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Volume and Surface Components of the Nuclear Symmetry Energy

A.N. Antonov[1], **M.K. Gaidarov**[1], **P. Sarriguren**[2], **E. Moya de Guerra**[3]

[1]Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria

[2]Instituto de Estructura de la Materia, IEM-CSIC, Serrano 123, E-28006 Madrid, Spain

[3]Departamento de Fisica Atomica, Molecular y Nuclear, Facultad de Ciencias Fisicas, Universidad Complutense de Madrid, E-28040 Madrid, Spain

Abstract. The Coherent Density Fluctuation Model (CDFM) is used to calculate the volume (a_A^V) and surface (a_A^S) contributions (and their ratio) to the nuclear symmetry energy (NSE). Starting with the global values for parameters for infinite nuclear matter, our approach makes it possible to derive their corresponding values in finite nuclei. Two energy-density functionals (EDF) for nuclear matter, those of Brueckner and Skyrme are used. The weight function in CDFM is calculated using the proton and neutron densities from the self-consistent HF+BCS method with Skyrme interactions. The obtained values of a_A^V, a_A^S, and their ratio for the Ni, Sn, and Pb isotopic chains, as well as isotopic sensitivity are presented and discussed. The results are compared with those of other theoretical models, as well as with available experimental data obtained from analyses of nuclear properties, such as binding energies, neutron-skin thicknesses, excitation energies to isobaric analog states and others.

1 Introduction

The nuclear matter symmetry energy that characterizes the isospin-dependent part of the equation of state of asymmetric nuclear matter (ANM) is important for studies of the NSE in finite nuclei. The latter is not a directly measurable quantity and is extracted indirectly from observables that are related to it (e.g., [1]). The NSE at saturation density has been often obtained by fitting ground state masses with various versions of the liquid-drop mass formula within the liquid-drop models (LDM) [2, 3]. In our works [4–6] the NSE has been calculated within the CDFM [7,8] using the Brueckner EDF [9] by making a transition from the symmetry energy of nuclear matter to that of finite nuclei.

In 1947 Feenberg [10] pointed out that the surface energy should contain a symmetry energy contribution as a consequence of the failure of the nuclear saturation at the edge of the nucleus and that the volume saturation energy also has a symmetry energy term. Cameron in 1957 [11] (see also Bethe [12]) suggested

a revised mass formula in which the volume energy was expressed as a sum of two contributions, the volume saturation energy proportional to the mass number A and a volume symmetry energy assumed proportional to $(A-2Z)^2/A$. In 1958 Green [13] estimated the values of the volume and surface components of the corresponding contributions to the symmetry energy. Myers and Swiatecki in 1966 [2] admitted that the ratio between the mentioned coefficients must be equal to the ratio between the surface and volume coefficients of the corresponding components of the mass formula.

In the present work (see also Ref. [14]) the CDFM is used to calculate the volume and surface components of the NSE and their ratio using the Brueckner as well as the Skyrme (e.g., [15]) EDF. The results are compared with results of other theoretical works as well as with available experimental data extracted from binding energies, neutron-skin thicknesses and excitation energies to isobaric analog states (IAS).

In Sect. II we present the main relationships for the NSE and its volume and surface components. Section III contains the CDFM formalism and provides a way to calculate the mentioned quantities. The main conclusions of the study are given in Sect. IV.

2 Relationships Concerning the Volume and Surface Contributions to Nuclear Symmetry Energy

The expression for the energy per particle has the form:

$$\begin{aligned} \bar{E} &= \frac{E}{A} = -c_1 + c_2 \frac{1}{A^{1/3}} + c_3' \left(\frac{N-Z}{A} \right)^2 \\ &+ \frac{1}{A}[\text{Coulomb term} + \text{shell corrections}], \end{aligned} \tag{1}$$

where the first three terms in the right-hand side of Eq. (1) correspond to the volume, surface, and symmetry components. The latter can be expressed by its volume and surface contributions by (see e.g., Ref. [12])

$$\frac{(N-Z)^2}{A}(c_3 - c_4 A^{-1/3}). \tag{2}$$

Estimations of c_3 and c_4 have been given in Ref. [13], while in Ref. [2] it has been admitted that the ratio c_4/c_3 can be taken to be equal to the ratio c_2/c_1 [12]:

$$\frac{c_4}{c_3} = \frac{c_2}{c_1} = \chi \tag{3}$$

with $\chi = 1.1838$ from [2] and $\chi = 1.14$ from [12]. By definition the NSE coefficient is

$$s = \frac{1}{2} \left. \frac{\partial^2 \bar{E}}{\partial \alpha^2} \right|_{\alpha=0}, \tag{4}$$

where

$$\alpha \equiv \frac{N-Z}{A}. \tag{5}$$

It follows from Eqs. (4), (2) and (3) that

$$s = c_3' = c_3 - \frac{c_4}{A^{1/3}} = c_3\left(1 - \frac{\chi}{A^{1/3}}\right). \tag{6}$$

Thus

$$c_3 = \frac{s}{1-\frac{\chi}{A^{1/3}}}, \qquad c_4 = \chi\left(\frac{s}{1-\frac{\chi}{A^{1/3}}}\right). \tag{7}$$

In modern times Danielewicz *et al.* (e.g., [16, 17] and references therein) proposed the following expression for the NSE:

$$E_{sym} = \frac{a_a(A)}{A}(N-Z)^2, \tag{8}$$

where the A-dependent coefficient $a_a(A)$ is expressed by means of the volume (a_A^V) and surface (a_A^S) coefficients by the form (see also [18]):

$$a_a(A) = \frac{a_A^V}{\left[1 + A^{-1/3}\frac{a_A^V}{a_A^S}\right]}. \tag{9}$$

It has been shown in the local density approximation that the ratio a_A^V/a_A^S can be expressed by means of the symmetry energy dependence on the density $s(\rho)$ [16, 19, 20]:

$$\frac{a_A^V}{a_A^S} = \frac{3}{r_0}\int dr \frac{\rho(r)}{\rho_0}\left\{\frac{s(\rho_0)}{s[\rho(r)]} - 1\right\}, \tag{10}$$

where $\rho(r)$ is the half-infinite nuclear matter density, ρ_0 is the nuclear matter equilibrium density and r_0 is the radius of the nuclear volume per nucleon $4\pi r_0^3/3 = 1/\rho_0$.

In the Danielewicz's approximation only the symmetry energy of a finite nucleus $a_a(A)$ has a mass dependence, while a_A^V, a_A^S, and their ratio a_A^V/a_A^S are A-independent quantities. The values of a_A^V and a_A^S differ for various Skyrme interactions in wide intervals. At the same time, as shown in [16], a combination of empirical data on skin sizes and masses of nuclei constrains the volume symmetry parameter to $27 \leq a_A^V \leq 31$ MeV and the ratio a_A^V/a_A^S to $2.0 \leq a_A^V/a_A^S \leq 2.8$.

3 The CDFM. Results of Calculations of NSE and Its Volume and Surface Contributions

In the CDFM [7, 8] the one-body density matrix $\rho(\mathbf{r}, \mathbf{r}')$ of the nucleus is a coherent superposition of the one-body density matrices $\rho_x(\mathbf{r}, \mathbf{r}')$ for spherical "pieces" of nuclear matter with radius x (so called "fluctons") with densities

$\rho_x(\mathbf{r}) = \rho_0(x)\Theta(x - |\mathbf{r}|)$ and $\rho_0(x) = 3A/4\pi x^3$. The Fermi momentum of such a formation is $k_F(x) = [3\pi^2\rho_0(x)/2]^{1/3}$. The nuclear density distribution in the CDFM has the form:

$$\rho(\mathbf{r}) = \int_0^\infty dx|\mathcal{F}(x)|^2\rho_0(x)\Theta(x - |\mathbf{r}|), \tag{11}$$

where the weight function $|\mathcal{F}(x)|^2$ in the case of monotonically decreasing local density ($d\rho/dr \leq 0$) can be obtained using a known density of a given nucleus:

$$|\mathcal{F}(x)|^2 = -\frac{1}{\rho_0(x)} \left.\frac{d\rho(r)}{dr}\right|_{r=x} \tag{12}$$

with the normalization $\int_0^\infty dx|\mathcal{F}(x)|^2 = 1$.

In the CDFM the symmetry energy s for finite nuclei is obtained [4–6] to be an infinite superposition of the corresponding ANM symmetry energy weighted by $|\mathcal{F}(x)|^2$:

$$s = \int_0^\infty dx|\mathcal{F}(x)|^2 s^{ANM}(x). \tag{13}$$

In our work we use for the matrix element $V(x)$ of the nuclear Hamiltonian the corresponding ANM energy from the Brueckner and Skyrme EDF's. The weight function $|\mathcal{F}(x)|^2$ from Eq. (12) is calculated by means of proton and neutron density distributions obtained from the self-consistent HF+BCS method from Ref. [21] with density-dependent Skyrme interactions [22] and pairing correlations.

Here we note that the coefficient $a_a(A)$ [Eqs. (8) and (9)], as expected, can be represented approximately at large A (at least for $A \geq 27$) (e.g., Refs. [16, 17, 19, 20] in the form of Eq. (2):

$$a_a(A) = \frac{a_A^V}{\left[1 + A^{-1/3}\frac{a_A^V}{a_A^S}\right]} \simeq c_3 - \frac{c_4}{A^{1/3}} \tag{14}$$

that corresponds to Eq. (6), if $c_3 = a_A^V$ and $c_4 = (a_A^V)^2/a_A^S$.

Using as a base Eq. (10), we develop in the CDFM another approach to calculate a_A^V/a_A^S, as well as a_A^V and a_A^S. Our motivation is that numerous analyses of the volume and surface components of the NSE using a wide range of data on the binding energies, neutron-skin thicknesses and excitation energies to IAS give estimations of these quantities as functions of the mass number A (e.g., Refs. [3, 20, 23–27]) that change in some intervals for different regions of nuclei. In the CDFM we take nuclear matter values of the parameters to deduce their values in finite nuclei (using the self-consistently calculated nuclear density) which become dependent on the considered nucleus. For this purpose, we start from Eq. (10) but in it we replace the density $\rho(r)$ for the half-infinite nuclear matter in the integrand by the density distribution of finite nucleus. Later,

using Eq. (11) we obtain approximately an expression that allows us to calculate the ratio a_A^V/a_A^S. It has the form:

$$\frac{a_A^V}{a_A^S} = \frac{3}{r_0\rho_0}\int_0^\infty dx|\mathcal{F}(x)|^2 x\rho_0(x)\left\{\frac{s(\rho_0)}{s[\rho_0(x)]} - 1\right\}. \tag{15}$$

The approximations made in the CDFM lead to one-dimensional integral over x, the latter being the radius of the "flucton" that is perpendicular to the nuclear surface. Here we would like to emphasize that in contrast to Eq. (10), in Eq. (15) we use the finite nuclei densities to calculate the weight function $|\mathcal{F}(x)|^2$. In Eq. (15) $s(\rho_0) = s^{ANM}(\rho_0)$ and the quantity $s[\rho_0(x)] = s^{ANM}[\rho_0(x)]$ is the NSE within the chosen approach for the EDF. From Eqs. (12) and (13) we obtain the CDFM value for the NSE

$$s \equiv a_a(A). \tag{16}$$

Let denote by

$$\kappa \equiv \frac{a_A^V}{a_A^S} \tag{17}$$

that can be calculated using Eq. (15). Then it follows from Eq. (9):

$$s = \frac{a_A^V}{1 + A^{-1/3}\kappa}. \tag{18}$$

Finally, as a next step we obtain from Eqs. (16)-(18) (having calculated within the CDFM the values of s and κ) the expressions from which we can estimate the values of a_A^V and a_A^S separately:

$$a_A^V = s(1 + A^{-1/3}\kappa), \tag{19}$$

$$a_A^S = \frac{s}{\kappa}(1 + A^{-1/3}\kappa). \tag{20}$$

In our work we performed calculations of a_A^V/a_A^S, a_A^V and a_A^S (as well as of s) for the Ni, Sn and Pb isotopic chains. In Figure 1 is presented, as an example, the ratio κ as a function of A calculated using SLy4, SGII, and SkIII forces in HF+BCS method in the case of the Brueckner EDF. Our results for κ in Ni, Sn, and Pb chains are within the range $2.10 \leq \kappa \leq 2.90$ that is similar to the estimations obtained by Danielewicz *et al.* from a wide range of available data on the already mentioned nuclear quantities. The ranges of the published values of the ratio κ extracted from nuclear properties and presented in Ref. [20] are: $2.6 \leq \kappa \leq 3.0$ from IAS and skins [28], $2.0 \leq \kappa \leq 2.8$ from masses and skins [16], and $1.6 \leq \kappa \leq 2.0$ from the analyses in Ref. [20] of masses and skins. As can be seen the empirical ranges are in a good agreement with our results.

As can be seen in Figure 1 there exists a "kink" in the curve of $\kappa \equiv a_A^V/a_A^S$ as a function of A for the double-magic ^{78}Ni nucleus. Such a "kink" exists also

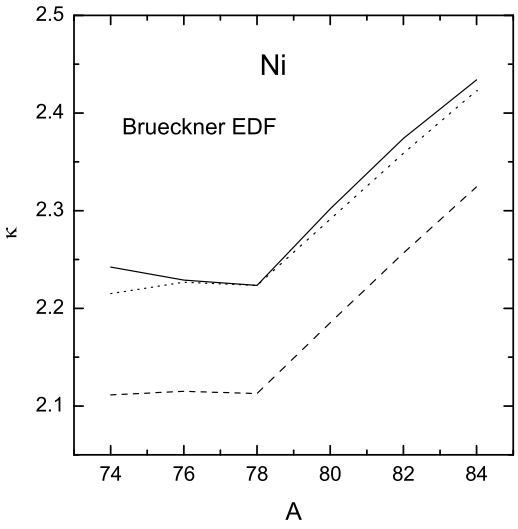

Figure 1. The ratio $\kappa = a_A^V/a_A^S$ as a function of A for the isotopic chain of Ni. The SLy4 (solid line), SGII (dashed line), and Sk3 (dotted line) forces have been used in the HF+BCS calculations of the densities in the case of Brueckner EDF.

for the double-magic ^{132}Sn nucleus. Here we would like to note that the origin of the "kinks" is in the different behavior of the density distributions $\rho(r)$ for given isotopes. Namely the derivative of $\rho(r)$ determines the weight function $|\mathcal{F}(x)|^2$ [Eq. (12)] that takes part in the integrand of the integral in Eq. (15) giving the ratio $\kappa \equiv a_A^V/a_A^S$. The peculiarities of $\rho(r)$ for the closed shells lead to the existence of "kinks". In the case of Pb isotopic chain such "kinks" do not exist and this reflects the smooth behavior without "kinks" of $s(A)$ and related quantities for the Pb isotopic chain.

In Figure 2 are presented a_A^V and a_A^S as functions of A for the Sn chain in the case of the Brueckner EDF. The CDFM results for a_A^V in the three chains (of Ni, Sn and Pb) are between 41.5 and 43 MeV, while for a_A^S they are between 14 and 20 MeV. These values are somewhat larger than those from other references given above. The differences are due mainly to the somewhat larger values of the NSE (s) for finite nuclei obtained within the CDFM using the Brueckner functional, because our values for the component $\kappa = a_A^V/a_A^S$ (that are between 2.1 and 2.9) are in the range obtained by other authors. Our results in this case can be compared with those obtained e.g., in Refs. [3, 16, 17, 19, 26–28] that are in the region $27 \leq a_A^V \leq 35$ MeV and for a_A^S they are between 8.5 and 11.3 MeV.

We would like to note that the same peculiarities (as for the ratio $\kappa \equiv a_A^V/a_A^S$), namely "kinks", appear in the cases of a_A^V and a_A^S as functions of the mass number A. In Figure 2(a) a "kink" appears for $a_A^V(A)$ not only for the double-magic ^{132}Sn, but also for the semi-magic ^{140}Sn nucleus. The latter is

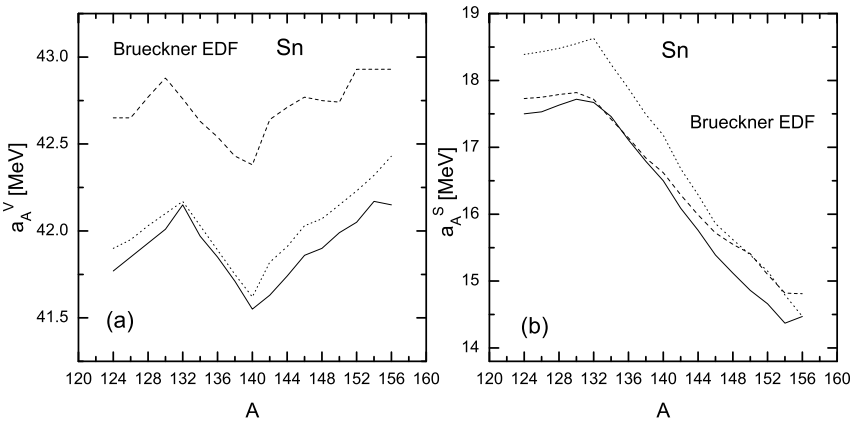

Figure 2. The values of a_A^V (a) and a_A^S (b) as functions of A for the isotopic chain of Sn. The SLy4 (solid line), SGII (dashed line), and Sk3 (dotted line) forces have been used in the HF+BCS calculations of the densities in the case of Brueckner EDF.

related to the closed $2f_{7/2}$ subshell for neutrons. As can be seen from Eqs. (19) and (20), the reason for "kinks" in the separate coefficients as functions of A is twofold. One of them is the already mentioned reason for the "kinks" in the ratio $\kappa \equiv a_A^V/a_A^S$, while the same reason causes also "kinks" in the NSE (s) at closed-shell nuclei.

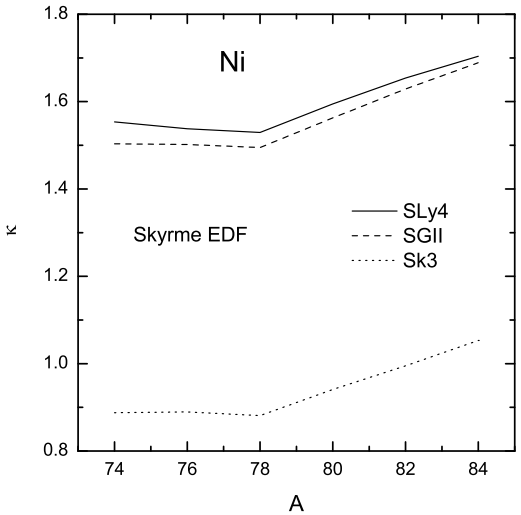

Figure 3. The ratio $\kappa = a_A^V/a_A^S$ as a function of A for the isotopic chain of Ni in the case of Skyrme EDF with use of SLy4, SGII, and Sk3 forces.

The second EDF that we use in the calculations is that one of Skyrme with different Skyrme forces (e.g., Ref. [15]). As an example, we present in Figure 3 the calculated value of κ for the Ni chain as a function of A in the case of the Skyrme EDF. In Figure 4 are shown the values of a_A^V and a_A^S for the Sn isotopic chain. The ranges of changes of both quantities for the three chains in the case of the Skyrme EDF with SLy4, SGII, and Sk3 forces obtained in the present work (see also Ref. [14]) are given in Table 1. The ranges of changes of κ are for the Ni isotopic chain: $1.5 \leq \kappa \leq 1.7$ (SLy4 and SGII forces) and $0.88 \leq \kappa \leq 1.05$ (Sk3 force), for the Sn isotopic chain: $1.52 \leq \kappa \leq 2.1$ (SLy4 and SGII forces) and $0.82 \leq \kappa \leq 1.14$ (Sk3 force), and for the Pb isotopic chain: $1.65 \leq \kappa \leq 1.75$ (SLy4 and SGII forces) and $0.84 \leq \kappa \leq 0.88$ (Sk3 force). We note that the ranges of κ for the SLy4 and SGII forces in the three chains are in agreement with those obtained in Ref. [20] $1.6 \leq \kappa \leq 2.0$ from analyses of masses and skins.

We find that our results for a_A^V are almost independent on A for a given isotopic chain and Skyrme force. They are also similar in the different chains for a given Skyrme force. The comparison of the results of our approach with those of other authors shows that our values of a_A^V for the isotopic chains of Ni, Sn, and Pb for the SGII and Sk3 forces are in agreement with those from, e.g., Refs. [16, 17, 19] given above, while the obtained values for the SLy4 force are comparable with the results in Ref. [3]. We note a "kink" in the behavior of κ for the Ni chain at $A = 78$, for the Sn chain at $A = 132$ and a lack of "kinks" for the Pb chain, like in the case when the Brueckner EDF is used. A "kink" at Ni chain at $A = 78$ can be seen also in the A-dependence of a_A^S, as well as a "kink" of a_A^S is seen at $A = 132$ in the case of the Sn chain. In the latter small

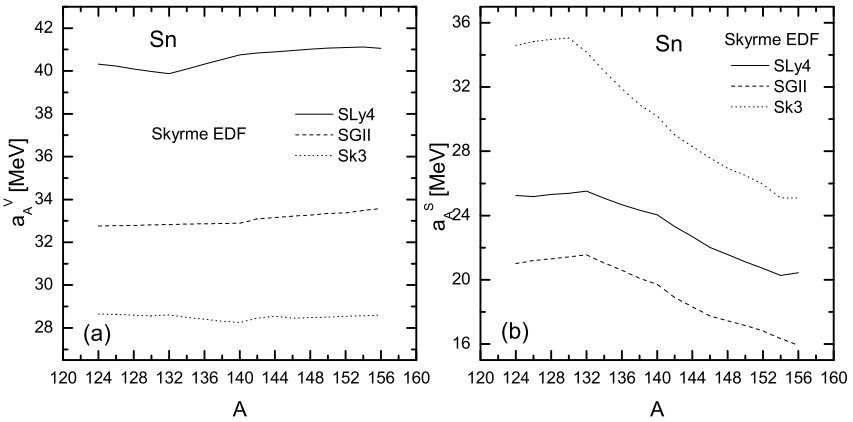

Figure 4. The values of a_A^V (a) and a_A^S (b) as functions of A for the isotopic chain of Sn in the case of Skyrme EDF with use of SLy4, SGII, and Sk3 forces.

Table 1. The ranges of changes of a_A^V and a_A^S and their average values for SLy4, SGII, and Sk3 forces used in the calculations with Skyrme EDF for the Ni, Sn, and Pb isotopic chains.

Isotopic chain	NSE component	SLy4	SGII	Sk3
Ni	a_A^V	$40.8 \div 42$	$33.5 \div 33.7$	$27.9 \div 28.3$
	a_A^S	$24.6 \div 26.7$	$20 \div 22.5$	$26.5 \div 32.1$
	$\bar{a}_A^V$	41.27	33.61	28.13
	$\bar{a}_A^S$	25.9	21.55	30.01
Sn	a_A^V	$40 \div 41.1$	$32.8 \div 33.6$	$28.3 \div 28.6$
	a_A^S	$20.3 \div 25.5$	$16 \div 21.5$	$25.1 \div 35$
	$\bar{a}_A^V$	40.6	33.06	28.51
	$\bar{a}_A^S$	23.36	19.21	30.24
Pb	a_A^V	$39.1 \div 39.6$	$32.3 \div 32.4$	$28.8 \div 29.1$
	a_A^S	$22.7 \div 23.5$	$18.8 \div 19.7$	$32.7 \div 34.6$
	$\bar{a}_A^V$	39.35	32.34	28.88
	$\bar{a}_A^S$	23.1	19.34	33.68

"kinks" can be observed also for a_A^V especially at $A = 132$ for the SLy4 force. There are no "kinks" of a_A^V and a_A^S in the Pb chain.

4 Conclusions

The results of the present work can be summarized as follows:

i) An approach within the CDFM is developed to calculate the symmetry energy (s), its volume (a_A^V) and surface (a_A^S) components and their ratio $\kappa = a_A^V/a_A^S$. The energy-density functionals of Brueckner and Skyrme are used. The weight function of CDFM is obtained by using the proton and neutron densities calculated within a self-consistent HF+BCS method with Skyrme forces. The approach makes it possible to start with the global values of parameters for infinite nuclear matter and to derive their corresponding values in finite nuclei which become A-dependent;

ii) The quantities a_A^V, a_A^S and κ are calculated for Ni, Sn and Pb isotopic chains. The results are compared with those of other theoretical methods, as well as with available experimental data obtained from analyses of binding energies, neutron-skin thicknesses, excitation energies to isobaric analog states and others. The existing agreement in various cases is discussed;

iii) "Kinks" of the A-dependence of κ, as well as of a_A^V and a_A^S when Brueckner EDF is used is found in the cases of double-magic nuclei ^{78}Ni and ^{132}Sn, as well as a "kink" of a_A^V for ^{140}Sn is observed. The "kinks" reflect the shell peculiarities. That one of a_A^V for ^{140}Sn is related to the closed $2f_{7/2}$ subshell of neutrons. The origin of the "kinks" is in the different behavior of the density distributions $\rho(r)$ for the isotopes and thus, of the weight function $|\mathcal{F}(x)|^2$;

iv) Similarly to the case when Brueckner EDF is used, "kinks" of $\kappa(A)$ and a_A^S for ^{78}Ni and ^{132}Sn exist in the case when the Skyrme EDF is used;

v) The obtained within the CDFM NSE coefficient $s = a_a(A)$ in finite nuclei (in the cases of both EDFs) avoids the problem related to fitting HF energies to LDM parametrization. It makes it possible to obtain additional information not only for a_A^V, but also about the surface symmetry energy a_A^S that is poorly constrained by empirical data. The obtained results provide a possibility to test the properties of nuclear energy-density functionals and characteristics related to NSE, e.g. the neutron-skin thickness of finite nuclei.

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