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Relation between Parity Shift Effects in the Spectra of Neighboring Even-Even and Odd-Mass Octupole Deformed Nuclei

P. Yotov, **N. Minkov**, **S. Drenska**

Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 72 Tzarigrad Road Blvd., BG-1784 Sofia, Bulgaria

Abstract. A systematic comparison between experimental spectra of neighboring odd-mass and even-even nuclei with octupole deformation has been done. As a result a difference in the angular momentum dependence of the parity shift in alternating-parity bands of even-even nuclei and of the parity-doublet splitting in odd-A nuclei was found. In even-even nuclei the parity shift is initially large and always decreases with the angular momentum. In odd-mass nuclei the parity-doublet splitting does not exhibit such uniform behavior, but shows several possible dependencies, namely continuous decrease, increase and decrease with subsequent increase. Also, it was found that the angular momentum dependence of the levels with given parity in the spectrum of an odd-mass nucleus is similar to the respective behavior of the levels with opposite parity in the spectra of the neighboring even-even nuclei. This finding gives an idea for possible explanation of the various dependencies of the parity-doublet splitting in odd-A nuclei.

1 Introduction

It is well known that the collective spectra of the double-even nuclei, built of states the parity of which alternates with the successive angular momenta, as well as, the parity doublets in the spectra of the odd-mass nuclei are indications for the presence of an octupole deformation in their shapes [1].

In the octupole deformed even-even nuclei, the states with a negative parity have much higher energies than these with a positive parity in the range of the low angular momenta. At higher angular momenta, however, the states with opposite parities merge into one sequence with energies regularly increasing with the increase of the angular momentum [2–5]. At a first glance, it looks like the states with opposite parities belong to two different bands but the presence of strong E1 transitions between them and the fact that they always emerge in conjunction justifies their consideration as one single band.

The difference between the levels with given parity and angular momentum and the levels with the same angular momentum lying on a curve interpolating the opposite-parity levels defines the parity shift in the even-even nuclei [4].

For the odd-mass nuclei, the collective spectra appear as parity doublets [2–5] consisting of states with consecutive values of the angular momenta and opposite parities $I^{\pm}$, $(I+1)^{\pm}$, $(I+2)^{\pm}$, $(I+3)^{\pm}, \ldots.$, where I is the lowest angular momentum equal to the angular momentum (half integer) of the odd particle. The energy difference between the states with the same angular momentum but opposite parities is called parity splitting.

The nowadays explanation of the parity shift/splitting effect, from a collective point of view, is based on the idea that the octupole shape of the considered nuclei oscillates between two opposite spatial directions [6–8]. This kind of dynamics is represented by a tunneling of the system through a potential barrier in a double well potential depending on an octupole deformation variable. The spectrum corresponding to this potential consists of couples of states with opposite parities and relatively close energies [9]. For the even-even nuclei, it is supposed that each part of the octupole band with certain parity is built on one of the two lowest lying oscillator states in the potential. Thus, the parity shift is generated by the energy difference of the oscillator states. It is assumed also that the height of the potential barrier increases with the increase of the angular momentum due to the centrifugal forces. Thus the energy difference between the oscillator states is decreasing as well as the parity shift.

For some of the octupole deformed even-even nuclei, the decrease of the parity shift is followed by an increase which occurs at high angular momenta and even another consequent decrease and increase at higher angular momenta. For explanation of these peculiarities some more detailed approaches are used [10–16].

The idea of the tunneling octupole shape is also applied for description of the parity doublets in the spectra of the odd-even nuclei. Each part of the doublet can be considered, again, as built on one of the lowest laying collective oscillator states. Thus, the parity splitting is a result of the tunneling and its magnitude and behavior on the angular momentum should be similar to that of the energy difference between the oscillator states. Then, we would expect that the splitting decreases with the increase of the angular momentum. However, examining the available experimental spectra of many odd-even octupole deformed nuclei, we found that for many of them the parity splitting is not only decreasing but also increasing, as well as decreasing and then increasing even at very low angular momentum. The aim of the present work is to give an idea how to describe the observed peculiarity. This is done by comparing the spectra of the odd-mass nuclei under consideration and their even-even neighbors.

In the next section, pictures representing the unexpected behavior of the splitting in the spectra of several odd-even nuclei are shown. We also introduce some basic definitions and do some remarks on the method of the investigation. A comparison of the spectra of the odd-mass nuclei under consideration and their neighboring even-even ones is presented. In the last section some conclusions based on the observations are made and an idea how to describe the behavior of the parity splitting is discussed.

2 Parity Shift and Parity Splitting: Similarities and Differences in the Spectra of Neighboring Double-Even and Odd-Mass Nuclei

In Figures 1 and 2, the dependence of the parity splitting on the angular momentum in the spectra of several odd-mass nuclei from the region of the rare-earths and the actinides is depicted. It is seen that the dependence is not strictly decreasing as it is expected according to the model of the tunneling octupole shape. For a large number of the considered nuclei the splitting increases, or decreases followed by an increase (in absolute values) at the beginning of the band.

In order to explain this observation we compare the spectra of isotopic and isotonic triplets of neighboring octupole deformed nuclei. For each nucleus we

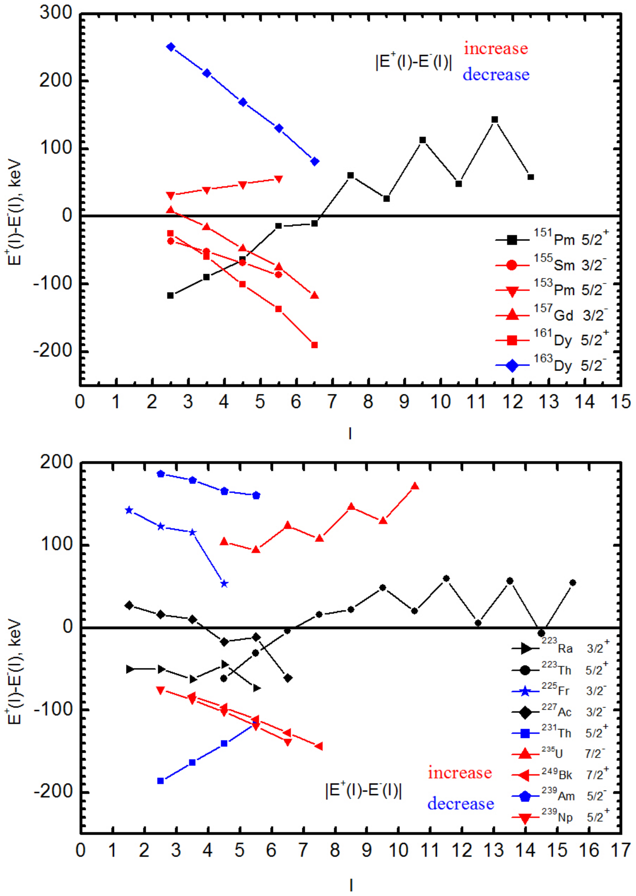

Figure 1. Experimental parity splitting in the spectra of some rare-earth (top) and actinide (bottom) nuclei.

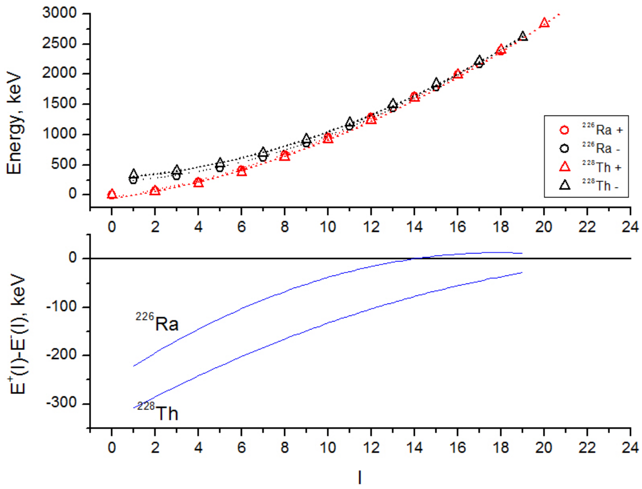

Figure 2. Experimental energies of the ground alternating parity bands and corresponding parity shifts for ^{226}Ra and ^{228}Th nuclei ($E^{\pm}(I)$ means $E^{\pm}_{\text{interp}}(I)$).

interpolate the sequences of states with positive and negative parities by separate parabolas

$$E^{\pm}_{\text{interp}} = a^{\pm}_0 + a^{\pm}_1 I + a^{\pm}_2 I^2. \tag{1}$$

Thus, we define the parity shift (in even-even nuclei) as a difference between the interpolation energies of the states with positive and negative parities at certain angular momentum, which is slightly different from the definition given in the introduction.

$$\Delta E_{shift}(I) = E^{+}_{\text{interp}}(I) - E^{-}_{\text{interp}}(I). \tag{2}$$

The experimental parity splitting in the spectra of the odd-mass nuclei is just a difference between the energies of the doublet states with opposite parities.

$$\Delta E_{split}(I) = E^{+}_{\text{exp}}(I) - E^{-}_{\text{exp}}(I). \tag{3}$$

It was mentioned in the introduction that for the double-even nuclei the states with a negative parity always lie much higher than these with a positive parity in the region of the low angular momenta. The parity shift is large in the beginning of the sequence and gradually decreases with the increase of the angular momenta. It should be stressed that this structure of the spectra is very stable with respect to the number of particles in the nuclei. As an illustration, the spectra of two doublets of neighboring double-even nuclei, ^{226}Ra, ^{228}Th and ^{238}U, ^{240}Pu, are shown in the next two pictures (Figures 3 and 4).

The circles and the triangles represent the experimental energies of the lighter and the heavier nucleus, respectively. The doted curves represent a quadratic interpolation of the dependence of the energy on the angular momentum. The

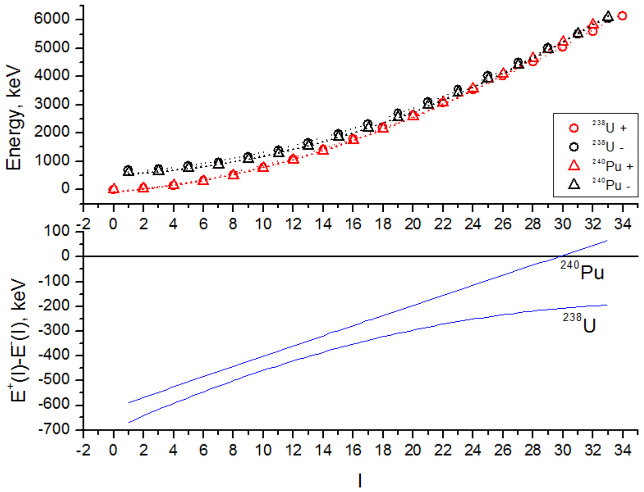

Figure 3. Experimental energies of the ground alternating parity bands and corresponding parity shifts for ^{238}U and ^{240}Pu nuclei ($E^{\pm}(I)$ means $E^{\pm}_{\rm interp}(I)$).

positive parity bands are depicted in red while the negative ones in black. The parity shift is obtained as a subtraction of the interpolation values of the energies at a given angular momentum. It is drawn at the bottom panel of each picture.

It is seen in Table 1 that the fit of the spectra of the even-even nuclei by parabolas is very good. The coefficients and the corresponding root mean square (RMS) values, for the considered nuclei, are also given in Table 1.

Figures 2 and 3 illustrate the fact that the energy curves for the states with opposite parities have different slopes and curvatures, merging at high angular momenta. This is an indication for a dependence of the rotation and vibration characteristics of the collective states on the corresponding parities. After rearranging (1) in the form of a rotator plus vibrator energy expression

$$E^{\pm}_{interp} = a^{\pm}_0 + (a^{\pm}_1 - a^{\pm}_2)I + a^{\pm}_2 I(I+1), \tag{4}$$

Table 1. Parameters of the interpolation parabolas and the $RMS^{\pm}$ deviations for the even/odd parity states: $a^{\pm}_0$ [keV], $a^{\pm}_1$ [keV/$\hbar$], $a^{\pm}_2$ [keV/$\hbar^2$], $RMS^{\pm}$ [keV]

	a^+_0	a^+_1	a^+_2	RMS^+	a^-_0	a^-_1	a^-_2	RMS^-
^{226}Ra	-37.2	51.3	4.7	23	212.2	21.9	5.5	11
^{228}Th	-53.5	49.8	4.7	31	278.7	25.5	5.2	13
^{238}U	-122.4	59.9	3.7	58	573.2	32.5	4.1	44
^{240}Pu	-72.3	40.6	4.5	36	537.2	19.5	4.6	23

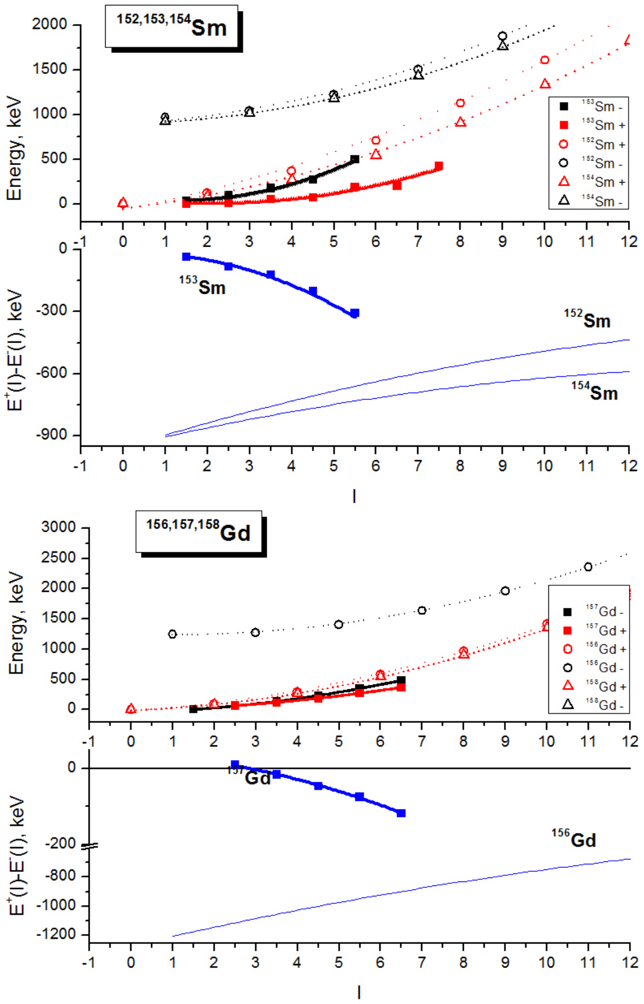

Figure 4. Comparison of the experimental energies of the ground alternating parity bands/parity doublets and corresponding parity shifts/splittings of two rare-earth isotopic triplets ($E^{\pm}(I)$ means $E^{\pm}_{\text{interp}}(I)$ for the blue curves and $E^{\pm}_{\text{exp}}(I)$ for the blue dots).

the corresponding coefficients in (4) can provide a measure for the difference between the rotation and vibration characteristics of the states with different parities.

The addition of an odd particle to the even-even octupole deformed core transforms the alternating parity band into a parity doublet. The spectra of two isotopic (^{152,153,154}Gd and ^{156,157,158}Sm), and two isotonic (^{226}Ra, ^{227}Ac, ^{228}Th and ^{238}U, ^{239}Np, ^{240}Pu) triplets are shown in Figures 4 and 5. The red

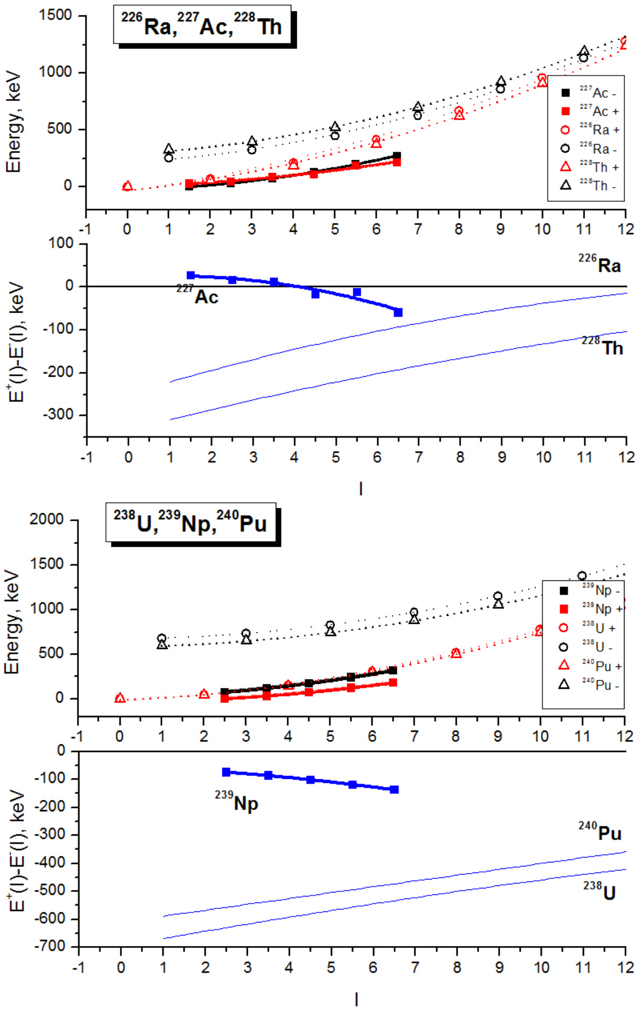

Figure 5. Comparison of the experimental energies of the ground alternating parity bands/parity doublets and corresponding parity shifts/splittings of two actinide isotonic triplets ($E^{\pm}(I)$ means $E^{\pm}_{\text{interp}}(I)$ for the blue curves and $E^{\pm}_{\text{exp}}(I)$ for the blue dots).

and black squares correspond to the experimental energies of the parity doublets. The interpolation curves of the corresponding experimental sequences are in black and red, too. The blue dots and the tick blue line correspond to the experimental parity splitting, and the parity-splitting obtained from the interpolation (just for completeness), respectively. The other objects are the same as in Figures 2 and 3. In Figure 4, the negative-parity states of the band and the corresponding interpolation and parity shift curve for the nucleus ^{158}Gd are not depicted because there are not enough experimental data. Comparing the spectra of the odd-even nucleus with these of the neighboring even-even nuclei, it is seen that the addition of the odd particle leads to a big change in the structure of the collective spectra. The structure is restored adding one more particle (to the next even-even nuclei in the isotopic/isotonic chain). Thus, it appears that the reason for this behavior could be the interaction of the odd particle with the octupole-deformed core.

For all even-even nuclei at low angular momenta, the curve representing the sequence of the negative-parity states lies higher and is less steep than the curve of the states with the positive parity. It is seen in Figures 4 and 5 that, for the odd-mass nuclei, the curve with the smaller steepness can be displaced so as to go under the other one (well seen in Figure 4 for nucleus ^{153}Sm). This configuration explains the increasing splitting (in absolute values) in the spectra of some odd-mass nuclei. The sign of the splitting depends on the mutual positions of the levels with positive and negative parity while its trend (increasing or decreasing) depends on the steepness of the curves.

There are several models [2, 17] explaining successfully the reduction of the parity shift after transition from an even-even to an odd-mass nucleus but they are not able to reproduce the above described swap of the curves.

It is also seen in the graphs of the considered nuclei, that the curves corresponding to the parts of the spectra with different parities in the double-even and in the odd-mass nuclei have similar curvatures and slopes two by two. This means that the motion of the odd particle does not interfere much with the collective rotation. On the other hand, the parity splitting is much smaller than the parity shift. Having in mind that the origin of both is the oscillation of the nuclear shape we can conclude that the single particle interacts much stronger with the vibration degrees of freedom than with the rotation.

3 Conclusion

On the base of the exposed observations we can do some generalizations. First of all, for the odd-mass nuclei under consideration, the increase of the splitting with the increasing angular momenta can be explained through the swap of the energy curves corresponding to the parts with different parities. The displacement causing the swap is more likely due to the interaction between the odd particle and the even-even core. The similarity of the curvature and the steepness of the energy curves for the odd-mass and their neighboring even-even nuclei suggests that the addition of the odd particle to the even-even core does not affect much the rotation motion of the nucleus. On the other hand the decrease of the parity splitting, compared with the parity shift in the closest even-even nuclei suggests a strong interaction between the odd particle and the oscillation of the shape.

Although, a model for description of the observed features was not introduced, we suggest that the remarks which have been made give a direction for a further investigations of the spectral structure and the properties of the odd-mass octupole-deformed nuclei. The work will be extended, in the future, in search of a proper mechanism and a particle-core interaction for description of the observed energy-curves swap.

Acknowledgments

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