NUCLEAR THEORY, Vol. 35 (2016)
eds. M. Gaidarov, N. Minkov, Heron Press, Sofia

Reduced E2-Transition Probabilities in the Excited Collective States of Triaxial Even-Even Nuclei

M.S. Nadirbekov

National University of Uzbekistan, Tashkent, Uzbekistan

Abstract. An intra-/inter-band reduced E2-transition probabilities in the excited states of lanthanide and actinide even-even nuclei has been considered within a free triaxiality approximation. These E2-transitions are studied in detailed in the spectra of nuclei: ^{154}Sm, ^{156}Gd, ^{158}Dy, ^{162,164}Er, ^{230,232}Th, ^{232,234,236}U. Comparison of results of calculations with the corresponding experimental data shows a very good agreement, including high angular momentum states. A comparison of the ratios reduced E2- transition probabilities with results of Alaga rules are pointed out. These comparisons allow to determine a sensitivity of E2-transition probabilities to surface multipole deformations.

1 Introduction

Nuclear triaxiality is associated with the breaking of axial symmetry of the quadrupole deformation. Note, experiments of Coulomb excitation [1–3] on the measurements of quadrupole moments and E2-transition probabilities give more direct indications of nuclear triaxiality. E2-transition rates obtained in these experiments may be related to some effective values of the Bohr deformation parameters β and γ [4].

In [5]a wide comparison of theoretical and experimental [6] values of energy levels of lanthanide and actinide even-even nuclei within non-adiabatic approximations is given. An approximation with free triaxiality [7] describes better an energy levels of the nuclei than others non-adiabatic approximations in the mass region 150≤A≤254 [5]. There a good agreement of theoretical and experimental [6] spectra for ^{154}Sm, ^{156}Gd, ^{158}Dy, ^{162,164}Er, ^{230,232}Th, ^{232,234,236,238}U and ^{240}Pu are presented.

A good agreement between experimental and theoretical values of the energy levels of different bands does not suggest that such agreement also exists for other observables quantities. Note, that the intra-/inter-band reduced E2-transition probabilities more detail tests the validity of non-adiabatic models. Because, these transitions connect identical and different bands of excited collective states. Therefore, the study above-mentioned properties of nuclei is very important.

The intra-/inter-band transitions in excited collective states of ground state, γ- and β-bands within non-adiabatic models were considered in [7–9]. But the analyses of the feasible reduced E2-transition probabilities in excited collective states of above mentioned bands [7–9] was not completed. Therefore, the aim of our paper is a detail study within free triaxiality model [5] the intra-/inter-band reduced E2-transition probabilities in the spectra of heavy nuclei. They have been calculated and results were compared with the corresponding experimental data and results of Alaga rules [10].

The study of energy levels for excited bands of deformed nuclei gives a information about its own rotational motion with presence of other collective vibrational degrees of freedom (for example: β and γ collective variables). Then, it is important to know contribution of the deformation parameters to different degree of freedoms in nuclei. This dynamics is determined by the intensity of different multipole transitions, described by intra-/inter-band reduced E2-transition probabilities.

2 Wave Functions

The wave functions $\Psi_{n_\gamma n_\beta IM\tau}(\beta,\gamma,\theta_i)$ of Hamilton operator for even-even nuclei with free triaxiality is [5, 7]:

$$\Psi_{n_\gamma n_\beta IM\tau}(\beta,\gamma,\theta_i) = F_{n_\beta}(\beta)\xi_{n_\gamma}(\gamma)\phi_{IM\tau}(\theta_i), \tag{1}$$

with following notations:

$$\xi_{n_\gamma}(\gamma) = \frac{N_{\gamma_0}\gamma^{p+1}}{\sqrt{|\sin 3\gamma|}} \exp\Big(-\frac{\gamma^2}{2b^2_{\gamma_0}}\Big) L^{p+1/2}_{n_\gamma}\Big(\frac{\gamma^2}{b^2_{\gamma_0}}\Big), \tag{2}$$

$$F_{n_\beta}(\beta) = N_{\beta_0}\beta^{q+1} \exp\Big(-\frac{\beta^2}{2b^2_{\beta_0}}\Big) L^{q+1/2}_{n_\beta}\Big(\frac{\beta^2}{b^2_{\beta_0}}\Big), \tag{3}$$

$$\phi_{IM\tau}(\theta_i) = \sqrt{\frac{2I+1}{16\pi^2(1+\delta_{0K})}}\left[D^I_{IK}(\theta_i) + (-1)^I D^I_{I,-K}(\theta_i)\right] A^\tau_{IK}, \tag{4}$$

for detail notations for quantities and quantum numbers in these formulas see Ref. [5].

3 Reduced E2-transition probabilities

Reduced E2-transition probabilities between $n'_\gamma n'_\beta I'\tau'$ and $n_\gamma n_\beta I\tau$ states [5] are determined by expression [7]

$$B(E2; n_\gamma n_\beta I\tau \to n'_\gamma n'_\beta I'\tau') = \frac{5}{16\pi(2I+1)} \sum_{MM'\mu} |< n'_\gamma n'_\beta I'\tau' | Q_{2\mu} | n_\gamma n_\beta I\tau >|^2, \tag{5}$$

where

$$Q_{2\mu} = \frac{\beta}{\beta_0}\left\{Q_0 D^2_{\mu 0}\cos\gamma + Q_2\left(D^2_{\mu 2} + D^2_{\mu,-2}\right)\frac{\sin\gamma}{\sqrt{2}}\right\}, \quad (6)$$

here Q_0 and Q_2-intrinsic quadrupole moments [11]. Q_0 is the quadrupole moment with respect to the axis, where θ, ϕ (θ, ϕ are the polar angles [11]) is the orientation of this axis, while Q_2 is a measure of the asymmetry in the shape with respect to this axis [11].

Reduced E2-transition probabilities between states $i \equiv \{n_\gamma n_\beta I\tau\}$ and $f \equiv \{n'_\gamma n'_\beta I'\tau'\}$ are presented as

$$B(E2; i \to f) = B_a(E2; I\tau \to I'\tau')S^2_{if\beta}S^2_{if\gamma}, \quad (7)$$

where first factor $B_a(E2; I\tau \to I'\tau')$–E2-transition probabilities in the excited states of the asymmetric rotator [7],

$$S_{if\beta} = \int_0^\infty F_i(\beta)\frac{\beta}{\beta_0}F_f(\beta)\beta^4 d\beta, \quad (8)$$

take into account a deformability of even-even nuclei. Furthemore, for the simplicity we take $B(E2) = B(n_\gamma n_\beta I\tau \to n'_\gamma n'_\beta I'\tau')$. Then matrix elements

$$S_{if\gamma} = \int_o^\infty \xi_i(\gamma)\phi_i(\theta_i)\left\{Q_0 D^2_{\mu 0}\cos\gamma + Q_2\left(D^2_{\mu 2} + D^2_{\mu,-2}\right)\frac{\sin\gamma}{\sqrt{2}}\right\} \times \xi_f(\gamma)\phi_f(\theta_i)|\sin 3\gamma|\sin\theta_2 d\gamma d\theta_1 d\theta_2 d\theta_3. \quad (9)$$

include a contribution of γ-vibrations on the reduced E2-transition probabilities.

$$S_{if\gamma} = \left\{Q_0\sqrt{2I+1}(I2K0|I'K')I_1 + Q_2\sqrt{\frac{2I+1}{2}}\left[\sqrt{\frac{1+\delta_{K0}}{1+\delta_{K'0}}}(I2K2|I'K') + \sqrt{\frac{1+\delta_{K'0}}{1+\delta_{K0}}}(I2K-2|I'K')\right]I_2\right\}\frac{2}{\lambda[\pi^2/(9b^2_{\gamma_0}), p+3/2]b^{p+p'+3}_{\gamma_0}}, \quad (10)$$

where $\lambda(x, a)$–incomplete gamma function; $(I2K0|I'K')$–Clebsh-Gordon coefficients, integrals I_1 and I_2 are expressed by

$$I_1 = \int_0^{\pi/3}\gamma^{p+p'+2}\exp\left(-\frac{\gamma^2}{b^2_{\gamma_0}}\right)\cos\gamma d\gamma, \quad (11)$$

$$I_2 = \int_0^{\pi/3}\gamma^{p+p'+2}\exp\left(-\frac{\gamma^2}{b^2_{\gamma_0}}\right)\sin\gamma d\gamma. \quad (12)$$

The reduced E2-transition probabilities for: $n_\beta = 0$, $n'_\beta = 0$, $n_\gamma = 0$, $n'_\gamma = 0$:

$$S_{if\beta} = \mu\sqrt{\frac{1}{\Gamma(q+\frac{7}{2})\Gamma(q'+\frac{7}{2})}}\Gamma\left(\frac{q+q'+7}{2}\right), \quad (13)$$

for E2-transition probabilities: $n_\beta = 1$, $n'_\beta = 1$, $n_\gamma = 0$, $n'_\gamma = 0$.

$$S_{if\beta} = \frac{\mu}{4}\sqrt{\frac{1}{(q+\frac{15}{2})(q'+\frac{15}{2})\Gamma(q+\frac{7}{2})\Gamma(q'+\frac{7}{2})}}\Gamma\Big(\frac{q+q'+8}{2}\Big) \times [(q+q'+10)(q+q'+8) - 2(q+q'+3)(q+q'+8)+(2q+3)(2q'+3)], \tag{14}$$

for E2-transition probabilities: $n_\beta = 1$, $n'_\beta = 0$, $n_\gamma = 0$, $n'_\gamma = 0$.

$$S_{if\beta} = \frac{\mu}{2}\sqrt{\frac{1}{(q+\frac{15}{2})\Gamma(q+\frac{7}{2})\Gamma(q'+\frac{7}{2})}}\Gamma\left(\frac{q+q'+8}{2}\right)(q-q'-5), \tag{15}$$

here $\Gamma(x)$ – Gamma function.

Such partial considerations have been done in [9], where reduced electric quadrupole transition probabilities between various rotational states of even nuclei divided into three types. The formulas (13-15) were obtained of calculation of the reduced E2↓-transition probabilities. The consept of detail balance [7] was used for calculation E2↑-transition probabilities. Note, the fitting parameters in [5] (the values of the papameters μ and γ_0) are used for the description reduced E2- transition probabilities. Since this work is a continuation of Ref. [5].

Note that in [5] the case with n_γ=1, n_β=1, τ=1 was not considered, which corresponds to a complex movement of γ- and β-vibrational-rotational nature and a contribution of parameter μ_γ to values of energy levels is missing [5]. But in formulas (13,14,15) for reduced E2-transition probabilities the variable p expressed by parameter μ_γ (see section 2).

4 Results and discussion

In Table 1 theoretical and experimental [6] values of the intra-/inter-band reduced E2-transition probabilities (as well as their ratios) of the excited collective states of nuclei ^{154}Sm, ^{156}Gd, ^{158}Dy, ^{162,164}Er, ^{230,232}Th, ^{232,234,236}U are presented and a good agreement with experimental data [6] are obtained (where, for convenience are suggested: $\tilde{Q}_0$=$Q_0^2/(16\pi^2)$, $\tilde{Q}_2$=$Q_2^2/(16\pi^2)$.

The ratios of the reduced transition probabilities are determined from relative intensities of E2-transitions [1,2]. In the last column of Table 1 the ratios reduced E2-transition probabilities are compared with the Alaga rules [10, 12]. These comparisons allow to determine a sensitivity of E2-transition probabilities to the presence of non-axial quadrupole deformations, because wave functions (1) are done by functions of deformations variables, which connect the shape of nucleus and an intrinsic quadrupole moments [11]. Alaga rules give a good agreement between experimental and theoretical values of the ratios of reduced E2- transition probabilities for ground state and γ-bands, but for β-bands, the agreement is poore.

Table 1. Comparison intra-/inter-band reduced E2-transitions probabilities between energy levels in ground, β and γ-bands with experimental data [6] and Alaga rules [10]

Nucleus	E2-transitions	Exp. [6]	Theory	[10]
^{154}Sm	$B(002^{+}1 \to 000^{+}1)$	0.843(21)	0.8312	
$\tilde{Q}_0$=3.5971	$B(004^{+}1 \to 002^{+}1)$	1.186(39)	1.206	
$\tilde{Q}_2$=9.8775	$B(006^{+}1 \to 004^{+}1)$	1.374(47)	1.3658	
μ_β=0.2702	$B(008^{+}1 \to 006^{+}1)$	1.49(15)	1.4877	
μ_γ=1.8921	$B(0010^{+}1 \to 008^{+}1)$	1.60(12)	1.6069	
γ=8.9^0	$\frac{B(004^{+}1 \to 002^{+}1)}{B(002^{+}1 \to 000^{+}1)}$	1.29(5)	1.4509	1.4286
	$\frac{B(006^{+}1 \to 004^{+}1)}{B(004^{+}1 \to 002^{+}1)}$	1.16(4)	1.1325	1.1014
	$\frac{B(008^{+}1 \to 006^{+}1)}{B(006^{+}1 \to 004^{+}1)}$	1.08(4)	1.0892	1.0468
	$\frac{B(0010^{+}1 \to 008^{+}1)}{B(008^{+}1 \to 006^{+}1)}$	0.99(10)	1.0801	1.0271
	$\frac{B(012^{+}1 \to 002^{+}1)}{B(012^{+}1 \to 000^{+}1)}$	2.1(5)	1.4798	1.4286
	$\frac{B(014^{+}1 \to 004^{+}1)}{B(014^{+}1 \to 002^{+}1)}$	1.08(27)	0.9387	9.1
	$\frac{B(012^{+}1 \to 004^{+}1)}{B(012^{+}1 \to 002^{+}1)}$	2.07(25)	1.6226	1.8
	$\frac{B(002^{+}2 \to 002^{+}1)}{B(002^{+}2 \to 000^{+}1)}$	1.54(42)	1.4947	1.4286
	$\frac{B(004^{+}2 \to 004^{+}1)}{B(004^{+}2 \to 002^{+}1)}$	18.1(19)	3.2711	2.95
	$\frac{B(002^{+}2 \to 004^{+}1)}{B(002^{+}2 \to 002^{+}1)}$	0.04(1)	0.0552	0.05
	$\frac{B(003^{+}1 \to 004^{+}1)}{B(003^{+}1 \to 002^{+}1)}$	0.4(1)	1.0188	0.4
^{158}Dy	$\frac{B(004^{+}1 \to 002^{+}1)}{B(002^{+}1 \to 000^{+}1)}$	1.35(13)	1.5151	1.4286
$\tilde{Q}_0$=3.5971	$\frac{B(006^{+}1 \to 004^{+}1)}{B(004^{+}1 \to 002^{+}1)}$	0.99(6)	1.2169	1.1014
$\tilde{Q}_2$=9.8775	$\frac{B(008^{+}1 \to 006^{+}1)}{B(006^{+}1 \to 004^{+}1)}$	0.94(6)	1.1896	1.0468
μ_β=0.2899	$\frac{B(0010^{+}1 \to 008^{+}1)}{B(008^{+}1 \to 006^{+}1)}$	0.97(7)	1.1776	1.0271

Table 1. Continued

Nucleus	E2-transitions	Exp. [6]	Theory	[10]
μ_γ=1.8921	$\frac{B(0012^+1\to0010^+1)}{B(0010^+1\to008^+1)}$	0.88(7)	1.1541	1.0177
γ=12.1^0	$\frac{B(0014^+1\to0012^+1)}{B(0012^+1\to0010^+1)}$	0.85(6)	1.1219	1.0125
	$\frac{B(0016^+1\to0014^+1)}{B(0014^+1\to0012^+1)}$	0.68(9)	1.0906	1.0093
	$\frac{B(0018^+1\to0016^+1)}{B(0016^+1\to0014^+1)}$	0.67(11)	1.0659	1.0072
	$\frac{B(0020^+1\to0018^+1)}{B(0018^+1\to0016^+1)}$	0.58(19)	1.0485	1.0057
	$\frac{B(012^+1\to002^+1)}{B(012^+1\to000^+1)}$	2.7(3)	1.3738	1.4286
	$\frac{B(014^+1\to004^+1)}{B(014^+1\to002^+1)}$	0.91(18)	0.6703	2.95
	$\frac{B(012^+1\to004^+1)}{B(012^+1\to002^+1)}$	>4.6	1.7987	1.8
	$\frac{B(014^+1\to006^+1)}{B(014^+1\to004^+1)}$	9.6(23)	2.4072	1.75
	$\frac{B(002^+2\to002^+1)}{B(002^+2\to000^+1)}$	1.7(6)	1.5373	1.4286
	$\frac{B(004^+2\to004^+1)}{B(004^+2\to002^+1)}$	4.79(100)	3.4795	2.95
	$\frac{B(002^+2\to004^+1)}{B(002^+2\to002^+1)}$	<0.1	0.0583	0.05
	$\frac{B(003^+1\to004^+1)}{B(003^+1\to002^+1)}$	0.7(3)	0.7947	0.4
	$\frac{B(005^+1\to006^+1)}{B(005^+1\to004^+1)}$	1.3(3)	0.8994	0.5714
^{156}Gd	$B(002^+1\to000^+1)$	—	0.8729	
$\tilde{Q}_0$=3.5971	$B(004^+1\to002^+1)$	1.299(52)	1.2841	
$\tilde{Q}_2$=9.8775	$B(006^+1\to004^+1)$	1.64(14)	1.4894	
μ_β=0.2823	$B(008^+1\to006^+1)$	1.57(15)	1.6745	
μ_γ=1.8921	$B(0010^+1\to008^+1)$	1.59(9)	1.8738	
γ=10.3^0	$B(002^+2\to000^+1$	0.0222(11)	1.0114	
	$\hookrightarrow 002^+1$	0.0355(19)	1.5292	
	$\hookrightarrow 004^+1$	0.0032(3)	0.0864	

Table 1. Continued

Nucleus	E2-transitions	Exp. [6]	Theory	[10]
	$B(003^{+}1 \to 002^{+}1)$	0.0364(17)	1.8109	
	$\hookrightarrow 004^{+}1)$	0.028(6)	1.4384	
	$B(004^{+}2 \to 002^{+}1)$	0.0078(9)	0.5595	
	$\hookrightarrow 004^{+}1)$	0.046(5)	1.8783	
	$B(005^{+}1 \to 004^{+}1)$	0.0295	1.5629	
	$\hookrightarrow 006^{+}1)$	0.041(4)	1.4018	
	$\frac{B(004^{+}1 \to 002^{+}1)}{B(002^{+}1 \to 000^{+}1)}$	1.412(56)	1.4711	1.4286
	$\frac{B(006^{+}1 \to 004^{+}1)}{B(004^{+}1 \to 002^{+}1)}$	1.261(108)	1.1599	1.1014
	$\frac{B(008^{+}1 \to 006^{+}1)}{B(006^{+}1 \to 004^{+}1)}$	1.06(11)	1.1243	1.0468
	$\frac{B(012^{+}1 \to 002^{+}1)}{B(010^{+}1 \to 002^{+}1)}$	0.32(9)	0.2902	2.8571
	$\frac{B(012^{+}1 \to 002^{+}1)}{B(012^{+}1 \to 000^{+}1)}$	2.7(7)	1.4508	1.4286
	$\frac{B(012^{+}1 \to 004^{+}1)}{B(012^{+}1 \to 000^{+}1)}$	2.2(11)	2.4142	2.5714
	$\frac{B(014^{+}1 \to 004^{+}1)}{B(014^{+}1 \to 002^{+}1)}$	2.90(191)	0.8546	1.8
	$\frac{B(016^{+}1 \to 006^{+}1)}{B(016^{+}1 \to 004^{+}1)}$	1.2(8)	0.6162	0.8089
	$\frac{B(0110^{+}1 \to 0010^{+}1)}{B(0110^{+}1 \to 008^{+}1)}$	>1.7	0.2154	0.744
	$\frac{B(002^{+}2 \to 002^{+}1)}{B(002^{+}2 \to 000^{+}1)}$	1.61(15)	1.5119	1.4286
	$\frac{B(002^{+}2 \to 004^{+}1)}{B(002^{+}2 \to 000^{+}1)}$	0.104(5)	0.0855	0.0714
	$\frac{B(003^{+}1 \to 004^{+}1)}{B(003^{+}1 \to 002^{+}1)}$	0.67(26)	1.0212	0.4
	$\frac{B(004^{+}2 \to 004^{+}1)}{B(004^{+}2 \to 002^{+}1)}$	5(3)	3.3570	2.95
	$\frac{B(004^{+}2 \to 006^{+}1)}{B(004^{+}2 \to 004^{+}1)}$	0.15(3)	0.0686	0.0864
	$\frac{B(005^{+}1 \to 006^{+}1)}{B(005^{+}1 \to 004^{+}1)}$	1.4(2)	1.1531	0.5714

Table 1. Continued

Nucleus	E2-transitions	Exp. [6]	Theory	[10]
	$\frac{B(006^+2\to006^+1)}{B(006^+2\to004^+1)}$	5.9(14)	4.5291	3.7143
	$\frac{B(007^+1\to008^+1)}{B(007^+1\to006^+1)}$	2.0(12)	1.2376	0.6667
	$\frac{B(009^+1\to0010^+1)}{B(009^+1\to008^+1)}$	2.5(12)	1.2975	0.7273
^{162}Er	$\frac{B(004^+1\to002^+1)}{B(002^+1\to000^+1)}$	1.01(14)	1.4878	1.4286
$\tilde{Q}_0$=7.5971	$\frac{B(006^+1\to004^+1)}{B(004^+1\to002^+1)}$	0.91(7)	1.1808	1.1014
$\tilde{Q}_2$=11.8775	$\frac{B(008^+1\to006^+1)}{B(006^+1\to004^+1)}$	0.80(7)	1.448	1.0468
μ_β=0.2999	$\frac{B(0010^+1\to008^+1)}{B(008^+1\to006^+1)}$	0.65(6)	1.286	1.0271
μ_γ=1.8921	$\frac{B(0012^+1\to0010^+1)}{B(0010^+1\to008+1)}$	0.38(9)	1.1067	1.0177
γ=12.8^0	$\frac{B(0014^+1\to0012^+1)}{B(0012^+1\to0010^+1)}$	1.01(1)	1.0811	1.0125
	$\frac{B(002^+2\to002^+1)}{B(002^+2\to000^+1)}$	2.35(13)	1.6372	1.4286
	$\frac{B(004^+2\to004^+1)}{B(004^+2\to002^+1)}$	12(2)	4.0548	0.4
	$\frac{B(006^+2\to006^+1)}{B(006^+2\to004^+1)}$	10.1(13)	6.0491	3.7143
	$\frac{B(002^+2\to004^+1)}{B(002^+2\to002^+1)}$	0.12(9)	0.066	1.4286
	$\frac{B(006^+2\to004^+2)}{B(006^+2\to004^+1)}$	330(55)	17.0117	2.4
	$\frac{B(003^+1\to004^+1)}{B(003^+1\to002^+1)}$	1.41(11)	1.2146	0.05
	$\frac{B(005^+1\to006^+1)}{B(005^+1\to004^+1)}$	1.75(18)	1.2358	0.5714
	$\frac{B(007^+1\to008^+1)}{B(007^+1\to006^+1)}$	1.09(25)	1.2811	0.6667
	$\frac{B(009^+1\to0010^+1)}{B(009^+1\to008^+1)}$	2.09(115)	1.3346	0.7273
	$\frac{B(007^+1\to005^+1)}{B(007^+1\to006^+1)}$	60(4)	1.789	0.8791
	$\frac{B(009^+1\to007^+1)}{B(009^+1\to008^+1)}$	39(4)	2.3824	1.0294

Table 1. Continued

Nucleus	E2-transitions	Exp. [6]	Theory	[10]
^{164}Er	$B(002^{+}1 \to 000^{+}1)$	–	0.7880	
$\tilde{Q}_0$=3.5971	$B(004^{+}1 \to 002^{+}1)$	1.38(14)	1.2164	
$\tilde{Q}_2$=9.8775	$B(006^{+}1 \to 004^{+}1)$	–	1.5228	
μ_β=0.2777	$B(008^{+}1 \to 006^{+}1)$	1.78(13)	1.8639	
μ_γ=1.8921	$B(0010^{+}1 \to 008+1)$	1.70(16)	2.2366	
γ=12.9^0	$B(0012^{+}1 \to 0010+1)$	1.89(19)	2.5951	
	$B(0014^{+}1 \to 0012^{+}1)$	–	2.8990	
	$B(0016^{+}1 \to 0014^{+}1)$	1.52(28)	3.1370	
	$B(0012^{+}2 \to 0010^{+}2)$	1.5(7)	0.2313	
	$B(0014^{+}2 \to 0012^{+}2)$	1.9(9)	0.2597	
	$\frac{B(004^{+}1 \to 002^{+}1)}{B(002^{+}1 \to 000^{+}1)}$	1.27(12)	1.5437	1.4286
	$\frac{B(006^{+}1 \to 004^{+}1)}{B(004^{+}1 \to 002^{+}1)}$	–	1.2519	1.1014
	$\frac{B(008^{+}1 \to 008^{+}1)}{B(006^{+}1 \to 004^{+}1)}$	–	1.2240	1.0468
	$\frac{B(0010^{+}1 \to 008^{+}1)}{B(008^{+}1 \to 006^{+}1)}$	1.13(14)	1.2	1.0271
	$\frac{B(0012^{+}1 \to 0010^{+}1)}{B(0010^{+}1 \to 008^{+}1)}$	1.11(10)	1.1603	1.0177
	$\frac{B(0014^{+}1 \to 0012^{+}1)}{B(0012^{+}1 \to 0010^{+}1)}$	1.23(17)	1.1171	1.0125
	$\frac{B(0016^{+}1 \to 0014^{+}1)}{B(0014^{+}1 \to 0012^{+}1)}$	0.652(120)	1.0821	1.0093
	$\frac{B(010^{+}1 \to 002^{+}2)}{B(010^{+}1 \to 002^{+}1)}$	2.(1)	0.9713	1
	$\frac{B(012^{+}1 \to 004^{+}1)}{B(012^{+}1 \to 002^{+}1)}$	0.70(17)	1.3720	0.4167
	$\frac{B(012^{+}1 \to 002^{+}1)}{B(012^{+}1 \to 000^{+}1)}$	2.3(5)	1.3120	1.4286
	$\frac{B(012^{+}1 \to 004^{+}1)}{B(012^{+}1 \to 002^{+}1)}$	0.70(17)	0.8775	1.8
	$\frac{B(012^{+}1 \to 002^{+}2)}{B(012^{+}1 \to 002^{+}1)}$	3.5(17)	0.3174	1.4286
	$\frac{B(014^{+}1 \to 004^{+}1)}{B(014^{+}1 \to 002^{+}1)}$	1.7	0.5538	9.1

Table 1. Continued

Nucleus	E2-transitions	Exp. [6]	Theory	[10]
	$\frac{B(002^+2 \to 002^+1)}{B(002^+2 \to 000^+1)}$	1.7(2)	1.5473	1.4286
	$\frac{B(002^+2 \to 004^+1)}{B(002^+2 \to 002^+1)}$	0.11(5)	0.0590	0.05
	$\frac{B(003^+1 \to 004^+1)}{B(003^+1 \to 002^+1)}$	0.89(7)	1.0175	0.4
	$\frac{B(004^+2 \to 004^+1)}{B(004^+2 \to 002^+1)}$	9.1(33)	3.5241	2.95
	$\frac{B(005^+1 \to 006^+1)}{B(005^+1 \to 004^+1)}$	1.3(3)	1.2927	0.5714
^{230}Th	$\frac{B(004^+1 \to 002^+1)}{B(002^+1 \to 000^+1)}$	1.40(14)	1.4767	1.4286
$\tilde{Q}_0$=3.5971	$\frac{B(012^+1 \to 002^+1)}{B(012^+1 \to 000^+1)}$	2.5(13)	1.444	1.4286
$\tilde{Q}_2$=9.8775	$\frac{B(012^+1 \to 004^+1)}{B(012^+1 \to 002^+1)}$	4.3(17)	1.672	1.8
μ_β=0.2873	$\frac{B(002^+2 \to 002^+1)}{B(002^+2 \to 000^+1)}$	1.75(15)	1.5168	1.4286
μ_γ=1.8921	$\frac{B(003^+1 \to 004^+1)}{B(003^+1 \to 002^+1)}$	0.56	1.0228	0.4
γ=10.6^0	$\frac{B(004^+2 \to 004^+1)}{B(004^+2 \to 002^+1)}$	8.1(12)	3.381	2.95
	$\frac{B(006^+2 \to 006^+1)}{B(006^+2 \to 004^+1)}$	>20	4.5737	3.7143
^{232}Th	$B(002^+1 \to 000^+1)$	–	.7378	
$\tilde{Q}_0$=3.5971	$B(004^+1 \to 002^+1)$	2.40	1.08	
$\tilde{Q}_2$=9.8775	$B(006^+1 \to 004^+1)$	2.64	1.2414	
μ_β=0.2577	$B(008^+1 \to 006^+1)$	2.77	1.3792	
μ_γ=1.8921	$B(0010^+1 \to 008^+1)$	2.81	1.5233	
γ=9.8^0	$\frac{B(004^+1 \to 002^+1)}{B(002^+1 \to 000^+1)}$	–	1.4636	1.4286
	$\frac{B(008^+1 \to 006^+1)}{B(006^+1 \to 004^+1)}$	0.98(8)	1.111	1.0468
	$\frac{B(0010^+1 \to 008^+1)}{B(008^+1 \to 006^+1)}$	1.03(6)	1.1045	1.0271
	$\frac{B(0012^+1 \to 0010^+1)}{B(0010^+1 \to 008^+1)}$	1.01(7)	1.1030	1.0177

Table 1. Continued

Nucleus	E2-transitions	Exp. [6]	Theory	[10]
	$\frac{B(0014^+1 \to 0012^+1)}{B(0012^+1 \to 0010^+1)}$	1.06(7)	1.0988	1.0125
	$\frac{B(0016^+1 \to 0014^+1)}{B(0014^+1 \to 0012^+1)}$	1.00(10)	1.0902	1.0093
	$\frac{B(0018^+1 \to 0016^+1)}{B(0016^+1 \to 0014^+1)}$	0.88(14)	1.0781	1.0072
	$\frac{B(012^+1 \to 002^+1)}{B(012^+1 \to 000^+1)}$	0.05(2)	1.4477	1.4286
	$\frac{B(002^+2 \to 002^+1)}{B(002^+1 \to 000^+1)}$	0.068(16)	1.5977	1.4286
	$\frac{B(002^+2 \to 002^+1)}{B(002^+2 \to 000^+1)}$	1.86(68)	1.4994	1.4286
	$\frac{B(002^+2 \to 004^+1)}{B(002^+2 \to 002^+1)}$	0.88(27)	0.0556	0.4167
^{232}U	$\frac{B(002^+2 \to 002^+1)}{B(002^+2 \to 000^+1)}$	1.4(4)	1.4962	1.4286
$\tilde{Q}_0$=3.5971	$\frac{B(002^+2 \to 004^+1)}{B(002^+2 \to 002^+1)}$	0.057(6)	0.0553	0.05
$\tilde{Q}_2$=9.8775	$\frac{B(002^+2 \to 012^+1)}{B(002^+2 \to 000^+1)}$	5.0(12)	1.234	1.4286
μ_β=0.2673	$\frac{B(002^+2 \to 012^+1)}{B(002^+2 \to 002^+1)}$	12.0(60)	1.2717	1
μ_γ=1.8921	$\frac{B(002^+2 \to 012^+1)}{B(002^+2 \to 010^+1)}$	3.12(79)	1.6937	1.4267
γ=9.2^0	$\frac{B(002^+2 \to 014^+1)}{B(002^+2 \to 004^+1)}$	59	12.8628	1
	$\frac{B(002^+2 \to 014^+1)}{B(002^+2 \to 012^+1)}$	0.15(5)	0.5599	0.05
	$\frac{B(003^+1 \to 004^+1)}{B(003^+1 \to 002^+1)}$	0.47(10)	1.0166	0.4
	$\frac{B(003^+1 \to 014^+1)}{B(003^+1 \to 004^+1)}$	40	6.7364	
	$\frac{B(004^+2 \to 004^+1)}{B(004^+2 \to 002^+1)}$	3.58(15)	3.279	2.95
^{234}U	$\frac{B(012^+1 \to 002^+2)}{B(012^+1 \to 000^+1)}$	2.2(17)	1.4825	1.4286
$\tilde{Q}_0$=7.1597	$\frac{B(012^+1 \to 004^+1)}{B(012^+1 \to 002^+1)}$	2.6(22)	1.2142	1.8

Table 1. Continued

Nucleus	E2-transitions	Exp. [6]	Theory	[10]
$\tilde{Q}_2$=3.8775	$\frac{B(002^+2 \to 002^+1)}{B(002^+2 \to 000^+1)}$	1.7(4)	1.5761	1.4286
μ_β=0.2232	$\frac{B(002^+2 \to 004^+1)}{B(002^+2 \to 002^+1)}$	0.47(11)	0.0618	0.05
μ_γ=2.8921	$\frac{B(003^+1 \to 004^+1)}{B(003^+1 \to 002^+1)}$	0.7(2)	0.7967	0.4
γ=8.3^0	$\frac{B(004^+2 \to 004^+1)}{B(004^+2 \to 002^+1)}$	3.1(3)	3.7291	2.95
	$\frac{B(004^+2 \to 006^+1)}{B(004^+2 \to 004^+1)}$	0.084(9)	0.1486	0.0864
	$\frac{B(005^+1 \to 006^+1)}{B(005^+1 \to 004^+1)}$	$2.0^{3}_{-1.4}$	0.9177	0.5714
	$\frac{B(006^+2 \to 006^+1)}{B(006^+2 \to 004^+1)}$	12(3)	5.4348	3.7143
	$\frac{B(007^+1 \to 008^+1)}{B(007^+1 \to 006^+1)}$	$1^{1.5}_{-0.7}$	1.0131	0.6667
^{236}U	$B(002^+1 \to 000^+1)$	2.16	2.1207	
$\tilde{Q}_0$=7.1597	$B(004^+1 \to 002^+1)$	3.03	3.0391	
$\tilde{Q}_2$=3.8775	$B(006^+1 \to 004^+1)$	3.28	3.3665	
μ_β=0.2323	$B(008^+1 \to 006^+1)$	3.42	3.5536	
μ_γ=2.8921	$B(0010^+1 \to 008^+1)$	3.11	3.6903	
γ=8.3^0	$B(0012^+1 \to 0010^+1)$	3.34	3.8065	
	$\frac{B(004^+1 \to 002^+1)}{B(002^+1 \to 000^+1)}$	1.27(16)	1.4331	0.0864
	$\frac{B(006^+1 \to 004^+1)}{B(004^+1 \to 002^+1)}$	1.08(700)	1.1077	0.5714
	$\frac{B(008^+1 \to 006^+1)}{B(006^+1 \to 004^+1)}$	1.04(9)	1.0556	3.7143
	$\frac{B(0010^+1 \to 008^+1)}{B(008^+1 \to 006^+1)}$	0.908(88)	1.0385	0.6667
	$\frac{B(0012^+1 \to 0010^+1)}{B(0010^+1 \to 008^+1)}$	$1.07^{+0.86}_{-0.31}$	1.0315	0.6667

Comparison of the calculation results with corresponding experimental data shows a very good agreement, including high angular momentum states.

E2-transitions are very sensitive to the specific structure of the wave functions and they provide an opportunity for detailed tests of nuclear wave functions. Therefore, the study of electromagnetic transition probabilities would make the model more sensitive to the presence of dynamical non-axial deformations. Such deformations were carry out in this work and the wave functions of Hamilton operator for even-even nuclei with free triaxiality are used with all dynamical variables of this operator. Therefore, wave function (2,3) determined in the solution of the Schrödinger equation [5] is important for analysis of E2-transition probabilities.

An accounting of the nucleus deformability, in addition to changing the location of the energy levels must substantially affect the reduced E2- transition probabilities of the excited states. In this work the intra-/inter-band reduced E2-transition probabilities of the excited collective states of heavy nuclei are considered in detail.

The obtained reduced E2-transition probabilities gives an important information about the nature of the collective excitations. The properties of the quadrupole vibrations are profoundly affected by the shell structure. In nuclei with many particles outside of closed shells and related to the instability phenomena manifested in the occurrence of nuclei with non spherical equilibrium shapes.

The present formalism is capable of providing model predictions reduced E2-transition probabilities between energy levels of different bands.

References

[1] Y. Toh *et al. Eur. Phys. J. A* **9** (2000) 353.
[2] M. Hasegawa, K. Kaneko and T. Mizusaki, *Phys. Rev. C* **71** (2005) 044301.
[3] Lu Guo, J. A. Maruhn and P.-G. Reinhard, *Phys. Rev. C* **76** (2007) 034317.
[4] G. Rosensteel, D.J. Rowe, *Ann. Phys.* (N.Y.) **104** (1977) 134.
[5] M. S. Nadirbekov and G. A. Yuldasheva, *IJMPE* **14** (2014) 1450034-1450049.
[6] http://www.nndc.bnl.gov/ensdf/.
[7] A. S. Davydov, *Excited States of Atomic Nuclei* (Atomizdat, Moscow, 1967).
[8] Sh. Sharipov, M. S. Nadirbekov, *Russian Acad. Sci. Bulletin* **59** (1995) 119-123.
[9] A. S. Davydov and V. S. Rostovsky, *Nucl. Phys.* **12** (1959) 58-68.
[10] G. Alaga *et al. Dan. Mat. Fys. Medd.* **29**(9) (1955).
[11] A. Bohr, *K. Dan. Vidensk. Selsk. Mat. Fys. Medd.* **26**(14) (1952).
[12] F. Becker *et al. Nucl. Phys. A* **770** (2006) 107.