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Emergence of SU(3) Symmetry in Heavy Deformed Nuclei

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Abstract. An approximate SU(3) symmetry appears in heavy deformed even-even nuclei. In each nuclear shell with $N \geq 3$, due to the spin orbit interaction, one set of orbitals has escaped to the lower shell and another has intruded from the upper shell. There is an one-to-one correspondence between the orbitals of the two sets, based on pairs of orbitals which have identical quantum numbers of orbital angular momentum, spin, and total angular momentum, but different size. Such relevant orbitals have Nilsson number differences $\Delta K[\Delta N \Delta n_z \Delta \Lambda] = 0[110]$. By omitting the intruder Nilsson orbital of highest total angular momentum and replacing the rest of the intruder orbitals by their relevant counterparts, an approximate SU(3) symmetry is reconstructed. The accuracy of this approximation is tested through calculations in the framework of the Nilsson model in the asymptotic limit of large deformations, taking into account the changes in the selection rules and in the avoided crossings caused by the opposite parity of the substitutes with respect to the substituted orbitals.

1 Introduction

The relation of SU(3) symmetry to nuclear deformation has been uncovered by J. P. Elliott [1, 2] in the sd shell nuclei, in which its microscopic origin has been demonstrated. The SU(3) also appears in the framework of the microscopic symplectic model [3], which can be seen as a generalization of the Elliott SU(3) scheme to more than one nuclear shells. Since then the SU(3) symmetry has been used in the framework of many phenomenological nuclear models, including the interacting boson model (IBM) [4], the fermion dynamical symmetry model (FDSM) [5], and the interacting vector boson model (IVBM) [6], for the description of heavy nuclei, in which the Elliott SU(3) symmetry is known to be broken by the strong spin-orbit interaction, making it necessary to abandon the LS coupling scheme in favor of the jj coupling scheme [7]. In addition, an approximate pseudo-SU(3) symmetry has been used in heavy nuclei, based on a relabelling of the normal parity orbitals only, using quantum numbers corresponding to the next lower major nuclear shell [8–11], being realized later that the pseudospin symmetry appears as a relativistic symmetry [12]. Furthermore, a quasi-SU(3) symmetry has been introduced [13, 14], based on the smallness of $\Delta j = 1$ matrix elements of the quadrupole operator in comparison to the $\Delta j = 2$ ones, leading to an approximate restoration of LS coupling in heavy nuclei.

On the other hand, the Nilsson model [15–17], despite its simplicity, being a harmonic oscillator with cylindrical symmetry supplied with a spin-orbit term and an angular momentum squared term, has been very successful in describing in detail many properties of heavy deformed nuclei. For large deformations, its wave functions reach an asymptotic limit, in which the number of oscillator quanta, N, the number of quanta along the cylindrical symmetry axis, n_z, and the projections of the orbital angular momentum, Λ, and of the spin, Σ, along the symmetry axis become good quantum numbers, remaining so even at intermediate deformation values [16]. As a consequence, Nilsson states for even nuclei are labelled by $K[Nn_z\Lambda]$, where $K = \Lambda + \Sigma$ is the projection of the total angular momentum along the symmetry axis.

As remarked by Ben Mottelson [18] on the occasion of the 50th anniversary of the Nilsson model, the asymptotic quantum numbers of the Nilsson model can be seen as a generalization of Elliott's SU(3), applicable to heavy deformed nuclei. Working along this line, we demonstrate in the present manuscript that a hidden approximate SU(3) symmetry of the Elliott type can be uncovered in heavy deformed nuclei. In order to achieve this, we take advantage of the largely overlapping $\Delta K[\Delta N \Delta n_z \Delta\Lambda] = 0[110]$ pairs, which have been found to play a key role in the development of nuclear deformation within a different context [19–21]. The steps taken are listed here, using a specific example.

1) The 50-82 nuclear shell consists of the 3s1/2, 2d3/2, 2d5/2, and 1g7/2 orbitals, which are the pieces of the full sdg shell remaining after the spin-orbit force lowering of the 1g9/2 orbitals down into the 28-50 nuclear shell. In addition, it contains the 1h11/2 orbitals, lowered into it from the pfh shell by the spin-orbit force.

2) The 1h11/2 orbital consists of the Nilsson orbitals 1/2[550], 3/2[541], 5/2[532], 7/2[523], 9/2[514], and 11/2[505]. As a first step in the approximation, in the 50-82 shell we omit the 11/2[505] orbital, i.e. the one with the highest total angular momentum, which, as one can see in the Nilsson diagrams [15, 16], lies at the very top of the 50-82 shell, thus its influence on the structure of the rest of the shell is expected to be minimal.

3) The 1g9/2 orbital consists of the Nilsson orbitals 1/2[440], 3/2[431], 5/2[422], 7/2[413], 9/2[404], which are 0[110] partners of the remaining 1h11/2 Nilsson orbitals listed in 2), in the same order. A pair of 0[110] partners shares exactly the same values of the orbital angular momentum, spin, and total angular momentum quantum numbers, i.e. it is expected to exhibit identical behavior as far as angular momentum related properties are concerned. This has been corroborated by calculating overlaps of orbitals in Ref. [20]. One can then think of replacing in the 50-82 shell the remaining 1h11/2 orbitals by their 1g9/2 coun-

terparts and checking numerically the accuracy of this approximation, taking carefully into account that during this replacement the N and n_z quantum numbers have been changed by one unit each, while the parity has changed sign. These changes will obviously affect the selection rules of various relevant matrix elements, as well as the avoided crossings [22] in the Nilsson diagrams, as we shall discuss in more detail below.

4) After these two approximations have been made, we are left with a collection of orbitals which is exactly the one of the full sdg shell. The sdg shell of the spherical harmonic oscillator is known to possess the U(15) symmetry, having an SU(3) subalgebra [23], therefore we can expect that some of the SU(3) features would appear within the approximate scheme. Of course one should bear in mind that in axially symmetric deformed nuclei the relevant symmetry is not spherical, but cylindrical [24]. Therefore the relevant algebras are not U(N) Lie algebras, but more complicated versions of deformed algebras, in which, among the angular momentum operators, only the L_z operator has the same physical content as the L_z operator in the Nilsson model [25–30]. Such deformed oscillators with commensurate ratios of frequencies have been studied in relation to the effect of superdeformation [31, 32], the ratio of frequencies 2:1 corresponding to deformation parameter $\epsilon = 0.6$ in the framework of the Nilsson model.

5) The same procedure can be applied to the 28-50, 82-126, 126-184 shells, leading to approximate pf, pfh, sdgi shells, corresponding to U(10), U(21), U(28) algebras having SU(3) subalgebras (see [23] and references therein).

6) Concerning level crossings, it should be remembered that orbits with different quantum numbers and/or parity do not interact, while in the case of identical angular momentum and parity, avoided crossings [22] appear, if the size of the relevant interaction matrix elements is small, as it is the case in the approximate scheme considered here, as we shall show in detail below. As a result, one major source of changes comes from the fact that in a nuclear shell the intruder levels do not interact with the normal parity levels, while the substitutes of the intruder levels will interact with the normal parity levels, if they have the same angular momentum. We shall show in the next section that the relevant additional non-vanishing matrix elements appearing in the latter case are few and small, thus influencing the Nilsson level schemes very little, without affecting their major features.

It should be noted that although the discussion above regards the description of even-even nuclei in the framework of Lie algebras, it has been found [33] that for odd nuclei a correspondence exists between the Nilsson scheme and the level scheme of odd nuclei described in the framework of the SU(3) symmetry limit of the U(6/12) symmetry of the interacting boson fermion model [34].

2 The Nilsson Hamiltonian for large deformations

The Nilsson single particle Hamiltonian [15, 16] reads

$$H = H_{osc} + v_{ls}\hbar\omega_0(\mathbf{l}\cdot\mathbf{s}) + v_{ll}\hbar\omega_0(\mathbf{l}^2 - \langle \mathbf{l}^2\rangle_N), \quad (1)$$

where

$$H_{osc} = \frac{\mathbf{p}^2}{2M} + \frac{1}{2}M(\omega_z^2 z^2 + \omega_\perp(x^2 + y^2)) \tag{2}$$

is the Hamiltonian of a harmonic oscillator with cylindrical symmetry,

$$\langle \mathbf{l}^2 \rangle_N = \frac{1}{2}N(N+3) \tag{3}$$

is the average of the square of the angular momentum $\mathbf{l}$ within the Nth oscillator shell, M is the nuclear mass, $\mathbf{s}$ is the spin, $\mathbf{p}$ is the momentum, while the rotational frequencies ω_z and $\omega_\perp$ are related to the deformation parameter ϵ by

$$\omega_z = \omega_0 \left(1 - \frac{2}{3}\epsilon\right), \qquad \omega_\perp = \omega_0 \left(1 + \frac{1}{3}\epsilon\right), \tag{4}$$

leading to

$$\epsilon = \frac{\omega_\perp - \omega_z}{\omega_0}, \tag{5}$$

with $\epsilon > 0$ corresponding to prolate shapes and $\epsilon < 0$ corresponding to oblate shapes The standard values of the constants v_{ls} and v_{ll}, determined from the available data on intrinsic nuclear spectra are reported, for example, in [35].

For large deformations, the asymptotic wave functions $|Nn_z\Lambda\Sigma\rangle$ are used [15, 16], where N is the total number of oscillator quanta, n_z is the number of the oscillator quanta along the z-axis, Λ is the z-projection of the orbital angular momentum, and Σ is the z-projection of the spin. Nilsson orbitals in even-even nuclei are then denoted by $K[Nn_z\Lambda]$, where K is the projection of the total angular momentum on the z-axis, with $K = \Lambda + \Sigma$. The eigenvalues of H_{osc} in this basis are

$$E_{osc} = \hbar\omega_0 \left(N + \frac{3}{2} - \frac{1}{3}\epsilon(3n_z - N)\right). \tag{6}$$

Taking advantage of the cylindrical symmetry, one can define creation and annihilation operators [16, 36]

$$\begin{aligned} R^+ &= \frac{1}{\sqrt{2}}(a_x^\dagger + ia_y^\dagger), \qquad & R &= \frac{1}{\sqrt{2}}(a_x - ia_y), \\ S^+ &= \frac{1}{\sqrt{2}}(a_x^\dagger - ia_y^\dagger), \qquad & S &= \frac{1}{\sqrt{2}}(a_x + ia_y), \end{aligned} \tag{7}$$

satisfying the commutation relations

$$[R, R^\dagger] = [S, S^\dagger] = 1, \tag{8}$$

thus going over to a $|n_z rs\Sigma\rangle$ basis, where r is the number of quanta related to the harmonic oscillator formed by $R^\dagger$ and R, and s is the number of quanta related to the harmonic oscillator formed by $S^\dagger$ and S, for which

$$n_\perp = r + s = N - n_z, \qquad \Lambda = r - s \tag{9}$$

hold, where $n_\perp$ is the number of quanta perpendicular to the z-axis. It is then a straightforward task [16] to calculate the matrix elements of the $\mathbf{l}\cdot\mathbf{s}$ and $\mathbf{l}^2$ operators in the new basis. Explicit results are given in Ref. [16] in a $|n_z n_\perp \Lambda\Sigma\rangle$ notation. Additional results for matrix elements in various notations can be found in Refs. [37–39].

The correspondence between states in the $|n_z r s \Sigma\rangle$ basis and the standard Nilsson orbitals $K[Nn_z\Lambda]$ can be easily obtained using Eq. (9) and $K = \Lambda + \Sigma$. As an example, the final results for the matrix elements of $\mathbf{l}\cdot\mathbf{s}$ for the 28-50 and pf shells are given in Tables 1 and 2, in which the 1g9/2 levels appearing in Table 1 have been replaced by the 1f7/2 levels in Table 2. It should be remembered that these matrix elements, as well as the matrix elements of $\mathbf{l}^2$, are independent from the deformation, since the effects of the deformation on them are neglected [35], due to the fact that the $\mathbf{l}\cdot\mathbf{s}$ and $\mathbf{l}^2$ terms are already relatively small perturbations

Table 1. Matrix elements $l \cdot s$ for Nilsson orbitals in the 28–50 shell

	$\frac{1}{2}$[301]	$\frac{1}{2}$[321]	$\frac{3}{2}$[312]	$\frac{1}{2}$[310]	$\frac{3}{2}$[301]	$\frac{5}{2}$[303]	$\frac{1}{2}$[440]	$\frac{3}{2}$[431]	$\frac{5}{2}$[422]	$\frac{7}{2}$[413]	$\frac{9}{2}$[404]
1/2[301]	-0.5	0	0	-1	0		0	0	0	0	0
1/2[321]	0	-0.5	0	1	0	0	0	0	0	0	0
3/2[312]	0	0	-1	0	0.707	0	0	0	0	0	0
1/2[310]	-1	1	0	0	0	0	0	0	0	0	0
3/2[301]	0	0	0.707	0	0.5	0	0	0	0	0	0
5/2[303]	0	0	0	0	0	-1.5	0	0	0	0	0
1/2[440]	0	0	0	0	0	0	0	0	0	0	0
3/2[431]	0	0	0	0	0	0	0	0.5	0	0	0
5/2[422]	0	0	0	0	0	0	0	0	1	0	0
7/2[413]	0	0	0	0	0	0	0	0	0	1.5	0
9/2[404]	0	0	0	0	0	0	0	0	0	0	2

Table 2. $l \cdot s$ matrix elements for Nilsson orbitals in the pf shell.

	$\frac{1}{2}$[301]	$\frac{1}{2}$[321]	$\frac{3}{2}$[312]	$\frac{1}{2}$[310]	$\frac{3}{2}$[301]	$\frac{5}{2}$[303]	$\frac{1}{2}$[330]	$\frac{3}{2}$[321]	$\frac{5}{2}$[312]	$\frac{7}{2}$[303]
1/2[301]	-0.5	0	0	-1	0		0	0	0	0
1/2[321]	0	-0.5	0	1	0	0	-1.225	0	0	0
3/2[312]	0	0	-1	0	0.707	0	0	-1.414	0	0
1/2[310]	-1	1	0	0	0	0	0	0	0	0
3/2[301]	0	0	0.707	0	0.5	0	0	0	0	0
5/2[303]	0	0	0	0	0	-1.5	0	0	-1.225	0
1/2[330]	0	-1.225	0	0	0	0	0	0	0	0
3/2[321]	0	0	-1.414	0	0	0	0	0.5	0	0
5/2[312]	0	0	0	0	0	-1.225	0	0	1	0
7/2[303]	0	0	0	0	0	0	0	0	0	1.5

of the oscillator potential. Results for higher shells will be published elsewhere [40].

The calculation of the energy eigenvalues of the full Hamiltonian becomes then a simple task of diagonalization of a matrix in which the diagonal matrix elements depend on the deformation, as given in Eq. (6), while the non-diagonal matrix elements remain invariant. Concerning the substitutes of the intruder 1g9/2 parity orbitals, coming from the 1f7/2 shell, in order to be brought to the place of the orbitals which they are going to substitute, they are uniformly pushed up by $1 - 2\epsilon/3$, as implied by Eq. (6), since both N and n_z have to be increased by one unit. As an example, numerical results for $\epsilon = 0.3$ are given for the 28-50 and pf shells in Tables 3 and 4. Results for higher shells and varying ϵ will be published elsewhere [40]. Since the results have been obtained by using the asymptotic wave functions, they are expected to be reliable for large

Table 3. H matrix elements for $\epsilon = 0.3$ and $u_{ls} = -0.16$ [35] for Nilsson orbitals in the 28–50 shell.

	$\frac{1}{2}$[301]	$\frac{1}{2}$[321]	$\frac{3}{2}$[312]	$\frac{1}{2}$[310]	$\frac{3}{2}$[301]	$\frac{5}{2}$[303]	$\frac{1}{2}$[440]	$\frac{3}{2}$[431]	$\frac{5}{2}$[422]	$\frac{7}{2}$[413]	$\frac{9}{2}$[404]
1/2[301]	4.88	0	0	0.16	0		0	0	0	0	0
1/2[321]	0	4.28	0	−0.16	0	0	0	0	0	0	0
3/2[312]	0	0	4.66	0	−0.113	0	0	0	0	0	0
1/2[310]	0.16	−0.16	0	4.50	0	0	0	0	0	0	0
3/2[301]	0	0	−0.113	0	4.72	0	0	0	0	0	0
5/2[303]	0	0	0	0	0	5.04	0	0	0	0	0
1/2[440]	0	0	0	0	0	0	4.70	0	0	0	0
3/2[431]	0	0	0	0	0	0	0	4.92	0	0	0
5/2[422]	0	0	0	0	0	0	0	0	5.14	0	0
7/2[413]	0	0	0	0	0	0	0	0	0	5.36	0
9/2[404]	0	0	0	0	0	0	0	0	0	0	5.58

Table 4. H matrix elements for $\epsilon = 0.3$ and $u_{ls} = -0.16$ [35] for Nilsson orbitals in the pf shell.

	$\frac{1}{2}$[301]	$\frac{1}{2}$[321]	$\frac{3}{2}$[312]	$\frac{1}{2}$[310]	$\frac{3}{2}$[301]	$\frac{5}{2}$[303]	$\frac{1}{2}$[330]	$\frac{3}{2}$[321]	$\frac{5}{2}$[312]	$\frac{7}{2}$[303]
1/2[301]	4.88	0	0	0.16	0		0	0	0	0
1/2[321]	0	4.28	0	−0.16	0	0	0.196	0	0	0
3/2[312]	0	0	4.66	0	−0.113	0	0	0.226	0	0
1/2[310]	0.16	−0.16	0	4.50	0	0	0	0	0	0
3/2[301]	0	0	−0.113	0	4.72	0	0	0	0	0
5/2[303]	0	0	0	0	0	5.04	0	0	0.196	0
1/2[330]	0	0.196	0	0	0	0	4.70	0	0	0
3/2[321]	0	0	0.226	0	0	0	0	4.92	0	0
5/2[312]	0	0	0	0	0	0.196	0	0	5.14	0
7/2[303]	0	0	0	0	0	0	0	0	0	5.36

and moderate deformations [16], but they are expected to fail completely for $\epsilon \leq 0.1$, where different approximate wave functions, providing different slopes of the energy levels as a function of ϵ, are appropriate [16, 35].

3 Discussion

3.1 The $\mathbf{l} \cdot \mathbf{s}$ and $\mathbf{l}^2$ matrix elements

In Table 1 the matrix elements of the spin-orbit term in the 28–50 shell are shown, in order to be compared to the relevant matrix elements appearing in the full pf shell, seen in Table 2, occurring after replacing the 1g9/2 levels of the 28–50 shell (the last 5 levels in the rows and columns of Table 1) by their 0[110] counterparts of the 1f7/2 levels (the last 4 levels in the rows and columns of Table 2). Each table is divided into four blocks by straight lines. The following comments apply.

1) Table 1 has one more column (the last one) and one more row (the last one) than Table 2, since the 9/2[404] level of the 28-50 shell has no 0[110] counterpart in the pf shell.

2) The upper left blocks of the two tables are obviously identical, since they refer to the same set of states.

3) The lower right blocks of the two tables are identical, since the 0[110] pairs possess the same orbital angular momentum and spin quantum numbers, taking also into account that the last level of 1g9/2, 9/2[404], has no counterpart in 1f7/2.

4) The lower left block and the upper right block in Table 1 are "empty", since all matrix elements vanish (because they connect states with different parity, while the $\mathbf{l} \cdot \mathbf{s}$ interaction is parity invariant), while in Table 2 a few non-vanishing matrix elements (3 out of 24 in each block) appear. These non-vanishing matrix elements represent the "damage" made by the approximation imposed.

The same comments apply to the spin-orbit matrix elements appearing in higher shells, to be shown elsewhere [40]. One can see that the percentage of matrix elements "damaged" by the approximation drops with increasing shell size.

Qualitatively similar results are obtained in the case of the matrix elements of the $\mathbf{l}^2$ operator, to be shown elsewhere [40], since no $\mathbf{l}^2$ term is used in the 28–50 and pf shells.

3.2 Matrix elements of the full Hamiltonian

In order to get a feeling of the number and magnitude of "damaged" matrix elements of the full Hamiltonian, we present results for the special case of $\epsilon = 0.3$ In the pf shell, shown in Table 4, six out of 100 matrix elements are "damaged", in comparison to the 28–50 shell, shown in Table 3.

Results for higher shells and for varying ϵ will be shown elsewhere [40]. In all cases the numerical values of the diagonal matrix elements are at least one order of magnitude larger than the numerical values of the non-diagonal matrix elements.

In conclusion, it can be shown that in all shells the changes inflicted on the Nilsson diagrams by the replacement of the intruder parity orbitals with their 0[110] counterparts do not change the main features of the diagrams (relevant figures will be shown elsewhere [40]). It can therefore be expected that several physical properties of the relevant heavy deformed nuclei could be correctly determined in the framework of the uncovered approximate SU(3) symmetry.

3.3 Comparison with the pseudo-SU(3) scheme

A detailed comparison between the present approximate SU(3) symmetry and the pseudo-SU(3) scheme can be carried out by considering in detail Table 5. Several comments apply.

1) In the pseudo-SU(3) scheme the intruder parity orbitals remain intact, while in our scheme the normal parity orbitals remain intact.

2) In the pseudo-SU(3) scheme all normal parity orbitals are replaced by pseudo-SU(3) counterparts. In our scheme, all but one (the one with the highest total angular momentum) of the intruder parity orbitals are replaced by 0[110] counterparts.

3) During the pseudo-SU(3) replacement, N is reduced by one unit, n_z remains intact, and Λ is also changed by one unit, either increasing or decreasing, but in both cases resulting in the inversion of the spin, i.e. in the change of the sign of Σ. In our scheme, N and n_z are reduced by one unit, but the quantum numbers related to angular momenta, Λ and Σ, remain intact. The total angular momentum K remains intact in both cases. It is clear that in the present scheme all properties dependent on angular momenta will be unaffected, something which is not *a priori* guaranteed within the pseudo-SU(3) scheme.

4) The result of the replacement in the case of the pseudo-SU(3) scheme is that the normal parity levels are replaced by a new set of levels, forming a complete harmonic oscillator shell with N reduced by one unit, obeying the relevant harmonic oscillator symmetry, while all the intruder parity levels remain alone, without obeying any harmonic oscillator symmetry. In our scheme, the result of the replacement is that the normal parity levels and the substitutes of the intruder parity levels form together a harmonic oscillator shell with quantum number N, while only one two-particles orbital, the highest lying one, remains alone and is ignored.

5) As far as parity is concerned, in the pseudo-SU(3) scheme the normal parity levels are lowered by one unit of N, ending up with the $N-1$ oscillator quantum number, while the intruder parity orbitals preserve their $N+1$ quantum number, the net result being that the parity of the normal levels is inverted and that in the final set we have states belonging to two different values of the oscillator quantum number, $N-1$ and $N+1$, sharing the same parity. In con-

Table 5. Levels appearing in the various major shells in the framework of the Nilsson model [15,16] (labelled by the size of the shell), in the present approximate SU(3) scheme (labelled by “present”), and in the pseudo-SU(3) scheme [8, 9] (labelled by “pseudo”). The orbitals been approximated in the last two cases are indicated by boldface.

28-50	present	pseudo	82-126	present	pseudo	126-184	present	pseudo
1/2[301]	1/2[301]	**1/2[220]**	1/2[501]	1/2[501]	**1/2[400]**	1/2[611]	1/2[611]	**1/2[510]**
1/2[321]	1/2[321]	**1/2[220]**	1/2[521]	1/2[521]	**1/2[420]**	1/2[600]	1/2[600]	**1/2[501]**
3/2[312]	3/2[312]	**3/2[211]**	3/2[512]	3/2[512]	**3/2[411]**	3/2[602]	3/2[602]	**3/2[501]**
1/2[310]	1/2[310]	**1/2[211]**	1/2[510]	1/2[510]	**1/2[411]**	1/2[631]	1/2[631]	**1/2[530]**
3/2[301]	3/2[301]	**3/2[202]**	3/2[501]	3/2[501]	**3/2[402]**	3/2[622]	3/2[622]	**3/2[521]**
5/2[303]	5/2[303]	**5/2[202]**	5/2[503]	5/2[503]	**5/2[402]**	5/2[613]	5/2[613]	**5/2[512]**
1/2[440]	**1/2[330]**	1/2[440]	1/2[541]	1/2[541]	**1/2[440]**	1/2[620]	1/2[620]	**1/2[521]**
3/2[431]	**3/2[321]**	3/2[431]	3/2[532]	3/2[532]	**3/2[431]**	3/2[611]	3/2[611]	**3/2[512]**
5/2[422]	**5/2[312]**	5/2[422]	5/2[523]	5/2[523]	**5/2[422]**	5/2[602]	5/2[602]	**5/2[503]**
7/2[413]	**7/2[303]**	7/2[413]	7/2[514]	7/2[514]	**7/2[413]**	7/2[604]	7/2[604]	**7/2[503]**
9/2[404]		9/2[404]	1/2[530]	1/2[530]	**1/2[431]**	1/2[651]	1/2[651]	**1/2[550]**
50-82	present	pseudo	3/2[521]	3/2[521]	**3/2[422]**	3/2[642]	3/2[642]	**3/2[541]**
1/2[400]	1/2[400]	**1/2[301]**	5/2[512]	5/2[512]	**5/2[413]**	5/2[633]	5/2[633]	**5/2[532]**
1/2[411]	1/2[411]	**1/2[310]**	7/2[503]	7/2[503]	**7/2[404]**	7/2[624]	7/2[624]	**7/2[523]**
3/2[402]	3/2[402]	**3/2[301]**	9/2[505]	9/2[505]	**9/2[404]**	9/2[615]	9/2[615]	**9/2[514]**
1/2[420]	1/2[420]	**1/2[321]**	1/2[660]	**1/2[550]**	1/2[660]	1/2[640]	1/2[640]	**1/2[541]**
3/2[411]	3/2[411]	**3/2[312]**	3/2[651]	**3/2[541]**	3/2[651]	3/2[631]	3/2[631]	**3/2[532]**
5/2[402]	5/2[402]	**5/2[303]**	5/2[642]	**5/2[532]**	5/2[642]	5/2[622]	5/2[622]	**5/2[523]**
1/2[431]	1/2[431]	**1/2[330]**	7/2[633]	**7/2[523]**	7/2[633]	7/2[613]	7/2[613]	**7/2[514]**
3/2[422]	3/2[422]	**3/2[321]**	9/2[624]	**9/2[514]**	9/2[624]	9/2[604]	9/2[604]	**9/2[505]**
5/2[413]	5/2[413]	**5/2[312]**	11/2[615]	**11/2[505]**	11/2[615]	11/2[606]	11/2[606]	**11/2[505]**
7/2[404]	7/2[404]	**7/2[303]**	13/2[606]		13/2[606]	1/2[770]	**1/2[660]**	1/2[770]
1/2[550]	**1/2[440]**	1/2[550]				3/2[761]	**3/2[651]**	3/2[761]
3/2[541]	**3/2[431]**	3/2[541]				5/2[752]	**5/2[642]**	5/2[752]
5/2[532]	**5/2[422]**	5/2[532]				7/2[743]	**7/2[633]**	7/2[743]
7/2[523]	**7/2[413]**	7/2[523]				9/2[734]	**9/2[624]**	9/2[734]
9/2[514]	**9/2[404]**	9/2[514]				11/2[725]	**11/2[615]**	11/2[725]
11/2[505]		11/2[505]				13/2[716]	**13/2[606]**	13/2[716]
						15/2[707]		15/2[707]

trast, in the present scheme the intruder parity levels are lowered from $N + 1$ to N, thus meeting the normal parity levels and forming a uniform set of levels characterized by a common value of N and sharing the same parity. Thus in matters concerning parity, one has to be equally careful within both approaches, since the parity of some of the levels is inverted in both cases.

6) In order to proceed, in the pseudo-SU(3) scheme one has to deal with the symmetry-obeying normal parity orbitals and the no-symmetry-obeying intruder parity orbitals, which have to be dealt with shell model methods, while in the present scheme one can proceed by taking into account only a set of symmetry-obeying orbitals.

7) An important difference is that the pseudo-SU(3) scheme is an approximation made to the levels of the Nilsson Hamiltonian at the energies at which they are predicted to be by the Hamiltonian, thus it represents an approximation scheme which can be used for simplified shell model calculations, while our scheme is the analog of a *Gedankenexperiment*: What would have happened if in the place of the intruder parity orbitals their 0[110] counterparts were appearing? From this consideration one can only draw conclusions regarding the extent in which the SU(3) features appear in the approximate shell, paving the way for calculations of various physical properties within the SU(3) symmetry, for example by appropriately modifying and simplifying the approach of Refs. [10,11] to heavy even-even nuclei. An example of application of the approximate SU(3) scheme will be briefly discussed in [41].

8) From Table 5 it is clear that the larger the shell, the smaller the percentage of orbitals approximated in the present scheme, while the opposite holds in the pseudo-SU(3) scheme, in which the larger the shell, the larger the percentage of orbitals being affected. This fact implies that the present scheme is expected to work best in the actinides, followed by the rare earths, while it is expected to be less satisfactory in lower shells.

4 Conclusions

In this manuscript we propose that an approximate SU(3) symmetry appears in heavy deformed nuclei, very similar to the Elliott SU(3) symmetry appearing in light (sd shell) nuclei. In order to demonstrate this fact, we use a simple and completely transparent Nilsson calculation, in which it becomes clear that the changes inflicted by replacing in each major shell the intruder parity orbitals by their 0[110] counterparts are small, therefore offering the basis for a reliable approximate scheme. The main reasons behind the success of this approximation are

1) The fact that the intruder parity orbitals have exactly the same orbital angular momentum, spin, and total angular momentum quantum numbers as their 0[110] substitutes.

2) The small number and small magnitude of the additional non-vanishing spin-orbit and angular-momentum-squared matrix elements appearing because of the approximation inflicted, which imply that the additional avoided crossings caused by the approximation are of small size, thus not affecting drastically the form of the Nilsson diagrams.

3)Because of 1) and 2), the real Nilsson diagrams have nearly the same structure as they would have had if the missing normal parity orbitals were present in the place of the intruder parity orbitals, completing an oscillator major shell with the appropriate U(N) symmetry algebra, having a SU(3) subalgebra.

The purpose of this approach is not to revise the Nilsson diagrams, but to justify the use of SU(3) in phenomenological collective models for the description of heavy deformed nuclei, a question remaining open for many years. The present approach implies that several properties of heavy deformed nuclei could be reliably described within the SU(3) symmetry.

The main open question is if this approximate SU(3) scheme can be of any practical use, in other words if the approximations made result in a SU(3) scheme from which reliable conclusions on physical quantities can be drawn. A first application is given in an accompanying manuscript [41], in which it is shown that the present scheme can predict the prolate over oblate dominance in deformed nuclei, as well as the location of the prolate-oblate shape phase transition in rare earth nuclei without using any free parameters.

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