such as $d\tilde{U}/d\gamma$=constant (the plateau condition). The maximum level is chosen such as $|x_i| \geq$3. Beyond this limit the contribution of more remoted levels is negligeable. Once the density of the smooth levels $\tilde{g}(\epsilon)$ is obtained by this smearing procedure, the smoothed part of the energy is given by

$$\tilde{u} = \tilde{U}/\hbar\omega = \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(\epsilon)\epsilon d\epsilon \tag{8}$$

where the Fermi level $\tilde{\lambda}$ for smoothed distribution is obtained from the conservation of the total number of nucleons:

$$N_e = \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(\epsilon) d\epsilon \tag{9}$$

By substituting the above expression for $\tilde{g}(\epsilon)$ one obtains

$$N_e = \frac{2}{\sqrt{\pi}} \sum_{1}^{\infty} \int_{-\infty}^{x_{iF}} f_m(x_i^2) \exp{(x_i^2)} dx_i\,, \tag{10}$$

where $x_{iF} = (\tilde{\lambda} - \epsilon_i)/\gamma$. The summation is in fact reduced to the levels around the Fermi limit. The latter equation yields the Fermi level for smoothed distribution $\tilde{\lambda}$, and is solved numerically. We consider a set of doubly degenerate energy levels $\{\epsilon_i\}$ expressed in units of $\hbar\omega_0^0$. Calculations for neutrons are similar with those for protons, hence for the moment we shall consider only protons. In the absence of a pairing field, the first $Z/2$ levels are occupied, from a total number of $n_t$ levels available. Only few levels below $(n)$ and above $(n')$ the Fermi energy are contributing to the pairing correlations. Usually $n' = n$. If $\tilde{g_s}$ is the density of states at Fermi energy obtained from the shell correction calculation $\tilde{g_s} = dZ/d\epsilon$, expressed in number of levels per $\hbar\omega_0^0$ spacing, the level density is half of this quantity: $\tilde{g_n} = \tilde{g_s}/2$.

We can choose as computing parameter, the cut-off energy (in units of $\hbar\omega_0^0$), $\Omega \simeq 1 \gg \tilde{\Delta}$. Let us take the integer part of the following expression

$$\Omega\tilde{g_s}/2 = n = n'\,. \tag{11}$$

When from calculation we get $n > Z/2$ we shall take $n = Z/2$ and similarly if $n' > n_t - Z/2$ we consider $n' = n_t - Z/2$.

The gap parameter $\Delta = |G| \sum_k u_k v_k$ and the Fermi energy with pairing corellations $\lambda$ (both in units of $\hbar\omega_0^0$) are obtained as solutions of a nonlinear system of two BCS equations

$$n' - n = \sum_{k=k_i}^{k_f} \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\,, \tag{12}$$

$$\frac{2}{G} = \sum_{k=k_i}^{k_f} \frac{1}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\,, \tag{13}$$

where $k_i = Z/2 - n + 1$; $k_f = Z/2 + n'$.

The pairing interaction strength $G$ is calculated from a continuous distribution of levels

$$\frac{2}{G} = \int_{\tilde{\lambda}-\Omega}^{\tilde{\lambda}+\Omega} \frac{\tilde{g}(\epsilon)d\epsilon}{\sqrt{(\epsilon - \tilde{\lambda})^2 + \tilde{\Delta}^2}}\,, \tag{14}$$

where $\tilde{\lambda}$ is the Fermi energy deduced from the shell correction calculations and $\tilde{\Delta}$ is the gap parameter, obtained from a fit to experimental data, usually taken as $\tilde{\Delta} = 12/\sqrt{A}\hbar\omega_0^0$. Both $\Delta_p$ and $\Delta_n$ decrease with increasing asymmetry $(N - Z)/A$. From the above integral we get

$$\frac{2}{G} \simeq 2\tilde{g}(\tilde{\lambda}) \ln\left(\frac{2\Omega}{\tilde{\Delta}}\right)\,. \tag{15}$$

Real positive solutions of BCS equations are allowed if

$$\frac{G}{2} \sum_k \frac{1}{|\epsilon_k - \lambda|} > 1\,, \tag{16}$$

i.e. for a pairing force (G-parameter) large enough at a given distribution of levels.

As a consequence of the pairing correlation, the levels situated bellow the Fermi energy are only partially filled, while those above the Fermi energy are partially empty; there is a given probability for each level to be occupied by a quasiparticle

$$v_k^2 = \frac{1}{2}\left[1 - \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\right] \tag{17}$$

or a hole

$$u_k^2 = 1 - v_k^2\,. \tag{18}$$

Only the levels in the near vicinity of the Fermi energy (in a range of the order of $\Delta$ around it) are influenced by the pairing correlations. For this reason, it is sufficient for the value of the cut-off parameter to exceed a given limit

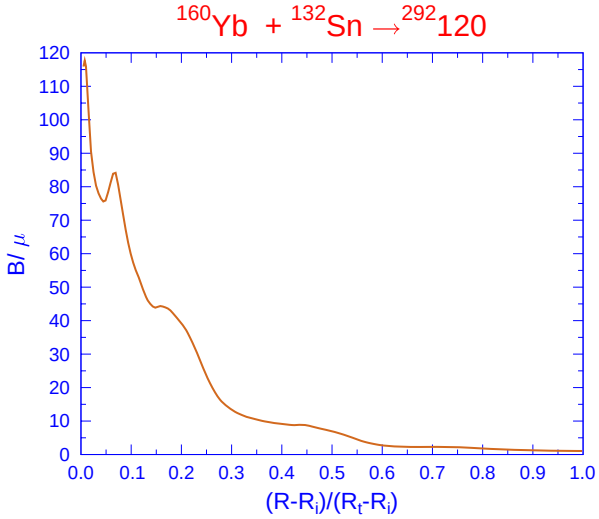


Figure 2. Variation of the total cranking inertia with the reduced distance between centers for the synthesis reaction $^{132}$Sn+$^{160}$Yb $\rightarrow^{292}$120.

$\Omega \gg \tilde{\Delta}$, the value in itself having no significance. The shell and pairing corrections calculated for the splitting of $^{292}$120 in the Sn fission channel are displayed in Figure 2 along the reduced distance between centers. One observes large fluctuations of the proton and neutron shell corrections, in counterphase with the corresponding pairing energy. Finally the pairing correction energy is obtained as the difference between the pairing correlation energies for the discrete level distribution $p$ and the one for the continuous level distribution $\tilde{p}$

$$\delta p = p - \tilde{p}\,, \tag{19}$$

where

$$p = \sum_{k=k_i}^{k_f} 2v_k^2\epsilon_k - 2\sum_{k=k_i}^{Z/2}\epsilon_k - \frac{\Delta^2}{G} \tag{20}$$

and

$$\tilde{p} = -(\tilde{g}\tilde{\Delta}^2)/2 = -(\tilde{g}_s\tilde{\Delta}^2)/4\,. \tag{21}$$

The smooth gap parameter is $\tilde{\Delta} = (12.0/A^{1/2})$, which gives a good agreement with the experimental gap throughout the periodic table. One has again: $\delta p = \delta p_p + \delta p_n$, and the total microscopic corrections are added: $\delta e = \delta u + \delta p$.

The macroscopic part is obtained using the Yukawa-plus-exponential method, specialized to binary processes. The Coulomb term $E_C$ [4] and the nuclear surface term $E_Y$ [5] are computed as

$$E_C = \frac{2\pi}{3}(\rho_{e_d}^2 F_{C_d} + \rho_{e_e}^2 F_{C_e} + 2\rho_{e_d}\rho_{e_e}F_{C_{T_e}}) \tag{22}$$

and

$$E_Y = \frac{1}{4\pi r_0^2}[c_{s_d} F_{EY_d} + c_{s_e} F_{EY_e} + 2(c_{s_d} c_{s_e})^{1/2} F_{EY_{de}}] \tag{23}$$

where $\rho_{ei}$ is the charge density and $c_{si}$ the surface coefficient. $F_{C_i}$ and $F_{EY_i}$ are shape dependent integrals.The peculiarity resides in the last term in both formulas, $F_{C_{de}}$ and $F_{EY_{de}}$, which account for the interaction between non-overlapped parts of the overlapping configuration. Details about these terms are given in [6].

The total deformation dependent macroscopic energy is calculated as the sum of the Coulomb and surface terms

$$E_{\text{macro}} = (E_C - E_C^{(0)}) + (E_Y - E_Y^{(0)})\,, \tag{24}$$

where $E_C^{(0)}$ and $E_Y^{(0)}$ are the values for the corresponding spherical compound nucleus. Finally the deformation energy is computed as the sum of the macroscopic part and the shell correction and pairing energies:

$$E_{\text{def}} = E_{\text{macro}} + E_{\text{sh}} + P. \tag{25}$$

## 3 Dynamics

In order to obtain the penetrabilities for different reaction channels, the action integral must be computed. Besides the usual deformation energy, the nuclear inertia tensor, which accounts for the reaction of the nucleus to the deformation along a given degree of freedom, is to be computed. This work uses the cranking approach to obtain the mass tensor components within the four-dimensional space of $(b_e, \chi_d, \chi_e, R)$. According to the cranking model, after including the BCS pairing correlations [7], the inertia tensor is given by [8]

$$B_{ij} = 2\hbar^2 \sum_{\nu\mu} \frac{\langle\nu|\partial H/\partial\beta_i|\mu\rangle\langle\mu|\partial H/\partial\beta_j|\nu\rangle}{(E_\nu + E_\mu)^3}(u_\nu v_\mu + u_\mu v_\nu)^2 + P_{ij}\,, \tag{26}$$

where $H$ is the two-center single-particle Hamiltonian allowing to determine the energy levels and the wave functions $|\nu\rangle$, $u_\nu$, $v_\nu$ are the BCS occupation probabilities, $E_\nu$ is the quasiparticle energy, and $P_{ij}$ gives the contribution of the occupation number variation when the deformation is changed (terms including variation of the gap parameter, $\Delta$, and Fermi energy, $\lambda$, $\partial\Delta/\partial\beta_i$ and $\partial\lambda/\partial\beta_i$). The binary configuration uses four independent geometric parameters, the small semiaxis of the projectile $b_p$, the ratio of the two semixes of the target nucleus $\chi_T$ and projectile $\chi_p$ and the distance between centers $R$. Consequently there are ten coupling components of the mass inertia tensor. In order to introduce the tensor into the action integral, we contract the components along the distance

between centers $R$, and we obtain:

$$B(R) = B_{b_P b_P}\left(\frac{db_P}{dR}\right)^2 + 2B_{b_P\chi_T}\frac{db_P}{dR}\frac{d\chi_T}{dR} + 2B_{b_P\chi_P}\frac{db_P}{dR}\frac{d\chi_P}{dR} + 2B_{b_P R}\frac{db_P}{dR} + B_{\chi_T\chi_T}\left(\frac{\chi_T}{dR}\right)^2 + 2B_{\chi_T\chi_P}\frac{d\chi_T}{dR}\frac{d\chi_P}{dR} + 2B_{\chi_T R}\frac{d\chi_T}{dR}B_{\chi_P\chi_P}\left(\frac{\chi_P}{dR}\right)^2 + 2B_{\chi_P R}\frac{d\chi_P}{dR} + B_{RR} \quad (27)$$

The final quantity to be computed is the penetrability, which is obtained as the result of the multidimensional minimization of the action integral within the space of deformation. The penetrability $P$ for a fusion path is given by:

$$P = \exp(-K_{\text{ov}})\,, \quad (28)$$

where the action integral is computed as

$$K_{\text{ov}}(b_P, \kappa_T, \kappa_P; R) = \frac{2}{\hbar}\int\limits_{\text{(fus)}} [2B(R)_{b_P,\kappa_T,\kappa_P} E_{\text{def}}(R)_{b_P,\kappa_T,\kappa_P}]^{1/2} dR\,. \quad (29)$$

## 4 Results

We calculated the penetrabilities for the sub-barrier fusion reactions towards the synthesis of the superheavy nuclei $^{292}120$ and $^{300}120$. For every superheavy system, the whole range of mass and charge asymmetry has been considered. The minimization has been performed numerically using a multidimensional grid, and calculating the deformation energy and the total mass inertia in every point. Then minimization is obtained by summing every successive step along the distance between centers with all the next steps. Finally, all possible sums are obtained, and the minimum is chosen as the value of the action integral. The path along the cranking inertia for the fusion favoured channel $^{132}$Sn+$^{160}$Yb $\rightarrow^{292}120$ is presented bellow. The other promising sub-barrier fusion channel is $^{132}$Sn+$^{168}$Yb $\rightarrow^{300}120$. One has to mention that these reactions benefy also from the low barrier due to the almost symmetric splitting of the binary configuration, as well as for the double magicity of the $^{132}$Sn projectile, which provides a negative shell correction energy in the deformation path.

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