

Prolate-Oblate Shape Transition in Neutron-Rich Heavy Rare Earths

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Abstract. A prolate to oblate shape transition is known to occur in neutron-rich rare earths at $N=116$, with ^{190}W , ^{192}Os , and ^{194}Pt identified as lying close to the transition point. We demonstrate that this transition is predicted within an approximate $SU(3)$ scheme for heavy deformed nuclei in a parameter-independent way.

1 Introduction

The existence of both prolate (cigar shaped) and oblate (pancake shaped) deformed nuclei, the possible transitions between the two shapes, as well as the experimentally observed dominance of prolate over oblate shapes in the ground state bands of even nuclei has been the focus of attention from many different viewpoints.

1) Microscopic calculations have evolved from early applications of the pairing plus quadrupole model in the prolate-oblate difference [1] and the prolate-oblate transition [2] to recent self-consistent Skyrme Hartree-Fock plus BCS calculations [3] and Hartree-Fock-Bogoliubov calculations [4–6] studying the structural evolution in neutron-rich Yb, Hf, W, Os, and Pt isotopes. reaching the conclusion that $N = 116$ nuclei in this region can be identified as the transition points between prolate and oblate shapes. In a related projected shell model study [7], a rotation-driven prolate-to-oblate shape phase transition has been found in ^{190}W .

2) The prolate-oblate shape phase transition has been considered [8–10] within the $O(6)$ symmetry of the interacting boson model [11]. In particular, the $O(6)$ symmetry has been considered [12, 13] as the critical point of the prolate to oblate shape phase transition within the interacting boson model.

3) An analytically solvable prolate to oblate shape phase transition has been found [14] within the $SU(3)$ limit of the interacting boson model. The collection of data of the chain of even nuclei (differing by two protons or two neutrons) ^{180}Hf , $^{182-186}\text{W}$, $^{188,190}\text{Os}$, $^{192-198}\text{Pt}$, considered in Ref. [14], suggests that the

transition occurs between ^{190}Os and ^{192}Pt , in agreement with their theoretical predictions.

4) From the experimental point of view, ^{192}Os [15] and ^{190}W [16] have been suggested as lying at the prolate-oblate border, with ^{194}Os [17] and ^{198}Os [18] having a clearly oblate character. The collection of data of the chain of even nuclei (differing by two protons or two neutrons) ^{180}Hf , $^{182-186}\text{W}$, $^{188-192}\text{Os}$, $^{194,196}\text{Pt}$, $^{198,200}\text{Hg}$, considered in Ref. [19], also suggests that the transition occurs between ^{192}Os and ^{194}Pt . Notice that all experimental information cited here is in agreement with the theoretical predictions of 1), since ^{190}W , ^{192}Os , and ^{194}Pt are $N = 116$ isotones.

5) The dominance of prolate over oblate nuclear shapes in the ground state bands of deformed even-even nuclei has been considered both in the framework of the Nilsson model [20], as well as by studying the effects of the spin-orbit potential within the framework of the Nilsson-Strutinsky method [21–23]. However, the almost complete dominance of prolate over oblate deformations in the ground states of deformed even-even nuclei is still considered as not adequately understood [24].

In the present work, we consider the prolate-oblate competition within the framework of the recently proposed hidden approximate $\text{SU}(3)$ symmetry in heavy nuclei, *without involving any free parameter*. Our main results are

1) The occurrence of the prolate-oblate transition at $N = 116$ comes out correctly in the W and Os chains of isotopes, while predictions are made for $Z < 74$ (i.e., below W).

2) The dominance of prolate over oblate deformation is obtained *without any free parameters*.

3) Predictions are made concerning the prolate-oblate transition in the region of the (yet unknown) neutron-deficient rare earths around $N = 72$.

2 The hidden/latent $\text{SU}(3)$ scheme

In the hidden approximate $\text{SU}(3)$ scheme [25], the protons of the 50-82 shell live in an approximate s_{dg} shell, having an approximate $\text{U}(15)$ symmetry, which is obtained by leaving out the (very high-lying) $11/2[505]$ orbital and replacing the rest of the $1h11/2$ subshell orbitals ($1/2[550]$, $3/2[541]$, $5/2[532]$, $7/2[523]$, $9/2[514]$) by their $0[110]$ counterparts [26–28] ($1/2[440]$, $3/2[431]$, $5/2[422]$, $7/2[413]$, $9/2[404]$), which form a $1g9/2$ subshell.

Similarly, in the same scheme [25], the neutrons of the 82-126 shell live in an approximate p_{fh} shell, having an approximate $\text{U}(21)$ symmetry, which is obtained by leaving out the (very high-lying) $13/2[606]$ orbital and replacing the rest of the $1i13/2$ subshell orbitals ($1/2[660]$, $3/2[651]$, $5/2[642]$, $7/2[633]$, $9/2[624]$, $11/2[615]$) by their $0[110]$ counterparts [26–28] ($1/2[550]$, $3/2[541]$, $5/2[532]$, $7/2[523]$, $9/2[514]$, $11/2[505]$), which form a $1h11/2$ subshell.

For the valence protons of each nucleus, the relevant $\text{SU}(3)$ irreducible representation (irrep) of the $\text{U}(15) \supset \text{SU}(3)$ decomposition, obtained by use of the

Table 1. Most leading SU(3) irreps [30, 31] contained in the U(15) and U(21) irreps for N protons or N neutrons, calculated using the code UNTOU3 [29].

N	1	2	3	4	5	6	7	8	9
irrep	[1]	[2]	[21]	[2 ²]	[2 ² 1]	[2 ³]	[2 ³ 1]	[2 ⁴]	[2 ⁴ 1]
U(15)	(4,0)	(8,0)	(10,1)	(12,2)	(15,1)	(18,0)	(18,2)	(18,4)	(19,4)
U(21)	(5,0)	(10,0)	(13,1)	(16,2)	(20,1)	(24,0)	(25,2)	(26,4)	(28,4)
N	10	11	12	13	14	15	16	17	18
irrep	[2 ⁵]	[2 ⁵ 1]	[2 ⁶]	[2 ⁶ 1]	[2 ⁷]	[2 ⁷ 1]	[2 ⁸]	[2 ⁸ 1]	[2 ⁹]
U(15)	(20,4)	(22,2)	(24,0)	(22,3)	(20,6)	(19,7)	(18,8)	(18,7)	(18,6)
U(21)	(30,4)	(33,2)	(36,0)	(35,3)	(34,6)	(34,7)	(34,8)	(35,7)	(36,6)
N	19	20	21	22	23	24	25	26	27
irrep	[2 ⁹ 1]	[2 ¹⁰]	[2 ¹⁰ 1]	[2 ¹¹]	[2 ¹¹ 1]	[2 ¹²]	[2 ¹² 1]	[2 ¹³]	[2 ¹³ 1]
U(15)	(19,3)	(20,0)	(16,4)	(12,8)	(9,10)	(6,12)	(4,12)	(2,12)	(1,10)
U(21)	(38,3)	(40,0)	(37,4)	(34,8)	(32,10)	(30,12)	(29,12)	(28,12)	(28,10)
N	28	29	30	31	32	33	34	35	36
irrep	[2 ¹⁴]	[2 ¹⁴ 1]	[2 ¹⁵]	[2 ¹⁵ 1]	[2 ¹⁶]	[2 ¹⁶ 1]	[2 ¹⁷]	[2 ¹⁷ 1]	[2 ¹⁸]
U(15)	(0,8)	(0,4)	(0,0)						
U(21)	(28,8)	(29,4)	(30,0)	(25,5)	(20,10)	(16,13)	(12,16)	(9,17)	(6,18)
N	37	38	39	40	41	42			
irrep	[2 ¹⁸ 1]	[2 ¹⁹]	[2 ¹⁹ 1]	[2 ²⁰]	[2 ²⁰ 1]	[2 ²¹]			
U(21)	(4,17)	(2,16)	(1,13)	(0,10)	(0,5)	(0,0)			

code UNTOU3 [29], can be seen in Table 1, using the standard Elliott notation (λ, μ) [30, 31], in which irreps with $\lambda > \mu$ correspond to prolate shapes, while irreps with $\lambda < \mu$ correspond to oblate shapes. In the same table, the relevant SU(3) irreducible representation (irrep) of the $U(21) \supset SU(3)$ decomposition, again obtained by use of the code UNTOU3 [29] corresponding to the valence neutrons of each nucleus, can be seen. By taking the sum of these two irreps for each nucleus, one can obtain the SU(3) irrep in which the ground state band (and possibly additional bands, according to the value of μ) is located. The results for the rare earths within the 50-82 proton shell and the 82-126 neutron shell are summarized in Table 2. Several comments are in place.

1) In the series of Os isotopes reported, one can see that starting from low N one finds prolate nuclei, the first oblate one being $^{192}\text{Os}_{116}$, being followed by additional oblate nuclei, in agreement with experimental evidence [12, 13, 15, 17, 18] and microscopic calculations [3–6].

2) In the series of W isotopes reported, one can again see that starting from low N one finds prolate nuclei, the first oblate one being $^{190}\text{W}_{116}$, in agreement with experimental evidence [16] and microscopic calculations [3–6]. No able experimental evidence exists yet beyond ^{190}W [32].

3) In the series of Hf isotopes reported, one can again see that starting from low N one finds prolate nuclei, but the experimentally known series [32] termi-

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Table 2. Most leading SU(3) irreps [30, 31] for nuclei with protons in the 50-82 shell and neutrons in the 82-126 shell. Boldface numbers indicate nuclei with $R_{4/2} = E(4_1^+)/E(2_1^+) \geq 2.8$, while * denotes nuclei with $2.8 > R_{4/2} \geq 2.5$, and ** labels a few nuclei with $R_{4/2}$ ratios slightly below 2.5, shown for comparison, while no irreps are shown for any other nuclei with $R_{4/2} < 2.5$. For the rest of the nuclei shown (using normal fonts and without any special signs attached) the $R_{4/2}$ ratios are still unknown [32].

		Ba	Ce	Nd	Sm	Gd	Dy
Z		56	58	60	62	64	66
N	val	6	8	10	12	14	16
88	6	(42,0)*	(42,4)*	(44,4)*			
90	8	(44,4)	(44,8)	(46,8)	(50,4)	(46,10)	(44,12)
92	10	(48,4)	(48,8)	(50,8)	(54,4)	(50,10)	(48,12)
94	12	(54,0)	(54,4)	(56,4)	(60,0)	(56,6)	(54,8)
96	14	(52,6)	(52,10)	(54,10)	(58,6)	(54,12)	(52,14)
98	16	(52,8)	(52,12)	(54,12)	(58,8)	(54,14)	(52,16)
100	18	(54,6)	(54,10)	(56,10)	(60,6)	(56,12)	(54,14)
102	20	(58,0)	(58,4)	(60,4)	(64,0)	(60,6)	(58,8)
104	22	(52,8)	(52,12)	(54,12)	(58,8)	(54,14)	(52,16)
106	24	(48,12)	(48,16)	(50,16)	(54,12)	(50,18)	(48,20)
108	26	(46,12)	(46,16)	(48,16)	(52,12)	(48,18)	(46,20)
110	28	(46,8)	(46,12)	(48,12)	(52,8)	(48,14)	(46,16)
112	30	(48,0)	(48,4)	(50,4)	(54,0)	(50,6)	(48,8)
114	32	(38,10)	(38,14)	(40,14)	(44,10)	(40,16)	(38,18)
116	34	(30,6)	(30,10)	(32,10)	(36,6)	(32,12)	(30,14)
118	36	(24,18)	(24,22)	(26,22)	(30,18)	(26,24)	(24,16)
120	38	(20,16)	(20,20)	(22,20)	(26,16)	(22,22)	(20,24)
		Er	Yb	Hf	W	Os	Pt
Z		68	70	72	74	76	78
N	val	18	20	22	24	26	28
88	6						
90	8	(44,10)*	(46,4)*	(38,12)*			
92	10	(48,10)	(50,4)	(42,12)*			
94	12	(54,6)	(56,0)	(48,8)	(42,12)	(38,12)*	
96	14	(52,12)	(54,6)	(46,14)	(40,18)	(36,18)*	
98	16	(52,14)	(54,8)	(46,16)	(40,20)	(36,20)*	
100	18	(54,12)	(56,6)	(48,14)	(42,18)	(38,18)	(36,14)*
102	20	(58,6)	(60,0)	(52,8)	(46,12)	(42,12)	(40,8)*
104	22	(52,14)	(54,8)	(46,16)	(40,20)	(36,20)	(34,16)*
106	24	(48,18)	(50,12)	(42,20)	(36,24)	(32,24)	(30,20)*
108	26	(46,18)	(48,12)	(40,20)	(34,24)	(30,24)	(28,20)*
110	28	(46,14)	(48,8)	(40,16)	(34,20)	(30,20)	(28,16)*
112	30	(48,6)	(50,0)	(42,8)	(36,12)	(32,12)	(30,8)**
114	32	(38,16)	(40,10)	(32,18)	(26,22)	(22,22)	(20,18)**
116	34	(30,12)	(32,6)	(24,14)	(18,28)*	(14,28)	(12,24)**
118	36	(24,24)	(26,18)	(18,26)	(12,30)	(8,30)*	(6,26)**
120	38	(20,22)	(22,16)	(14,24)	(8,28)	(4,28)*	(2,24)**

nates at present at $^{184}\text{Hf}_{112}$, which is still in the prolate region, the first oblate nucleus expected to be $^{190}\text{Hf}_{118}$.

4) The Pt series of isotopes is *not* expected to exhibit the SU(3) symmetry, ^{196}Pt being the textbook example of the O(6) symmetry [11, 33]. However, if one blindly ascribes SU(3) irreps to the series of Pt isotopes, the first oblate one appears to be $^{194}\text{Pt}_{116}$, in agreement with empirical observations [12, 13] and theoretical predictions [3–6]. This is also in rough agreement with the empirical observations and theoretical findings of Ref. [14], carried out within the SU(3) limit of the interacting boson model, locating $^{192}\text{Pt}_{114}$ near the prolate-oblate transition point. It is also in rough agreement with the expectation that the O(6) symmetry represents the critical point of a prolate to oblate shape phase

Table 3. Same as Table II, but for the most leading SU(3) irreps [30, 31] for nuclei with protons in the 50-82 shell and neutrons in the 50-82 shell.

		Ba	Ce	Nd	Sm	Gd	Dy
	Z	56	58	60	62	64	66
N	val	6	8	10	12	14	16
56	6	(36,0)	(36,4)	(38,4)	(42,0)	(38,6)	(36,8)
58	8	(36,4)	(36,8)	(38,8)	(42,4)	(38,10)	(36,12)
60	10	(28,4)	(38,8)	(40,8)	(44,4)	(40,10)	(38,12)
62	12	(42,0)	(42,4)	(44,4)	(48,0)	(44,6)	(42,8)
64	14	(38,6)	(38,10)	(40,10)	(44,6)	(40,12)	(38,14)
66	16	(36,8)	(36,12)	(38,12)	(32,8)	(38,14)	(36,16)
68	18	(36,6)	(36,10)	(38,10)	(42,6)	(38,12)	(36,14)
70	20	(38,0)*	(38,4)	(40,4)	(44,0)	(40,6)	(38,8)
72	22	(30,8)*	(30,12)*	(32,12)	(36,8)	(32,14)	(30,16)
74	24	(24,12)*	(24,16)*	(26,16)*	(30,12)*	(26,18)*	(24,20)
76	26		(20,16)*	(22,16)*	(26,12)*	(22,18)*	(20,20)*
78	28						
		Er	Yb	Hf	W	Os	Pt
	Z	68	70	72	74	76	78
N	val	18	20	22	24	26	28
56	6	(36,6)	(38,0)	(30,8)	(24,12)	(20,12)	(18,8)
58	8	(36,10)	(38,4)	(30,12)	(24,16)	(20,16)	(18,12)
60	10	(38,10)	(40,4)	(32,12)	(26,16)	(22,16)	(20,12)
62	12	(42,6)	(44,0)	(36,8)	(30,12)	(26,12)	(24,8)
64	14	(38,12)	(40,6)	(32,14)	(26,18)	(22,18)	(20,14)
66	16	(36,14)	(38,8)	(30,16)	(24,20)	(20,20)	(18,16)
68	18	(36,12)	(38,6)	(30,14)	(24,18)	(20,18)	(18,14)
70	20	(38,6)	(40,0)	(32,8)	(26,12)	(22,12)	(20,8)
72	22	(30,14)	(32,8)	(24,16)	(18,20)	(14,20)	(12,16)
74	24	(24,18)	(26,12)	(18,20)	(12,24)	(8,24)	(6,20)
76	26	(20,18)	(22,12)	(14,20)	(8,24)	(4,24)	(2,20)
78	28	(18,14)	(20,8)	(12,16)	(6,20)	(2,20)	(0,16)

transition [12, 13], since $^{196}\text{Pt}_{118}$ is the textbook example [11, 33] of the $O(6)$ symmetry in the interacting boson model.

5) The results reported in Table 2 exhibit clearly that no particle-hole symmetry appears within the framework of the approximate $SU(3)$ symmetry [24]. From the mathematical point of view, this fact is already made clear by Table 1.

Using the same method one can also consider the rare earths with protons in the 50-82 shell and neutrons also in the 50-82 shell, the results being summarized in Table 3. Several comments apply.

1) A prolate to oblate transition appears at the lower right part of the table.

2) In the W, Os, and Pt series of isotopes, the first oblate nuclei appear at $N = 74$, i.e. they are ^{148}W , ^{150}Os , ^{152}Pt , all of them lying far away from the region experimentally accessible at present [32].

3) In the Hf series of isotopes, the first oblate nucleus is ^{148}Hf , having $N = 76$.

4) These predictions should be considered with extreme care, since in this shell protons and neutrons occupy the same major shell, thus the role of formation of proton-neutron pairs by protons and neutrons occupying identical or very similar orbitals should be examined before any conclusions could be reached.

On the other hand, the prolate over oblate dominance is clear in both tables, since in both cases the oblate nuclei are limited to the lower right part of the tables, i.e. just below the filling of the proton shell and the simultaneous filling of the neutron shell.

It should be noticed that the prolate over oblate dominance in heavy $N = Z$ nuclei has been recently obtained in the framework of the quasi- $SU(3)$ symmetry [34, 35], focused in the region from $^{56}_{28}\text{Ni}_{28}$ to $^{96}_{48}\text{Cd}_{48}$ [35].

3 Discussion

In the present work the prolate over oblate dominance in deformed rare earth nuclei is obtained within the framework of an approximate $SU(3)$ symmetry, using the symmetry properties alone and not involving any free parameters. In addition, within the same $SU(3)$ framework, the point of the prolate-oblate shape phase transition is predicted to be at $N = 116$, in complete agreement with the existing experimental data and recent microscopic calculations.

The success of the hidden approximate $SU(3)$ scheme in predicting the prolate dominance, as well as the location of the prolate-oblate transition, without using any free parameters, may be understood by the fact that the Nilsson levels are *not* changed much by the approximation involved [25]. In particular, downwards leading prolate orbits remain downwards leading, while upwards moving oblate orbits remain upwards moving. As a result, it is expected that the approximate scheme should provide correct results concerning nuclear properties related to the prolate or oblate character of deformed nuclei.

The present work suggests that it is worth investigating how far one can go in the description of the properties of heavy deformed nuclei taking advantage of the approximate SU(3) symmetry scheme. Work in this direction is in progress.

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