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Two Decay Paths for Calculating Nuclear Matrix Element of Neutrinoless Double-Beta Decay

J. Terasaki

Institute of Experimental and Applied Physics, Czech Technical University in Prague, Horská 3a/22, 128 00, Prague 2, Czech Republic

Abstract. It is possible to calculate nuclear matrix elements of neutrinoless double-β decay using virtual decay paths with two-particle transfer under well-known the closure approximation. The nuclear matrix elements are calculated using the proton-neutron quasiparticle random-phase approximation (QRPA) for the original double-β path and the like-particle QRPA for the two-particle-transfer path. I determine the strength of the isoscalar proton-neutron pairing interaction so as to obtain the same nuclear matrix elements by the two calculations. The consistency of the QRPA approach is improved by this method.

1 Introduction

Since the discovery of the neutrino oscillation [1–4], the determination of the neutrino masses has been one of the most important subjects in physics. One of quite few methods for the determination is the method using the neutrinoless double-β ($0\nu\beta\beta$) decay of nuclei. This decay occurs, if the neutrino is a Majorana particle, *i.e.*the neutrino itself is the antineutrino. In addition, if this decay is observed, it would be the first evidence that the lepton number is not conserved. Moreover, information on the right-handed neutrino would be obtained. Many experiments are under operation or in preparation for detecting the $0\nu\beta\beta$ decay using the underground facilities.

The principle for determining the neutrino mass by the $0\nu\beta\beta$ decay is given by a basic equation of quantum mechanics, that is, the one that the decay probability is proportional to the squared absolute value of the transition matrix element. In the $0\nu\beta\beta$ decay, this squared one can be written as the product of the squared absolute value of the nuclear matrix element, phase-space factor, and the squared effective neutrino mass scaled by the electron mass, and this effective neutrino mass provides us with the mass scale of the neutrino. The phase-space factor is calculated from the wavefunctions of the emitted electrons and established better (see Ref. [5] and references therein) than the nuclear matrix elements. Establishment of the nuclear matrix elements is more difficult because the accurate nuclear wavefunctions are necessary. Several methods have been used for calculating the nuclear wavefunctions, and currently the obtained

nuclear matrix elements are distributed in the range of a factor of $2-3$ [6]. This is the most serious problem in the theoretical part of the $0\nu\beta\beta$-decay approach for determining the effective neutrino mass.

The purpose of this study is to improve the reliability of the method using the quasiparticle random-phase approximation (QRPA). Two decay paths are considered; one is the original $\beta\beta$-decay path, and another one is the virtual decay path with the two-particle transfer. The latter is possible under the closure approximation, and naturally, the agreement is necessary in the nuclear matrix elements between the two methods. Fulfillment of this condition improves the reliability of the QRPA approach. The detail of this study is found in Ref. [7]; this paper is a brief report.

2 Nuclear Matrix Element and Two Decay Paths

The matrix element of the $0\nu\beta\beta$ decay $M^{(0\nu)}$ is calculated by, e.g. [8],

$$M^{(0\nu)} = \sum_{K\pi} \sum_{a_F^{K\pi} a_I^{K\pi}} \sum_{\alpha\alpha':p} \sum_{\beta\beta':n} \langle -\alpha\alpha' | \hat{O}^{(0\nu)} | \beta - \beta' \rangle \times \langle F | c_\beta c^\dagger_{-\alpha} | a_F^{K\pi} \rangle \langle a_F^{K\pi} | a_I^{K\pi} \rangle \langle a_I^{K\pi} | c^\dagger_{\alpha'} c_{-\beta'} | I \rangle, \quad (1)$$

where $\hat{O}^{(0\nu)}$ is the operator causing the $0\nu\beta\beta$ decay defined by

$$\hat{O}^{(0\nu)} = h_+(r_{12}, \bar{E}) \{ -\underline{\sigma}(1) \cdot \underline{\sigma}(2) + g_V^2/g_A^2 \} \tau^+(1) \tau^+(2). \quad (2)$$

$h_+(r_{12}, \bar{E})$ is the neutrino potential, which is a function of the inter nucleon distance r_{12} and the average energy $\bar{E}$ used in the closure approximation. The vector symbol $\underline{\sigma}$ is the Pauli spin operator, and its arguments 1 and 2 distinguish the nucleons on which the operator acts. Parameters g_V and g_A are the vector and axial-vector current coupling constants, respectively. Symbols α and α' (β and β') stand for the protons (neutrons). Their creation and annihilation operators are written as $c^\dagger_\alpha$ and c_β. The notation of $-\alpha$ indicates that the z component of the nucleon angular momentum has the sign opposite to that of α. $|I\rangle$ and $|F\rangle$ are the initial and final states, that is, the ground states of the parent and daughter nuclei. States $|a_F^{K\pi}\rangle$ and $|a_I^{K\pi}\rangle$ are the intermediate states of the $0\nu\beta\beta$ decay, and they are obtained by the proton-neutron (pn) QRPA calculation on the basis of the initial or final states. K and π are the z component of the nuclear angular momentum and parity, respectively. Note that $|a_I^{K\pi}\rangle \neq |a_F^{K\pi}\rangle$ because the QRPA is an approximation. The overlap $\langle a_F^{K\pi} | a_I^{K\pi} \rangle$ is calculated according to the method developed in Ref. [9].

The intermediate-state dependence appears only in the projectors of those states, and the summation is taken over all of those states. Thus, one can derive the following equation:

$$M^{(0\nu)}_{2p2n} \equiv \sum_{K\pi}\sum_{m_F^{K\pi}m_I^{K\pi}}\sum_{\alpha\alpha':p}\sum_{\beta\beta':n}\langle -\alpha\alpha|\hat{O}^{(0\nu)}|\beta-\beta'\rangle \times\langle F|c^\dagger_{-\alpha}c^\dagger_{\alpha'}|m_F^{K\pi}\rangle\langle m_F^{K\pi}|m_I^{K\pi}\rangle\langle m_I^{K\pi}|c_\beta c_{-\beta'}|I\rangle = M^{(0\nu)}. \quad (3)$$

This equation indicates that the virtual decay path is used in which two neutrons are removed, and then two protons are added. The like-particle (lp) QRPA is used for calculating the intermediate states $|m_F^{K\pi}\rangle$ and $|m_I^{K\pi}\rangle$.

3 Calculation

3.1 $0\nu\beta\beta$ decay

The calculation was performed for the decay of ^{150}Nd$\rightarrow^{150}$Sm using the Skyrme interaction with the parameter set SkM* and the contact volume pairing interaction, of which the strengths were adjusted so as to fit the pairing gaps of the protons and neutrons obtained from the experimental nuclear masses by the Hartree-Fock-Bogoliubov (HFB) calculation of the initial and final ground states. In addition, I use the proton-neutron pairing interaction

$$V^{\rm pair} = g^{\rm pnpair}_{T=0}\delta(\underline{r}_1-\underline{r}_2)P_{S=1}P_{T=0} + g^{\rm pnpair}_{T=1}\delta(\underline{r}_1-\underline{r}_2)P_{S=0}P_{T=1}, \quad (4)$$

where T and S indicate the absolute values of the isospin and spin of a nucleon pair, and P_T and P_S are the projectors to those specified isospin and spin pair subspaces. Parameter $g^{\rm pnpair}_T$ is the strength of the interaction. There is no proton-neutron pairing gap in the HFB ground states of ^{150}Nd and ^{150}Sm. In this case, $V^{\rm pair}$ affects only the pnQRPA calculation. Therefore, the solutions of the pn and lp QRPAs have different many-body correlations, and $M^{(0\nu)} = M^{(0\nu)}_{2p2n}$ is not automatically guaranteed (for more detailed discussion, see Ref. [7]). By assuming that all of the interactions except for the proton-neutron pairing one are well established, that equivalence condition of the two paths can be used for determining the strengths of the proton-neutron pairing interaction. This interaction is relatively uncertain because the experimental evidence of the effect of this interaction is not as clear as that of other interactions.

The setup of the calculation, the space size and others, is explained in detail in Ref. [10]. I emphasize that very large single-particle spaces are used, and the convergence of the results with respect to the dimension of those spaces has been confirmed. The effective-operator methods often applied for the decay operator is not used in my calculation. The result obtained using $g_A = 0.254$ (the bare value) and $g_V = 1.0$ is shown in Table 1. If an effective value of g_A smaller than the bare one is used, a smaller nuclear matrix element would be obtained. The lowest limit is unfortunately unknown because a firm theory is not yet established for determining the effective g_A. The value of the proton-neutron pairing interaction used for getting 3.697 is $g^{\rm pnpair}_{T=0} = -197.44$, and $g^{\rm pnpair}_{T=1}$ was omitted.

Table 1. $M^{(0\nu)}$ and $M^{(0\nu)}_{2p2n}$ for ^{150}Nd$\rightarrow^{150}$Sm

Equation and interaction	Nuclear matrix element
$M^{(0\nu)}_{2p2n}$ without V^{pnpair}	5.324
$M^{(0\nu)}_{2p2n}$ with V^{pnpair}	3.697
$M^{(0\nu)}$	3.604

There is an argument that the latter should be determined so as to retrieve the isospin symmetry. In my calculation, the breaking of the isospin symmetry is not significant (see next section). That $g^{\text{pnpair}}_{T=0}$ value is actually the average of the proton and neutron (like-particle) pairing interactions. If the mechanism causing the proton-neutron and like-particle pairing correlations is not completely different, the similarity of the $g^{\text{pnpair}}_{T=0}$ and strengths of the like-particle pairing interactions is not surprising. The values of the nuclear matrix elements calculated by other groups are shown in Table 2. Our value is close to the largest one by the pnQRPA approach of other groups.

Table 2. Nuclear matrix element of the $0\nu\beta\beta$ decay of ^{150}Nd$\rightarrow^{150}$Sm by other groups. IBM-2 indicates interacting boson model-2, and GCM is the generator coordinate method. The values obtained with $g_A = 1.25$ or close to it are listed, as long as that value is used. $M^{(0\nu)}$ = 3.14 and 2.71 of PnQRPA (Skyrme, volume pairing) were obtained with parameter set SkM* and modified SkM*, respectively

Method	$0\nu\beta\beta$ nuclear matrix element	Reference
PnQRPA (CD-Bonn, G matrix)	3.34	[13]
PnQRPA (Skyrme, volume pairing)	3.14, 2.71	[14]
IBM-2	2.321	[15, 16]
Projected HFB	3.24±0.44	[17]
Energy density functional (Gogny, GCM, projection)	2.190	[18]
Relativistic (GCM, projection)	5.60	[19]

3.2 $2\nu\beta\beta$ decay

The nuclear matrix element of the two-neutrino double-β ($2\nu\beta\beta$) decay is calculated using [8]

$$M^{(2\nu)} = \frac{M^{(2\nu)}_{GT}}{\mu_0} - \frac{g_V^2}{g_A^2}\frac{M^{(2\nu)}_F}{\mu_{0F}}, \tag{5}$$

$$\frac{M^{(2\nu)}_{GT}}{\mu_0} = \sum_{a_I^K, a_F^K} \frac{1}{\mu_a} \langle F| \sum_n \tau^+(n)(-)^K [\sigma(n)]_{-K} |a_F^K\rangle \langle a_F^K | a_I^K\rangle \times \langle a_I^K | \sum_{n'} \tau^+(n')[\sigma(n')]_K |I\rangle \begin{cases} 2, & K=1, \\ 1, & K=0, \end{cases} \tag{6}$$

$$\frac{M_F^{(2\nu)}}{\mu_{0F}} = \sum_{a_I,a_F} \frac{1}{\mu_a} \langle F| \sum_n \tau^+(n)|a_F\rangle\langle a_F|a_I\rangle\langle a_I| \sum_{n'} \tau^+(n')|I\rangle, \quad (7)$$

$$\mu_a = \frac{1}{m_e c^2}\Big\{E_{aK,I} - \frac{1}{2}(M_F + M_I)\Big\}. \quad (8)$$

The indexes n and n' indicate the nucleons. The intermediate states of the Gamow-Teller (GT) decay are those of $K = 0$ and 1 with positive parity, and those of the Fermi (F) decay have only $K = 0$ and positive parity. $E_{aK,I}$ is the energy of the intermediate state. M_F and M_I are the masses of the final and initial states, respectively, and m_e is the electron mass.

Table 3. $M^{(2\nu)}$ calculated for ^{150}Nd$\rightarrow^{150}$Sm and its components. Semi-experimental values are also shown, which were obtained from the experimental half-life [11], theoretical phase-space factor [12], and g_A. $|K|$ indicates those of the intermediate states. For $K \neq 0$, the components of the nuclear matrix element shown here are summations of those for $K = \pm|K|$

$\|K\|$	$M_{GT}^{(2\nu)}(\|K\|)/\mu_0$	$M_F^{(2\nu)}(\|K\|)/\mu_{0F}$	$M^{(2\nu)}(\|K\|)$	
			$g_A = 1.254$	1.000
0	0.0271	−0.0092	0.0329	0.0363
1	0.0486	0	0.0486	0.0486
	$M_{GT}^{(2\nu)}/\mu_0$	$M_F^{(2\nu)}/\mu_{0F}$	$M^{(2\nu)}$	
Total	0.0757	−0.0092	0.0816	0.0849
Semi-exp.			0.0368	0.0579

The result of calculation is summarized in Table 3. The typical effective g_A of 1.0 is also used as reference. The calculated values of $M^{(2\nu)}$ are 0.0816 ($g_A = 1.254$) and 0.0849 ($g_A = 1.000$), and they are larger than the corresponding semi-experimental values by 122% and 47%, respectively; improvement is necessary. If the isospin symmetry is conserved, the Fermi component $M_F^{(2\nu)}$ vanishes. The table shows that the $M_F^{(2\nu)}/\mu_{0F}$ is -11% and -8 % of the $M^{(2\nu)}$ calculated with $g_A = 1.254$ and $g_A = 1.000$, respectively. Thus, the isospin symmetry is not seriously broken in my calculation without special treatment.

4 Summary

In this study, I introduced a new condition for the calculation of the nuclear matrix element of the $0\nu\beta\beta$ decay. The nuclear matrix element should not be different between the calculation using the original $\beta\beta$ path and that using the virtual two-particle-transfer path. The QRPA is an approximation choosing parts of the many-body correlations, thus this condition is not automatically fulfilled. The condition of the equivalence of the two paths is a constraint to the effective interactions used in the QRPA. The most uncertain interaction used in the cal-

culation of this study is the isoscalar proton-neutron pairing interaction, and its strength was determined so as to fulfill that condition. The validity of this new idea is shown by the result that the strength is not strange at all. The consistency of the QRPA approach to the $0\nu\beta\beta$ decay was strengthened by this study. The remaining most serious uncertainty is the effective g_A as shown by the comparison of $M^{(2\nu)}$ between the calculated and semi-experimental values. This problem is difficult to solve by the QRPA alone.

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