

Electron Captures and Neutron Emissions in Magnetic White Dwarfs and Magnetars

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Abstract. Electron captures and neutron emissions by atomic nuclei in dense matter are among the most important processes governing the late evolution of stars. Not only do these processes limit the stability of massive white dwarfs, but also lead to the neutronization of matter in the interior of neutron stars. Although these processes have been known for a long time, most studies have focused on nonmagnetized matter. On the other hand, some white dwarfs and neutron stars are endowed with very strong magnetic fields, of the order of 10^9 G and 10^{15} G respectively. As a matter of fact, much stronger fields may potentially exist in their interiors. In this paper, we review our recent studies of the role of a strong magnetic field on the onset of electron captures and neutron emissions in dense stellar matter.

1 Introduction

Soon after the discovery of the neutron (predicted by Rutherford in 1920) by James Chadwick in February 1932, it was realized that cold high-density matter is predominantly composed of neutrons [1] (see also Ref. [2]). In December 1933, during a meeting of the American Physical Society at Stanford, Walter Baade and Fritz Zwicky predicted the existence of *neutron stars* formed from the catastrophic gravitational collapse of stars during supernova explosions [3]. Baade and Zwicky were apparently unaware of the studies about white dwarfs, which owe their existence to the presence of a highly degenerate electron gas in their core. The connection was first made by Landau [4] and Gamow [5]. At a conference in Paris in 1939, Chandrasekhar also pointed out: “If the degenerate core attains sufficiently high densities, the protons and electrons will combine to form neutrons. This would cause a sudden diminution of pressure resulting in the collapse of the star to a neutron core” [6]. Electron captures and neutron emissions by atomic nuclei in dense matter thus play a very important role in the late stages of stellar evolution (see *e.g.* Ref. [7] for a recent review).

Electron captures and neutron emissions have been mainly studied in unmagnetized matter. However, about 600 magnetic white dwarfs and 170 magnetic cataclysmic variables have been detected so far [8]. Whereas the strongest observed surface magnetic fields (as inferred from Zeeman spectroscopy and polarimetry, as well as cyclotron spectroscopy) are of the order 10^9 G [8], much stronger fields may exist in the stellar core [9]. In particular, it has been recently proposed that white dwarfs endowed with strongly quantizing magnetic fields $B \gg B_{\text{crit}}$, where $B_{\text{crit}} = m_e^2 c^3 / (e\hbar) \approx 4.4 \times 10^{13}$ G (with m_e the electron mass, c the speed of light, e the proton electric charge, and \hbar Planck-Dirac constant), could be the progenitors of overluminous type Ia supernovae like SN 2006gz and SN 2009dc [10]. Actually, such strongly magnetized white dwarfs were studied a long time ago [11]. The stability of these peculiar white dwarfs would be also limited by electron captures [12–14]. Even stronger magnetic fields $\sim 10^{16} - 10^{17}$ G could be generated in hot newly-born neutron stars with initial periods of a few milliseconds [15]. Soft gamma-ray repeaters and anomalous X-ray pulsars are believed to be the best candidates of these so called *magnetars* (see *e.g.* Ref. [16] for a review). Their surface magnetic fields, as inferred from spin-down and spectroscopic studies, are of the order of $10^{14} - 10^{15}$ G [17–19]. Recent observations of slow pulse phase modulations suggest the presence of internal magnetic fields of order 10^{16} G [20]. Theoretical considerations and numerical simulations show that the magnetic field in neutron-star cores could reach $\sim 10^{18}$ G (see *e.g.* Ref. [21] and references therein).

In this paper, we review our recent studies of electron captures and neutron emissions by nuclei in the core of a strongly magnetized white dwarf, and in the outer crust of a magnetar.

2 Model of Dense Magnetized Stellar Matter

In the core of a magnetic white dwarf and in the outer crust of a magnetar, atoms are fully ionized by the gravitational pressure. We further assume that the temperature T is lower than the crystallization temperature T_m and that ions are arranged in a regular crystal lattice. Considering ions ${}^A_Z X$ with proton number Z and mass number A , the crystallization temperature is given by (see *e.g.* Ref. [22])

$$T_m = \frac{e^2}{a_e k_B \Gamma_m} Z^{5/3}, \quad (1)$$

where $a_e = (3/(4\pi n_e))^{1/3}$ is the electron-sphere radius, n_e is the electron number density, k_B is Boltzmann's constant, and $\Gamma_m \simeq 175$ is the Coulomb coupling parameter at melting. Since T_m is generally much lower than the electron Fermi temperature defined by

$$T_F = \frac{\mu_e - m_e c^2}{k_B}, \quad (2)$$

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where μ_e is the Fermi energy, electrons are highly degenerate. In the following, we shall take $T = 0$. To a very good approximation, electrons can be treated as an ideal Fermi gas (see *e.g.* Ref. [23] for a discussion).

In the presence of a strong magnetic field B , the electron motion perpendicular to the field is quantized into Landau levels (see *e.g.* Ref. [22]). General expressions for the electron energy density \mathcal{E}_e and electron pressure P_e can be found in Ref. [22]. The maximum index ν_{\max} of occupied Landau levels is determined by the electron number density

$$n_e = \frac{2B_\star}{(2\pi)^2\lambda_e^3} \sum_{\nu=0}^{\nu_{\max}} g_\nu \sqrt{\frac{\mu_e}{m_e c^2} - 1 - 2\nu B_\star}. \quad (3)$$

where $B_\star = B/B_{\text{crit}}$, g_ν is the degeneracy of the levels ($g_\nu = 1$ for $\nu = 0$ and $g_\nu = 2$ for $\nu \geq 1$), and $\lambda_e = \hbar/(m_e c)$ is the electron Compton wavelength. According to the Bohr-van Leeuwen theorem [24], the lattice energy density is independent of the magnetic field and is given by

$$\mathcal{E}_L = C e^2 n_e^{4/3} Z^{2/3}, \quad (4)$$

where $C \approx -1.444$ is the crystal structure constant (assuming that ions are arranged in a body-centered cubic lattice). In this expression, the finite-size of the ions is neglected, as well as the small contribution due to quantum zero-point motion of ions off their equilibrium position [25]. The lattice contribution to the pressure can be readily obtained from Eq. (4) and is given by $P_L = \mathcal{E}_L/3$.

The quantum effects of the magnetic field on dense matter properties are most important when $\nu_{\max} = 0$. This situation arises when $n_e < n_{eB}$ and $T < T_B$ with

$$n_{eB} = \frac{B_\star^{3/2}}{\sqrt{2}\pi^2\lambda_e^3}, \quad (5)$$

$$T_B = \frac{m_e c^2}{k_B} B_\star. \quad (6)$$

In the regime of ultrarelativistic electrons, $\mu_e \gg m_e c^2$, we obtain

$$\mu_e \approx \frac{2\pi^2 m_e c^2 \lambda_e^3 n_e}{B_\star}, \quad (7)$$

$$P = P_e + P_L \approx \frac{B_\star \mu_e^2}{4\pi^2 \lambda_e^3 m_e c^2} \left[1 + \left(\frac{4B_\star}{\pi^2} \right)^{1/3} \left(\frac{m_e c^2}{\mu_e} \right)^{2/3} C \alpha Z^{2/3} \right], \quad (8)$$

where $\alpha = e^2/(\hbar c)$ is the fine structure constant. Ions do not contribute to the pressure, but do contribute to the energy density

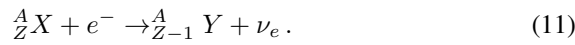
$$\mathcal{E}_X = n_X M(A, Z) c^2, \quad (9)$$

where n_X is the ion number density, and $M(A, Z)$ the ion mass (including the rest mass of Z protons, $A - Z$ neutrons and Z electrons¹). We assume that ion masses remain unchanged under the presence of the magnetic field. The average baryon number density is given by $n = An_X = (A/Z)n_e$. In the strongly quantizing regime, using Eq. (7), we find

$$n \approx \frac{A}{Z} \frac{\mu_e B_\star}{2\pi^2 m_e c^2 \lambda_e^3}, \quad (10)$$

3 Electron Capture and Neutron Emission

At some density n_β , the nucleus ${}^A_Z X$ becomes unstable against the capture of an electron with the emission of a neutrino :



The daughter nucleus ${}^A_{Z-1} Y$ itself may be unstable, and capture another electron. Inside the star the pressure has to vary continuously so that the process (11) occurs at a fixed pressure P_β (on the contrary, the density n varies discontinuously). Since the temperature is also fixed ($T = 0$), the suitable thermodynamic potential for studying the stability of dense matter is the Gibbs free energy per nucleon g (this still remains the case even in the presence of a strong magnetic field, as shown in Ref. [23]). The threshold pressure P_β is of particular astrophysical interest as it determines the highest possible pressure that can be found in white dwarf cores (see *e.g.* Ref. [26]). Indeed, as electrons combine with nuclei, further compression of matter does not increase the pressure thus leading to a global instability. Consequently, P_β sets an upper limit to the maximum possible mass of white dwarfs (their stability may be further limited by general relativity depending on the stellar composition, see *e.g.* Ref. [23] and references therein). The onset of electron captures can be determined by requiring the Gibbs free energy per nucleon of matter made of ions ${}^A_Z X$ to be equal to that of matter made of the daughter nuclei ${}^A_{Z-1} Y$. The Gibbs free energy per nucleon is defined by

$$g = \frac{\mathcal{E} + P}{n}, \quad (12)$$

where \mathcal{E} is the average energy density given by

$$\mathcal{E} = \mathcal{E}_X + \mathcal{E}_e + \mathcal{E}_L - n_e m_e c^2. \quad (13)$$

The last term in Eq.(13) is introduced to avoid double counting since the electron mass is already included in the ion mass. The threshold pressure P_β can thus be determined from the condition

$$g(A, Z, P_\beta) = g(A, Z - 1, P_\beta), \quad (14)$$

¹The reason for including the electron rest mass in $M(A, Z)$ is that experimental *atomic* masses are generally tabulated rather than *nuclear* masses.

which can be approximately expressed as [23]

$$\mu_e + Ce^2 n_e^{1/3} \left[Z^{5/3} - (Z-1)^{5/3} + \frac{Z^{2/3}}{3} \right] = \mu_e^\beta(A, Z), \quad (15)$$

where

$$\mu_e^\beta(A, Z) \equiv M(A, Z-1)c^2 - M(A, Z)c^2 + m_e c^2. \quad (16)$$

Equation (15) generalizes the usual criterion for the onset of electron captures (see *e.g.* Ref. [26]) by taking into account electron-ion interactions. Note that Eq. (15) remains the same whether the matter is magnetized or not. On the contrary, the threshold baryon density n_β and pressure P_β do depend on the magnetic field. For the known magnetic white dwarfs, ν_{\max} is typically of the order of $10^6 - 10^7$ assuming that the internal magnetic field is of the same order as the observed surface field. In this case, Landau quantization effects are negligible. The threshold density and pressure are thus approximately given by [23]

$$n_\beta \approx \frac{A}{Z} \frac{\mu_e^\beta(A, Z)^3}{3\pi^2(\hbar c)^3} \left[1 + \frac{C\alpha}{(3\pi^2)^{1/3}} \left(Z^{5/3} - (Z-1)^{5/3} + \frac{Z^{2/3}}{3} \right) \right]^{-3}, \quad (17)$$

$$P_\beta \approx \frac{\mu_e^\beta(A, Z)^4}{12\pi^2(\hbar c)^3} \left[1 + \frac{4C\alpha}{(81\pi^2)^{1/3}} Z^{2/3} \right] \times \left[1 + \frac{C\alpha}{(3\pi^2)^{1/3}} \left(Z^{5/3} - (Z-1)^{5/3} + \frac{Z^{2/3}}{3} \right) \right]^{-4}. \quad (18)$$

On the other hand, the authors of Ref. [10] have speculated that strongly quantizing magnetic fields $B_\star \gg 1$ could be present in the core of some white dwarfs. Let us consider the most extreme situation for which $\nu_{\max} = 0$, as in Ref. [10]. In such case, the threshold density and pressure are approximately given by [23]

$$n_\beta \approx \frac{A B_\star \mu_e^\beta(A, Z)}{Z 2\pi^2 m_e c^2 \lambda_e^3} \left[1 - C\alpha \left(\frac{B_\star}{2\pi^2} \right)^{1/3} \times \left(\frac{m_e c^2}{\mu_e^\beta(A, Z)} \right)^{2/3} \left(Z^{5/3} - (Z-1)^{5/3} + \frac{Z^{2/3}}{3} \right) \right], \quad (19)$$

$$P_\beta \approx \frac{B_\star \mu_e^\beta(A, Z)^2}{4\pi^2 \lambda_e^3 m_e c^2} \left[1 - C\alpha \left(\frac{4B_\star}{\pi^2} \right)^{1/3} \times \left(\frac{m_e c^2}{\mu_e^\beta(A, Z)} \right)^{2/3} \left(Z^{5/3} - (Z-1)^{5/3} - \frac{2}{3} Z^{2/3} \right) \right]. \quad (20)$$

In the weakly quantizing regime for which several Landau levels are populated, Eq. (15) must be solved numerically. The threshold density is found to exhibit

typical quantum oscillations as a function of the magnetic field strength [23]. The previous analysis can be easily extended to ionic mixtures [23]. As shown in Refs. [13, 14], white dwarfs endowed with strongly quantizing magnetic fields become unstable against electron captures if the magnetic field is too strong. The argument is the following. If, as argued in Ref. [10] the central pressure in the most massive super-Chandrasekhar magnetic white dwarfs is limited by $P_B = P_e(n_{eB}) + P_L(Z, n_{eB})$, the core will thus become unstable against electron captures whenever $P_\beta < P_B$. In turn, this condition leads to an upper limit on the magnetic field strength in the stellar core [14]

$$B_\star^\beta \approx \frac{1}{2} \left(\frac{\mu_e^\beta(A, Z)}{m_e c^2} \right)^2 \left[1 + \left(\frac{4}{\pi} \right)^{2/3} \frac{C\alpha}{3} \left(Z^{5/3} - (Z-1)^{5/3} \right) \right]^{-2}. \quad (21)$$

At sufficiently high pressures, the quantum-zero point fluctuations of ions about their equilibrium position may become large enough to trigger pycnonuclear fusion reactions



The threshold pressure $P_\beta(2A, 2Z)$ for the onset of electron capture by the daughter nucleus $\frac{2A}{2Z}Y$ is generally lower than the corresponding pressure $P_\beta(A, Z)$ of the parent nucleus $\frac{A}{Z}X$. For this reason, pycnonuclear fusion reactions, if they occur at a pressure $P_{\text{pyc}} < P_\beta(2A, 2Z)$, would drastically reduce the maximum strength of the magnetic field in the core of white dwarfs, from $B_\star = 383$ to 74 for the fusion of ^{12}C into ^{24}Mg , from 240 to 9.7 for the fusion ^{16}O in ^{32}S , or from 115 to 6.5 for the fusion of ^{20}Ne into ^{40}Ca [14]. In this way, we showed that the strongly magnetized super-Chandrasekhar white dwarfs proposed in Ref. [10] are highly unstable [13, 14].

Electron captures are also of relevance for neutron stars (see *e.g.* Ref. [22]). Indeed, during the collapse of massive stars (with a mass $M \gtrsim 8M_\odot$, M_\odot being the mass of the Sun), electrons in the stellar core are captured by nuclei leading to progressively more neutron rich matter as the pressure increases. Other processes are expected to take place in the hot newly born neutron star so that matter is expected to remain close to the nuclear equilibrium corresponding to the minimum of the Gibbs free energy per nucleon g at given temperature T (decreasing) and pressure P . The star rapidly cools down by powerful neutrino emission. After about $10^4 - 10^5$ years, the cooling is governed by the emission of thermal photons due to the diffusion of heat from the interior to the surface. Eventually, the interior of the star becomes cold and fully “catalyzed”. We have shown in a series of works [27–30] that the presence of a strong magnetic field can change substantially the equilibrium composition of the outer crust of a magnetar. For this purpose, we have made use of experimental atomic masses. For the masses that have not yet been measured, we have made use of the Brussels-Montreal microscopic mass tables [31], which are based on the self-consistent Hartree-Fock-Bogoliubov (HFB) method (see *e.g.* Ref. [32] for a short review of this method, and Ref. [33] for a review of the latest Brussels-Montreal models).

Whereas the composition of the shallowest crustal layers is completely determined by experimental masses, the composition of the deeper layers is more uncertain. As an example, we have shown in Table 1 the predictions from two HFB atomic mass models: DIM [35] based on a finite-range Gogny interaction and HFB-27* [36] based on a zero-range Skyrme interaction. Note that we used a refined version of the HFB-27* table from BRUSLIB [31]. For this reason, the results obtained here are slightly different from those presented in Ref. [30]. For comparison, we have also shown the results obtained with the phenomenological model of Duflo and Zuker [37]. If the magnetic field is strong enough, the crustal composition may be further altered due to changes of the nuclear masses [39], an effect which we have not considered here. For the strongest possible magnetic fields $B \sim 10^{18}$ G, almost the entire outer crust is made of only one element [39]. With our crust model, this element is found to be ^{90}Zr for all three atomic mass models (the mass of this nuclide has been experimentally measured). The magnetic field not only impacts the composition, but also makes the crustal matter much more incompressible. Near the stellar surface, the density n in a layer at pressure P is approximately given by [29]

$$n \approx n_s \left(1 + \sqrt{\frac{P}{P_0}} \right), \quad (23)$$

$$P_0 = m_e c^2 \frac{n_s^2 \pi^2 \lambda_e^3}{B_\star} \left(\frac{Z}{A} \right)^2, \quad (24)$$

$$n_s \approx \frac{A_s}{\lambda_e^3} \left(\frac{|C| \alpha B_\star^2}{4\pi^4 Z_s} \right)^{3/5}, \quad (25)$$

where Z_s is the proton number of ions at the surface, and A_s their mass number, while Z and A are the proton and mass numbers of ions at pressure P . At a given pressure P , the magnetic field makes the matter less neutron rich. As a consequence, the magnetic field changes various crustal properties like the shear modulus [28, 29]. More recently, we have shown that the presence of a strong magnetic field impacts the neutron-drip transition, at which point nuclei $^A_Z X$ become unstable against the capture of Z electrons accompanied by the emission of A neutrons and Z neutrinos. In the strongly quantizing regime, the neutron-drip density and pressure are approximately given by [34]

$$n_{\text{drip}} \approx \frac{A B_\star \mu_e^{\text{drip}}(A, Z)}{Z 2\pi^2 \lambda_e^3 m_e c^2} \left[1 - \frac{4}{3} C \alpha Z^{2/3} \left(\frac{B_\star}{2\pi^2} \right)^{1/3} \left(\frac{m_e c^2}{\mu_e^{\text{drip}}(A, Z)} \right)^{2/3} \right], \quad (26)$$

$$P_{\text{drip}} \approx \frac{B_\star \mu_e^{\text{drip}}(A, Z)^2}{4\pi^2 \lambda_e^3 m_e c^2} \left[1 - \frac{1}{3} C \alpha Z^{2/3} \left(\frac{4B_\star}{\pi^2} \right)^{1/3} \left(\frac{m_e c^2}{\mu_e^{\text{drip}}(A, Z)} \right)^{2/3} \right], \quad (27)$$

where

$$\mu_e^{\text{drip}}(A, Z) \equiv \frac{-M(A, Z)c^2 + A m_n c^2}{Z} + m_e c^2. \quad (28)$$

In the weakly quantizing regime, the variations of the neutron drip density with magnetic field strength exhibits typical quantum oscillations whose amplitudes are universal [34]. As shown in Table 1, all three atomic mass models Duflo&Zuker [37], D1M [35] and HFB-27* [36] predict very similar values for the pressure at which neutrons start to drip out of nuclei although the nuclei are different.

Table 1. Sequence of equilibrium nuclides with increasing depth in the outer crust of a magnetar for different atomic mass models: Duflo-Zuker (DZ) [37], D1M [35] and HFB-27* [36]. The magnetic field strength is $B_* = 2000$. The nuclides with experimentally measured masses from the 2012 Atomic Mass Evaluation [38] are indicated in boldface. The maximum pressure at which each nuclide can be found is indicated in parenthesis in units of MeV fm^{-3} .

DZ	D1M	HFB-27*
^{56}Fe (3.12×10^{-7})	^{56}Fe (3.12×10^{-7})	^{56}Fe (3.12×10^{-7})
^{62}Ni (1.23×10^{-5})	^{62}Ni (1.23×10^{-5})	^{62}Ni (1.23×10^{-5})
^{88}Sr (2.68×10^{-5})	^{88}Sr (2.68×10^{-5})	^{88}Sr (2.68×10^{-5})
^{86}Kr (7.06×10^{-5})	^{86}Kr (7.06×10^{-5})	^{86}Kr (7.06×10^{-5})
^{84}Se (1.46×10^{-4})	^{84}Se (1.46×10^{-4})	^{84}Se (1.46×10^{-4})
^{82}Ge (2.44×10^{-4})	^{82}Ge (2.44×10^{-4})	^{82}Ge (2.44×10^{-4})
^{132}Sn (2.56×10^{-4})	^{132}Sn (2.56×10^{-4})	^{132}Sn (2.56×10^{-4})
^{80}Zn (2.92×10^{-4})	^{80}Zn (3.29×10^{-4})	^{80}Zn (3.31×10^{-4})
^{82}Zn (3.25×10^{-4})	—	—
—	—	^{130}Cd (3.54×10^{-4})
^{78}Ni (4.82×10^{-4})	—	—
—	^{128}Pd (5.20×10^{-4})	^{128}Pd (5.11×10^{-4})
^{126}Ru (5.90×10^{-4})	^{126}Ru (6.47×10^{-4})	^{126}Ru (6.18×10^{-4})
^{124}Mo (7.16×10^{-4})	^{124}Mo (7.88×10^{-4})	^{124}Mo (7.94×10^{-4})
^{122}Zr (9.32×10^{-4})	^{122}Zr (9.68×10^{-4})	^{122}Zr (8.90×10^{-4})
^{120}Sr (1.12×10^{-3})	^{120}Sr (1.14×10^{-3})	^{120}Sr (1.09×10^{-3})
—	—	^{122}Sr (1.14×10^{-3})
—	—	^{124}Sr (1.14×10^{-3})
^{118}Kr (1.15×10^{-3})	—	—

4 Conclusion

We have reexamined the threshold density and pressure for the onset of electron captures in both magnetic and nonmagnetic white dwarfs, generalizing the instability condition originally formulated in the context of the simple Chandrasekhar model. We have shown that the recently proposed super-Chandrasekhar massive white dwarfs endowed with strongly quantizing magnetic fields are highly unstable.

We have also studied the role of a strong magnetic field on the structure of the outer crust of a magnetar. We have shown that the magnetic field changes the

composition and makes matter more incompressible. We have also shown that the neutron drip density and pressure increase almost linearly with the magnetic field strength in the strongly quantizing regime for which electrons lie in the lowest Landau level. For weaker magnetic fields, the neutron drip density can be either increased or decreased. These results may have some implications for the physical interpretation of timing irregularities and quasiperiodic oscillations detected in soft gamma-ray repeaters and anomalous X-ray pulsars, as well as for the cooling of strongly magnetized neutron stars.

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