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Neutrino Mass and Forbidden Beta Decays

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Abstract. A possibility to exploit the first-, second-, and third- unique and the first non-unique forbidden beta decays to estimate the neutrino mass is discussed. Our findings show that the corresponding Kurie function in the vicinity of the endpoint is within a good accuracy linear in the limit of massless neutrinos like the Kurie function of the superallowed beta decay of tritium.

1 Introduction

The absolute neutrino mass scale is important for the physics beyond the Standard Model. Various experiments with neutrino oscillations proved that neutrinos are massive particles. The absolute value of the neutrino masses cannot be determined by the oscillation experiments being kind of the interference experiments.

A precise and model independent way to obtain the direct information on the neutrino mass is the kinematical analysis of the endpoint of the electron energy spectrum in single β decays such as the tritium β decay. The first measurement was performed by Hanna and Pontecorvo in 1949 [1] and a limit of $\sim$ 1 keV was obtained. Currently, from the Mainz and Troitsk experiments for the upper bound on the effective neutrino mass m_β we have $m_\beta <$ 2.2 eV [2]. The KATRIN experiment [3] aims at reaching a sensitivity of 0.2 eV. Another promising way to determine the absolute neutrino mass scale is to exploit the first unique forbidden β decay of ^{187}Re due to its low transition energy of 2.47 keV [4].

The aim of this contribution is to study the theoretical electron energy spectrum in the first, second, and third unique and the first non-unique forbidden beta decays. The attention is paid to the Kurie function near the endpoint as a function of the neutrino mass m_β.

2 Forbidden Beta Decays

The width of the first non-unique ($\Delta J^\pi = 0^-$) forbidden β decay for the spin of the initial nucleus $J_i = 0$ takes the same form,

$$\frac{d\Gamma}{dE_e} = \frac{2}{\pi^2} G_F^2 V_{ud}^2 \sum_j p_e E_e p_\nu E_\nu |U_{ej}|^2 \theta(E_0 - E_e - m_j) \mathcal{B}_A \mathcal{G}_A(E_e), \qquad (1)$$

as the differential decay rates of the first ($\Delta J^\pi = 2^-$), second ($\Delta J^\pi = 3^+$), and third ($\Delta J^\pi = 4^-$) unique forbidden β decays. Here, G_F is the Fermi constant and V_{ud} is the element of the Cabibbo-Kobayashi-Maskawa mixing matrix. p_e, E_e, and E_0 are the momentum, energy, and the maximal endpoint energy (in the case of zero neutrino mass) of the electron, respectively. The neutrino energy and momentum, respectively, are $E_\nu = E_0 - E_e$ and $p_\nu = \sqrt{(E_0 - E_e)^2 - m_j^2}$. U_{ej} and m_j are the element of neutrino mixing matrix and the neutrino mass, respectively. $\theta(x)$ is a theta (step) function.

We stress that there is only one nuclear matrix element $\mathcal{B}_A$ which enters the decay rate given in Eq. 1. For a given particular forbidden β transition, i.e. for the first non-unique ($A = 0^-$) and the first ($A = 2^-$), second ($A = 3^+$), and third ($A = 4^-$) unique forbidden β decay the nuclear matrix element is given as

$$\begin{aligned}
\mathcal{B}_{0^-} &= g_A^2 \left| \langle 0^{\pi_f} \parallel \sum_n \frac{r_n}{R} \tau_n^+ \{\sigma_1(n) \otimes Y_1(n)\}_0 \parallel 0^{\pi_i} \rangle \right|^2 , \\
\mathcal{B}_{2^-} &= \frac{g_A^2}{2J_i + 1} \left| \langle J_f^{\pi_f} \parallel \sum_n \frac{r_n}{R} \tau_n^+ \{\sigma_1(n) \otimes Y_1(n)\}_2 \parallel J_i^{\pi_i} \rangle \right|^2 , \\
\mathcal{B}_{3^+} &= \frac{g_A^2}{2J_i + 1} \left| \langle J_f^{\pi_f} \parallel \sum_n \frac{r_n^2}{R^2} \tau_n^+ \{\sigma_1(n) \otimes Y_2(n)\}_3 \parallel J_i^{\pi_i} \rangle \right|^2 , \\
\mathcal{B}_{4^-} &= \frac{g_A^2}{2J_i + 1} \left| \langle J_f^{\pi_f} \parallel \sum_n \frac{r_n^3}{R^3} \tau_n^+ \{\sigma_1(n) \otimes Y_3(n)\}_4 \parallel J_i^{\pi_i} \rangle \right|^2 .
\end{aligned} \tag{2}$$

Here, g_A denotes an axial-vector coupling constant. The functions $\mathcal{G}_A(E_e)$ depend solely on the electron energy.

$$\begin{aligned}
\mathcal{G}_{0^-}(E_e) &= \frac{1}{9} \Big(F_{p_{1/2}}(E_e, R)(p_e R)^2 + 2F_{sp_{1/2}}(E_e, R) \frac{p_e p_\nu^2}{E_\nu} R^2 \\
&\quad + F_{s_{1/2}}(E_e, R)(p_\nu R)^2 \Big) , \\
\mathcal{G}_{2^-}(E_e) &= \frac{1}{9} \left(F_{p_{3/2}}(E_e, R)(p_e R)^2 + F_{s_{1/2}}(E_e, R)(p_\nu R)^2 \right) , \\
\mathcal{G}_{3^+}(E_e) &= \frac{1}{45} \Big(\frac{1}{5} F_{d_{5/2}}(E_e, R)(p_e R)^4 + \frac{2}{3} F_{p_{3/2}}(E_e, R)(p_e p_\nu)^2 R^4 \\
&\quad + \frac{1}{5} F_{s_{1/2}}(E_e, R)(p_\nu R)^4 \Big) , \\
\mathcal{G}_{4^-}(E_e) &= \frac{1}{105^2} \big(F_{f_{7/2}}(E_e, R)(p_e R)^6 + 7F_{d_{5/2}}(E_e, R) p_e^4 p_\nu^2 R^6 \\
&\quad + 7F_{p_{3/2}}(E_e, R) p_e^2 p_\nu^4 R^6 + F_{s_{1/2}}(E_e, R)(p_\nu R)^6 \big) .
\end{aligned} \tag{3}$$

The Fermi functions associated with the emission of $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{5/2}$, and $f_{7/2}$-state electron, respectively, are defined as

$$\begin{aligned} F_{s_{1/2}}(E_e,R) &= g_{-1}^2(E_e,R)+f_{+1}^2(E_e,R), \\ F_{p_{1/2}}(E_e,R) &= \frac{g_{+1}^2(E_e,R)+f_{-1}^2(E_e,R)}{(p_eR)^2/3^2}, \\ F_{p_{3/2}}(E_e,R) &= \frac{g_{-2}^2(E_e,R)+f_{+2}^2(E_e,R)}{(p_eR)^2/3^2}, \\ F_{d_{5/2}}(E_e,R) &= \frac{g_{-3}^2(E_e,R)+f_{+3}^2(E_e,R)}{(p_eR)^4/15^2}, \\ F_{f_{7/2}}(E_e,R) &= \frac{g_{-4}^2(E_e,R)+f_{+4}^2(E_e,R)}{(p_eR)^6/105^2}. \end{aligned} \tag{4}$$

The β transitions with the $\Delta J^\pi = 0^-$ are non-unique first forbidden. Therefore, the decay rate consists of three contributions (see Eq. 3) associated with emission of the electron in $s_{1/2}$-state, $p_{1/2}$-state and the interference between $s_{1/2}$-state and $p_{1/2}$-state, respectively. The corresponding Fermi function for the interference term is defined as

$$F_{sp_{1/2}}(E_e,R) = \frac{g_{+1}(E_e,R)f_{+1}(E_e,R)-g_{-1}(E_e,R)f_{-1}(E_e,R)}{p_eR/3}. \tag{5}$$

Important ingredients in the Fermi functions are the radial electron wave functions, $g_{-k}(E_e,R)$ and $f_k(E_e,R)$, which satisfy the radial Dirac equations. The input is the potential of Coulomb field of daughter nucleus distorted with screening potential of electrons of daughter atom. The electron radial wave functions are evaluated by means of the subroutine package RADIAL [5]. Here we

Table 1. Nuclei which beta decays are classified as first, second, and third unique and first non-unique are listed. Both transitions to the ground state with spin and parity J^π and the first excited state J_1^π of final nucleus are considered. We note that $Q = E_0 - m_e$

Parent($J_i^{\pi_i}$)	Daughter($J_f^{\pi_f}$)	ΔJ^π	Q-value, keV	$T_{1/2}$, yrs
^{10}Be(0^+)	^{10}B(3^+)	3^+	556	1.51×10^6
^{40}K(4^-)	^{40}Ca(0^+)	4^-	1311.07	1.248×10^9
^{79}Se($7/2^+$)	^{79}Br($3/2^-$)	2^-	150.6	3.26×10^5
^{90}Sr(0^+)	^{90}Y(2^-)	2^-	546	28.79
^{93}Zr($5/2^+$)	^{93}Nb($1/2_1^-$)	2^-	60	1.61×10^6
^{107}Pd($5/2^+$)	^{107}Ag($1/2^-$)	2^-	34.1	6.5×10^6
^{138}La(5^+)	^{138}Ce(2_1^+)	3^+	255.3	1.05×10^{11}
^{140}Ba(0^+)	^{140}La(0_1^-)	0^-	468.9	3.49×10^{-2}
^{144}Ce(0^+)	^{144}Pr(0^-)	0^-	318.7	7.8×10^{-1}
^{187}Re($5/2^+$)	^{187}Os($1/2^-$)	2^-	2.663	4.35×10^{10}

have made a reasonable assumption that in β decay of nuclei participate mostly nucleons close to the Fermi level, i.e. the radial electron wave functions are evaluated at the nuclear surface $R = 1.2A^{1/3}$ fm.

In Table 1, we present nuclei which undergo β decays that are classified as the first, second, and third unique and the first non-unique forbidden. Our attention has been focused primarily on the decays with small Q value and lifetime larger than 10 days.

3 Kurie Functions

The current upper limit on neutrino mass holds in the degenerate neutrino mass region, i.e. $m_1 \simeq m_2 \simeq m_3 \simeq m_\beta = \sum_{j=1}^{3} |U_{ej}|^2 m_j$. The Kurie function for the allowed transitions takes the following form,

$$K(E_e, m_\beta) \sim (E_0 - E_e)\sqrt[4]{1 - \frac{m_\beta^2}{(E_0 - E_e)^2}}. \tag{6}$$

We define the Kurie functions for the first non-unique and the first, second , and third unique forbidden β decays, respectively, in such a way,

$$\begin{aligned}
K_{0^-}(E_e, m_\beta) &= \sqrt{\frac{d\Gamma/dE_e}{p_e E_e \left(F_{p_{1/2}}(E_e, R)(p_e R)^2/3^2\right)}} \\
&= G_F V_{ud}\sqrt{\frac{2}{\pi^2}\mathcal{B}_{0^-}}(E_0 - E_e)\sqrt[4]{1 - \frac{m_\beta^2}{(E_0 - E_e)^2}}\sqrt{S_{0^-}(E_e)}, \\
K_{2^-}(E_e, m_\beta) &= \sqrt{\frac{d\Gamma/dE_e}{p_e E_e \left(F_{p_{3/2}}(E_e, R)(p_e R)^2/3^2\right)}} \\
&= G_F V_{ud}\sqrt{\frac{2}{\pi^2}\mathcal{B}_{2^-}}(E_0 - E_e)\sqrt[4]{1 - \frac{m_\beta^2}{(E_0 - E_e)^2}}\sqrt{S_{2^-}(E_e)}, \\
K_{3^+}(E_e, m_\beta) &= \sqrt{\frac{d\Gamma/dE_e}{p_e E_e \left(F_{d_{5/2}}(E_e, R)(p_e R)^4/15^2\right)}} \\
&= G_F V_{ud}\sqrt{\frac{2}{\pi^2}\mathcal{B}_{3^+}}(E_0 - E_e)\sqrt[4]{1 - \frac{m_\beta^2}{(E_0 - E_e)^2}}\sqrt{S_{3^+}(E_e)}, \\
K_{4^-}(E_e, m_\beta) &= \sqrt{\frac{d\Gamma/dE_e}{p_e E_e \left(F_{f_{7/2}}(E_e, R)(p_e R)^6/105^2\right)}} \\
&= G_F V_{ud}\sqrt{\frac{2}{\pi^2}\mathcal{B}_{4^-}}(E_0 - E_e)\sqrt[4]{1 - \frac{m_\beta^2}{(E_0 - E_e)^2}}\sqrt{S_{4^-}(E_e)},
\end{aligned} \tag{7}$$

that they resemble the Kurie function for the allowed β decays. The shape factors $S_A(E_e)$ are given as

$$\begin{aligned}
S_{0^-}(E_e) &= \Big(1 + 2\frac{F_{sp_{1/2}}(E_e,R)p_\nu^2}{F_{p_{1/2}}(E_e,R)p_e E_\nu} + \frac{F_{s_{1/2}}(E_e,R)p_\nu^2}{F_{p_{1/2}}(E_e,R)p_e^2}\Big),\\
S_{2^-}(E_e) &= \Big(1 + \frac{F_{s_{1/2}}(E_e,R)p_\nu^2}{F_{p_{3/2}}(E_e,R)p_e^2}\Big),\\
S_{3^+}(E_e) &= \Big(1 + \frac{10F_{p_{3/2}}(E_e,R)p_\nu^2}{3F_{d_{5/2}}(E_e,R)p_e^2} + \frac{F_{s_{1/2}}(E_e,R)p_\nu^4}{F_{d_{5/2}}(E_e,R)p_e^4}\Big),\\
S_{4^-}(E_e) &= \Big(1 + 7\frac{F_{d_{5/2}}(E_e,R)p_\nu^2}{F_{f_{7/2}}(E_e,R)p_e^2} + 7\frac{F_{p_{3/2}}(E_e,R)p_\nu^4}{F_{f_{7/2}}(E_e,R)p_e^4} + \frac{F_{s_{1/2}}(E_e,R)p_\nu^6}{F_{f_{7/2}}(E_e,R)p_e^6}\Big).
\end{aligned} \tag{8}$$

We performed numerical analysis of the shape factors $S_A(E_e)$ for the particular nuclei listed in Table 1. They are plotted in Figure 1 against the electron energy close to the endpoint except ^{187}Re due to an extremely low Q value. Our findings show that the shape factors can be approximated as $S_A(E_e) \sim 1$ close to the endpoint for all nuclei given in Table 1. Then the Kurie functions for the first non-unique and the first, second, and third unique forbidden β decays can be approximated with the Kurie function for the allowed transitions,

$$K_A(E_e, m_\beta) \cong G_F V_{ud}\sqrt{\frac{2}{\pi^2}\mathcal{B}_A}(E_0 - E_e)\sqrt[4]{1 - \frac{m_\beta^2}{(E_0-E_e)^2}}. \tag{9}$$

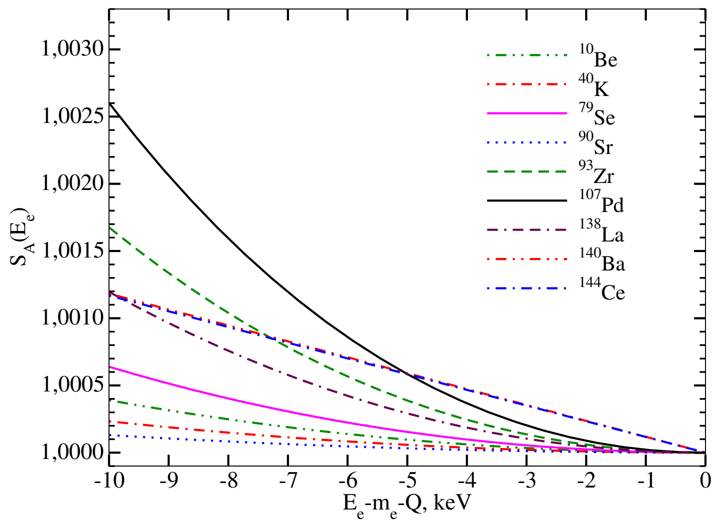

Figure 1. The shape factors as a function of the electron energy.

The goal is that the Kurie plot is linear only in the case of massless neutrinos. However, the linearity is lost for the non-zero neutrino mass.

4 Conclusions

The kinematical measurement of the neutrino mass has been performed in the laboratory by taking the advantage of the superallowed β decay of tritium. We found that this goal can be achieved also with the forbidden β decays. It is shown that the Kurie function for the first non-unique and the first, second, and third unique forbidden β decays close to the endpoint coincides up to a factor to the Kurie function of the superallowed β decay of tritium having the same dependence on the effective neutrino mass m_β.

References

[1] G. Hanna, B. Pontecorvo, *Phys. Rev.* **75** (1949) 983.

[2] E.W. Otten, C. Weinheimer, *Rept. Prog. Phys.* **71** (2008) 086201.

[3] C. Weinheimer, *Nucl. Phys. Proc. Suppl.* **168** (2007) 5.

[4] R. Dvornický, K. Muto, F. Šimkovic, and A. Faessler, *Phys. Rev. C* **83** (2011) 045502.

[5] D. Štefánik, R. Dvornický, F. Šimkovic, and P. Vogel, *Phys. Rev. C* **92** (2015) 055502.

[6] R. Dvornický, F. Šimkovic, *Acta Physica Polonica B Proc. Supp.* **8** (2015) 559-565.