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Few-Body Models in the Dark Matter Problem

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Abstract. Models of dark particles interaction with massive gravitational bodies are proposed and investigated. The models describe the resonant amplification of effective interaction between massive bodies at large distances between them. The phenomenon is explained by the catalytic action of dark particles in the system consisted of two and more heavy bodies (galaxies). Resonant amplification of the effective interaction between these heavy bodies imitates the increase of mass of the bodies, while their true gravitational mass remains unchanged. Such increasing of interactions leads besides to the gravitational lensing of passing light.

The interaction between a dark particle and a heavy body was taken in the separable form. Such separable form allows us getting the analytical solutions and simple analysis of the three-body solutions. The gravitational interaction between two heavy bodies was taken in the ordinary form.

1 Introduction

The dark matter problem emerged from the analysis of astronomical data, which showed abnormal high orbital velocities of galaxies at periphery of galactic clusters. It seems that the periphery galaxies gained additional invisible mass named as dark matter or dark particles. Discovered effect of gravitational lensing supported such assumption. This problem is still not solved and remains intensively studied [1–3].

In this paper we do not consider the nature of dark matter or dark particles (for that see, for instance, [3–7]). May be the dark particles exist in form of stretch-out strings with special properties, which can be described in the frame of SUSY. There are a lot of ideas on the nature of the dark matter [8, 9].

Below we consider the following questions: Why the dark particles interact more intensively with massive bodies at large distances, but demonstrate the insignificant impact on massive bodies at relatively small astronomical distances? Why within the solar system the observed action of dark particles is vanishing?

In this problem we can see the analogy with the catalytic action of an additional third particle which intensify the interaction in the system of two initial particles.

In comparison with a system of interacting two particles, the systems of three particles have some exceptional phenomena. Thomas' effect describes the collapse in the system of three identical particles, when the interaction in the every pair of the particles has zero range (point-like interactions) [10]. Then, V. Efimov predicted the effects of the condensation of levels in the three-particle spectrum and the logarithmic growth of their number when the ratio of the pair scattering length to the radius of pair forces is increasing [11].

The Efimov effect was confirmed in the some independent experiments [12, 13]. The opposite movements of ordinary bound states and the Efimov levels in the energy plane that happened at changing of the coupling constant of two-body interactions have been described in [14]. The effects demonstrate the different nature of these sort of levels. The phenomena take place in four body and more complicated few-body systems [12, 13]. So, three-body (and few-body also) systems demonstrate the more rich phenomena than simple two-body systems [15, 16]. For example, three-body resonance peaks depend on the distance between the heavy bodies [17, 18].

It is remarkable that the impulse of a light particle and the distance between heavy bodies do not submit to the uncertainty relation, because these main variables are correspond to different objects. Therefore, we can determine the resonant energies and the resonant distances with enough accuracy.

So then, we try to use specific properties of the three-body physics in order to create a model of dark matter action. We consider that dark particles exist and we suppose they do not interact with each other. However, the dark particles interact with heavy bodies. This interaction is to be very small at low energies and becomes a little more intense, then decreases again with increasing of energy. In this model the heavy bodies represent itself the galaxy in the galaxy clusters.

The three-body model gives the resonance amplification of interaction between the two heavy bodies (galaxies) situated on relative huge distances between them. The exact solutions may be interesting for a number of real problems in modern astrophysics.

2 Effective Interactions Between Heavy Bodies in Presence of Dark Particles

We consider the problem in the frame of quantum scattering theory of three-body and four-body systems. For simplicity, let us assume that the interaction between a dark particle and a heavy body has the separable form: $V_{DH} = |\nu > \lambda_{DH} < \nu|$, where V_{DH} is the potential of interaction between a dark particle and a heavy body, denoted with DH, λ_{DH} is the corresponding coupling constant. The form-factor of the potential in the impulse representation is $< \nu|\vec{p} >= \nu(\vec{p})$, $m = M_1$ is the mass of the dark particle, but M_i are the heavy body masses, where $i = 2, 3$ and $M_i >> m$. For simplicity, we guess $\hbar = c = 1$. The two-body T-matrix has the separable form $T_{DH} = |\nu > \eta_{DH} < \nu|$, where

$$\eta_{DH}^{-1} = \lambda_{DH}^{-1} + I(E)\,, \quad I(E) = -\int d\vec{p}\frac{\nu^2(\vec{p})}{E - E_p + i\gamma}\,. \tag{1}$$

For simplicity we consider the S-wave and the potential form-factor in the form: $\nu(p) = Nt/(1+t^2)$, where $t = p/\beta$, $N^2 = 4\pi/m\beta$, $t_0 = p_0/\beta$. Then $I(E) = I(t_0) = (1-2it_0)/(1-it_0)^2$, $M = M_i >> m$. We take $-1 < \lambda_{DH} < 0$, it means that the two-body system has a virtual state only. In common case, we can take into account more complicated interactions.

2.1 Interactions Between Two Heavy Bodies Created by Dark Particles

The problem of dark particle interaction with two heavy bodies can be solved in the frame of Faddeev's quantum mechanical equations:

$$T_{ij}(E) = T_{DH,i}\delta_{ij} + T_{DH,i}\sum_l \overline{\delta}_{il} G_0(E) T_{lj}(E)\,,\ i,j = 1,2,3 \quad , \qquad (2)$$

where $\overline{\delta}_{il} = I - \delta_{il}$ and i, j, l - the numbers of interaction pares. For example, $i = 1$ marks the interaction between two heavy bodies with numbers 2 and 3, $i = 2$ means the interactions between the dark particle with heavy body 3, and $i = 3$ means the interactions between the dark particle with heavy body 2. Total T-matrix is $T = \sum_{i,j} T_{i,j}$.

At first, we determine the solutions without gravity forces. Then the solution for T-matrix takes the form:

$$T_{ij}(E) = T_{DH,i}\delta_{ij} + |\nu_i > \eta_i M_{ij} \eta_j < \nu_j|\,,\ i,j = 2,3\ , \qquad (3)$$

where

$$M_{ij}(\vec r, \vec r\,') = J_{ij}(\vec r; p_0)\delta(\vec r + \vec r\,') + \sum_k J_{ik}(\vec r; p_0)\eta_k(p_0) M_{k,j}(-\vec r, \vec r\,')\,, \quad (4)$$

$$J_{ik}(\vec r; p_0) = 2m \int d\vec p \exp(i\vec r\vec p) \frac{\nu_i(p)\overline{\delta}_{ik}\nu_k(p)}{p_0^2 - p^2 + i0}\ \ , k = 2,3\,. \qquad (5)$$

The expressions $M_{ij}(\vec r, \vec r\,') = M^+_{ij}(\vec r)\cdot\delta(\vec r + \vec r\,')$, if $j \neq i$ and $M_{ii}(\vec r, \vec r\,') = M^-_{ii}(\vec r)\cdot\delta(\vec r - \vec r\,')$ in the case $j = i$, then

$$M^+_{ij}(\vec r) = [I - K(\vec r)]^{-1}_{ii} J_{ij}(\vec r; p_0)\ \ j \neq i\ \ , \qquad (6)$$

$$M^-_{ii}(\vec r) = [I - K(\vec r)]^{-1}_{ii} K_{ii}(\vec r; p_0)\cdot\eta_i^{-1}(p_0)\ \ . \qquad (7)$$

Here, $K_{ii}(\vec r, p_0) = J_{ij}(\vec r; p_0)\eta_j(p_0)J_{ji}(\vec r; p_0)\eta_i(p_0)$, p_0 is the initial momentum of the dark particle, $d = 2r$ is the distance between two heavy bodies. Note that $D = det[I - K(\vec r, p_0)] = 0$ corresponds to the pole in the amplitudes $M^+_{ij}(\vec r)$ and $M^-_{ii}(\vec r)$ (see Figure 1).

We take into account the two important values as given: p_0 is the impulse of an incident dark particle and $d = 2r$ is the distance between the two heavy bodies.

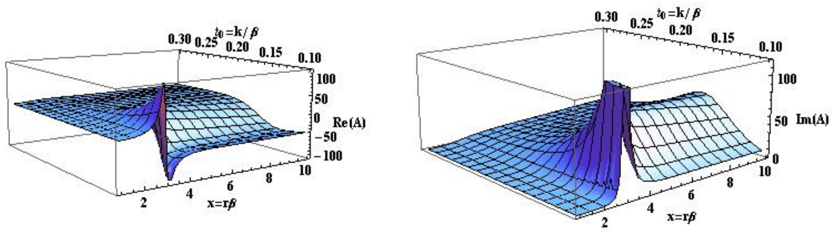

Figure 1. Real and imaginary parts of the amplitude $A(t_0; x) = M^+ + M^-$; $t_0 = p_0/\beta$; $x = r\beta$.

2.2 Interactions of Dark Particles with Three Heavy Bodies

It is very important to take into account the solutions of more complex objects, particularly the four-body systems which consist of three heavy bodies and one light dark particle. As in the previous simple case, the problem of a dark particle scattering on three body subsystem can be solved in an analytical form.

Omitting reduction procedure of Yakubovsky-Faddeev [20–22], we can write an expression for the connected part T^c of the total four-body T-matrix as

$$T^c_{i,j;n,n} = |\nu_i > \eta_i R^l_{ij} \eta_j < \nu_j| \ , \tag{8}$$

and write down the equation:

$$R^l_{ij} = Q^j_{il} \cdot \eta_l \cdot \left\{P_{lj} + R^i_{lj}\right\} \ , \tag{9}$$

where

$$Q^j_{il} = \left\{\Lambda_{il} + P^j_{ii}\eta_j\Lambda_{il} + P_{ij}\eta_j\Lambda_{jl}\right\}\bar\delta_{li}\bar\delta_{lj} \ . \tag{10}$$

We keep the previous notations, and mark by $i, j, l = 2, 3, 4$ at P^j_{ii}, Q^j_{il} and R^i_{lj} the number of scattering centers (the number of heavy bodies).

To simplify this, we can consider that all three heavy bodies are identical. Then, for diagonal amplitudes:

$$T^c_{i,i;n,n} = |\nu_i > \eta_i R^l_{ii} \eta_i < \nu_i| \ , \tag{11}$$

we can get

$$R^l_{ii} = P^l_{ii}\bar\delta_{lj} + \Lambda_{il}\bar\delta_{li}\bar\delta_{lj}\eta_l R^j_{li} \ . \tag{12}$$

Transform the equations to the coordinate space:

$$R^l_{ij}(\vec r_l, \vec r_j; \vec r'_l, \vec r'_i,) = \int\cdots\int d\vec p_l \vec p_j \vec p'_l \vec p'_i \exp\left\{-i\vec p_l \vec r_l - i\vec p_j \vec r_j\right\} \cdot \tag{13}$$
$$\cdot \exp\left\{i\vec p'_l \vec r'_l + i\vec p'_i \vec r'_i\right\} R^l_{ij}(\vec p_l \vec p_j; \vec p'_l \vec p'_i) \ ,$$

then, we have:

$$Q^j_{il}(\vec{r}_j,\vec{r}_l;\vec{r}'_i,\vec{r}'_j) = \frac{1}{I - B^j_{ii}}[J_{il}(\vec{r}_l)\cdot\delta(\vec{r}_l+\vec{r}'_l)\cdot\delta(\vec{r}_j-\vec{r}'_j) + \quad (14)$$
$$+J_{ij}(\vec{r}_j)\eta_j J_{jl}(\vec{r}_l)\cdot\delta(\vec{r}_l+\vec{r}'_j)\cdot\delta(\vec{r}_l+\vec{r}'_i)]\cdot\bar{\delta}_{lj}\bar{\delta}_{li} \ ,$$

where

$$B^j_{ii}(\vec{r}) = J_{ij}(\vec{r})\eta_j(p_0)J_{ji}(-\vec{r})\eta_i(p_0) \ . \quad (15)$$

Here, p_0 is the initial impulse of the dark particle.

$$R^l_{ij}(\vec{r}_j,\vec{r}_l) = Q^l_{ij}(\vec{r}_j,\vec{r}_l) + \Upsilon_1\cdot R^l_{ij}(\vec{r}_j,\vec{r}_l) + \quad (16)$$
$$\Upsilon_2\cdot R^l_{ij}(-\vec{r}_j,\vec{r}_l) + \Upsilon_3\cdot R^l_{ij}(\vec{r}_j,-\vec{r}_l) \ ,$$

where

$$\Upsilon_k = \frac{1}{I - B^j_{ii}(\vec{r}_j)}\cdot\Omega_k\cdot\eta_i \ , \quad (17)$$

and

$$\Omega_1 = J_{il}(\vec{r}_l)\cdot\eta_l\cdot\frac{1}{I - B^j_{ll}(\vec{r}_j)}\cdot J_{li}(-\vec{r}_l) + \quad (18)$$
$$J_{ij}(\vec{r}_j)\cdot\eta_j\cdot J_{jl}(\vec{r}_l)\cdot\eta_l\cdot\frac{1}{I - B^j_{ll}(-\vec{r}_l)}\cdot J_{lj}(-\vec{r}_l)\cdot\eta_j\cdot J_{li}(-\vec{r}_j) \ ,$$

$$\Omega_2 = J_{il}(\vec{r}_l)\cdot\eta_l\cdot\frac{1}{I - B^j_{ll}(\vec{r}_j)}\cdot J_{lj}(\vec{r}_j)\cdot\eta_j\cdot J_{ji}(-\vec{r}_l) \ , \quad (19)$$

$$\Omega_3 = J_{ij}(\vec{r}_j)\cdot\eta_j\cdot J_{jl}(\vec{r}_l)\cdot\eta_l\cdot\frac{1}{I - B^j_{ll}(-\vec{r}_l)}\cdot J_{li}(-\vec{r}_j) \ . \quad (20)$$

It is important, that

$$R^l_{ij}(\vec{r}_j,\vec{r}_l)\to R^l_{ij}(\vec{r}_j,-\vec{r}_l) \ , \quad if \quad \vec{r}_l\to\vec{r}_j \ , \ \ \vec{r}_j\to\vec{r}_l \ , \quad (21)$$

$$R^l_{ij}(\vec{r}_j,\vec{r}_l)\to R^l_{ij}(-\vec{r}_j,\vec{r}_l) \ , \quad if \quad \vec{r}_l\to-\vec{r}_j \ , \ \ \vec{r}_j\to\vec{r}_l \ , \ \ etc. \quad (22)$$

As an example, we consider the simple case when distances between three heavy bodies are the same $\vec{r}=\vec{r}_i=\vec{r}_j=\vec{r}_l$. Then the amplitude $R=R^l_{ij}$ in S-wave can be written in the form:

$$R = R(p_0,r) = \frac{J(1-J\cdot\eta)}{1-2J\cdot\eta} \ , \quad (23)$$

where

$$J = J_{ij}(p_0,|\vec{r}_j|) \ , \ \ i\neq j \ . \quad (24)$$

Following the equation (23), we can see that the resonances appear at other values of distances between the heavy bodies and the dark particles impulses than in the case of the three-body system. But these quantities must be close to each other on the order of magnitude.

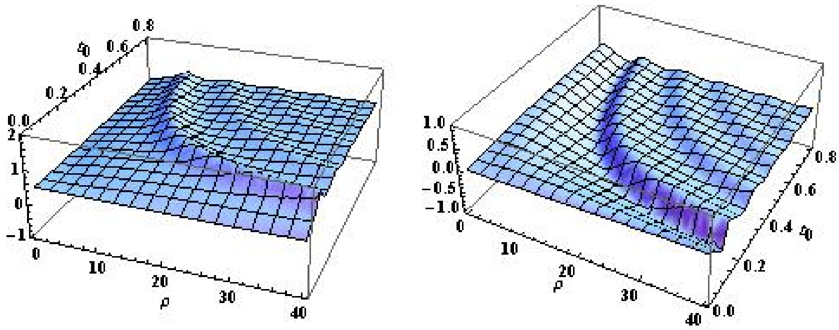

Figure 2. Real and imaginary parts of the amplitude $R = R(t_0; \rho)$; $t_0 = p_0/\beta$; $\rho = r\beta$.

3 Gravity Interactions on Background of Dark Matter

We mark as V_1 the gravity interaction between two heavy bodies; and we denote with index DH the interactions and solutions where the dark matter particles take part. In non-relativistic quantum mechanics the Hamiltonian of two heavy bodies can be written in the form :

$$\hat{H} = -\frac{1}{2M}\nabla^2 + V_1(r) \, , \tag{25}$$

where V_1 is the Newtonian gravitational potential [1,2]. Then, including this potential in the Lippmann-Schwinger equation we can write

$$|\Psi >= |\Phi > + G_\Phi V_1 |\Psi >= |\Phi > + G_0 V_{ef} |\Psi > \, , \tag{26}$$

where $\Psi(r)$ is the total wave function for the three-body system, Φ and T_Φ are the solutions without gravity forces. The indices i, j are omitted for simplicity.

The effective potential equals $V_{ef} = [I + T_\Phi G_0] V_1$ and can be written as

$$V_{ef} = V_1 + |\nu_{DH} > \eta_{DH} [I + (M^+_{DH} + M^-_{DH})\eta_{DH}] < \nu_{DH}| G_0 | V_1 \, . \tag{27}$$

We determine the enhancement factor for the gravity force as

$$\Xi = \frac{< \nu_{DH} | V_{ef} | \nu_{DH} >}{< \nu_{DH} | V_1 | \nu_{DH} >} \approx (M^+_{DH} + M^-_{DH}) \, . \tag{28}$$

Note that the second addendum of V_{ef} in (28) depends on $r = d/2$, where d is the distance between two heavy bodies. Let us then estimate the characteristic distances. If the resonance takes place at following parameters: $\lambda_{DH} = -0.99$; $\rho = r{\cdot}\beta = 3.4$, then, assuming that $r = 3.4{\cdot}10^{22} m$, one can get $\beta \approx 10^{-22} m^{-1}$ and $p_0 \geq 10^{-23} m^{-1}$, and a real part of the enhancement factor can achieve $\Xi(r) = 100$.

The orbital velocity of the peripheral body becomes higher than that at normal gravity owing to the enhancement factor. Moreover, the flux of dark matter

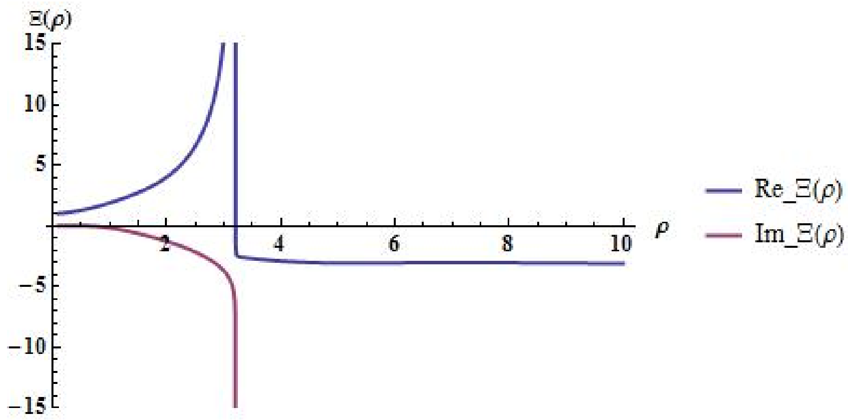

Figure 3. Real part (left) and imaginary part (right) of the enhancement factor Ξ at $\lambda_{DH} = -0.99$.

particles would attract these two heavy bodies and, with respect to other particles and fields (gamma quanta, for instance). The system would have the effective mass much higher than its own gravitational mass. Such mechanism can contribute to the gravitational lensing of electromagnetic radiation.

Noticeably, one can include in equations (8) – (10), (25) – (27) the additional interactions with gammas and obtain the enhancement factor for these interactions.

4 Conclusion

The obtained above solutions demonstrate that the effective interaction between two gravitational objects can behave resonantly at the huge distances between them. This resonant amplication is stimulated by additional interactions of massive bodies via dark matter particles. At small distances the amplication is negligibly small and the effective interaction coincides in this case with the direct interaction between the heavy bodies.

It is remarkable that the enhancement factor in this model acts only between two heavy bodies at large distances but is not so effective in the case of interactions between a dark matter particle and three or more heavy bodies simultaneously.

In reality, the interactions between dark matter particles and heavy bodies may be more complex.

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