NUCLEAR THEORY, Vol. 35 (2016)
eds. M. Gaidarov, N. Minkov, Heron Press, Sofia

Numerical Generalization of Bethe–Weizsacker Mass Formula

In memory of Bulgarian matematician Lubomir Alexandrov

S.Cht. Mavrodiev

Institute of Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Tzarigrad Road 72, BG–1784 Sofia, Bulgaria

Abstract. In this paper is presented the explicit improved numerical generalization of Bethe-Weizsacker mass formulae which describes the values of all 2654 measured nuclei masses in AME2012 nuclear database with accuracy less than 2.2 MeV, starting from the number of protons $Z = 1$ and the number of neutrons $N = 1$. In the obtained generazation of the Bethe-Weizsacker formula the influence of magic numbers and boundaries of their influence between them is defined for nine proton (2, 8, 14, 20, 28, 50, 82, 108, 124) and ten neutron (2, 8, 14, 20, 28, 50, 82, 124, 152, 202) magic numbers.

1 Introduction

The history and development of Bethe-Weizsacker (BW) mass formulae was presented in details in paper [1] - accuracy for nuclei masess bether or equale to 2.6 MeV. The purpose of the present work is to obtain the improved, compared with [1], explicit form of BW formulae as function of Z and N, which describes the values of nuclei masses from most recent evaluation database AME2012 (December 2012 – [2,3]. The masses extrapolated from systematics and marked with the symbol in the error column [4] are not taken into account here. These aim have been reached using Alexandrov dynamic autoregularization method (FORTRAN code REGN-Dubna [5–17]) for solving the overdetermined algebraic systems of equations, which is constructive development of Tikhonov regularization method [18–20]. One have to note that the use of procedure LCH (developed by Alexandrov and Mavrodiev) permits to dicover the explicit form of unknown function [21–26]. The basis for the classic BW mass formula is sketched in Sec.1 of this paper. The explicit form of numerical generalization of BW mass formula is described in Section 2. The results and graphical presentation of residuals $Res = Expt - Th$ like functions of variables A, Z and N is presented in Section 3. The Fortran source code of the generalized BW mass formula and the description of the experimental nuclear mass values from AME2012 database are presented in paper [38]. The predicted values of the binding energy, nuclear mass, atomic mass and mass excess for supper-havy nuclei analyzed in paper [37] are presented in Table 2 of Appendix.

2 The Bethe–Weizsacker Mass Formula and Binding Energy of the Nucleus

The nuclear masses can be calculated from the formula:

$$M_{Nucl}(A, Z) = Zm_p + Nm_n - AE_{B(A,Z)} \; , \tag{1}$$

where Z and N are the numbers of protons and neutrons, m_p and m_n are their masses correspondingly, $A = Z + N$ and $E_{B(A,Z)}$ is the binding energy per nuclei.

In the theory of the liquid drop model, proposed by George Gamov [27], the BW formulae for binding energy per nucleonis given by

$$\begin{aligned} E_{B(A,Z)} &= Volume - Surface\frac{1}{A^{(1/3)}} - Charge\frac{Z(Z-1)}{A^{(4/3)}} \\ &- Symmetry\frac{(N-Z)^2}{A^2} + Pairing\frac{\delta(A,Z)}{A^{(3/2)}} \; , \end{aligned} \tag{2}$$

where $\delta(A, Z) = +1$ for even N,Z, $\delta(A, Z) = -1$ for odd N, Z and $\delta(A, Z) = 0$ for odd $A = Z + N$. The improving of Eq. (1) had been proposed in many papers: [20–28]. For performing the digital generalization of BW mass formula we accept that the Volume, Surface, Charge, Symmetry, Pairing and powers of $A : 1/3, 4/3, 2, 3/2$ from formula (2) are s unknown functions of A, Z and unknown parameters $a = (ai, i = 1, N)$. If we accept the notation $Vol = Volume$, $Cha = Charge$, $Sym = Symmetry$, $Wig = Pairing$, magic numbers correction function $K_{MN}(A, Z, a)$ and for powers $P_1(A, Z, a)$, $P_2(A, Z, a)$, $P_3(A, Z, a)$, $P_4(A, Z, a)$ the formula of the binding energy will has a form

$$\begin{aligned} E_B(A,Z,a) &= Vol(A,Z,a) - Sur(A,Z,a)\frac{1}{A^{P_1(A,Z,a)}} \\ &- Cha(A,Z,a)\frac{Z(Z-1)}{A^{P_2(A,Z,a)}} - Sym(A,Z,a)\frac{(N-Z)^2}{A^{P_3(A,Z,a)}} \\ &+ Wig(A,Z,a)\frac{\delta(A,Z)}{A^{P_4(A,Z,a)}} + K_{MN}(A,Z,a). \end{aligned} \tag{3}$$

For more convenient start of iteration procedure the inicial values of new unknown functions are choosen near to the values of constants from papers [4], [27–37] as well as the values of powers in fomula (2). The explicit form of this ten unknown functions will be discovered by the solution of inverse problems, defined from overdetermined systems of nonlinear equations for binding energy,

$$E_B^{Expt(A_j,Z_j)} = E_B^{Th(A_j,Z_j,a)} \tag{4}$$

nuclear mass

$$M_{Nucl}^{Expt(A_j,Z_j)} = M_{Nucl}^{Th(A_j,Z_j,a)} \tag{5}$$

atomic masses

$$M_{At}^{Expt(A_j,Z_j)} = M_{At}^{Th(A_j,Z_j,a)} \tag{6}$$

and mass exccess

$$M_{Exc}^{Expt(A_j,Z_j)} = M_{Exc}^{Th(A_j,Z_j,a)} \tag{7}$$

where $j = 1, \ldots, 2564$ and a is a set unknown digital parameters. The relations between the values of nuclei mass $M_{Nucl}(A,Z)$, atomic mass $M_{At}(A,Z)$, mass excess $M_{Exc}(A,Z)$, hydrogen atom mass m_H, proton mass m_P and the neutron mass m_N [2]- [4] are:

$$M_{At}(A,Z) = Zm_H + Nm_N + AE_B(A,Z), \tag{8}$$

$$M_{Nucl}(A,Z) = M_{At}(A,Z)(Zm_e + A_{el}Z^{2.39} + B_{el}Z^{5.35}) \tag{9}$$

and

$$M_{Exc}(A,Z) = M_{At}(A,Z) - Au \tag{10}$$

where $A_{el} = 1.4438110^{-5}$ MeV, and $B_{el} = 1.5546810^{-12}$ MeV, $a_{N-1} = 2.39$, $a_N = 5.35$ [19, 30], the mass of the Hydrogen atom $m_H = 938.782303(0.084)$ MeV, $m_n = 939.56538(4.56)$ MeV, $m_p = 938.272046(21)$ MeV, $m_{el} = 0.510998928(11)$ MeV, and $u = 931.494061(21)$ MeV.

The $1 - \sigma$ uncertainties in the last digits of the above values are given in parentheses after the values.

The error analysis of the digital parameters a_i need more power computer facilities and work time. So, the correlations between parameters and their exclusion from unknown function will be done in the next research.

The using the LCH procedure, realized in the REGN program, permits us to choose the "better" function out of two functions with the same χ^2.

3 The Explicit Form of the Numerical Generalization of BW Mass Formula

The linearly independent arguments of numerical generalization $v = v_i$, $i = 1, \ldots, 9$ can be choosen as follow:

$$v_1 = \frac{Z}{A}, \quad v_2 = \frac{N}{A}, \quad v_3 = \frac{N-Z}{A},$$
$$v_4 = \frac{Z}{N+1}, \quad v_5 = \ln(A+1), \quad v_6 = 1/v_5$$

where Z and N are the numbers of protons and neutrons in nuclei, $A = Z + N$ and $v_7 = 0$, for odd A $v_7 = 1$, for even A, $v_8 = 0$, for odd Z and $v_8 = 1$, for even Z, $v_9 = 0$, for odd N, $v_9 = 1$, for even N.

Solving the overdetemined nonlinear algebraic systems (3 and 23) with condition $Expt = Theory$ for binding energy, nuclear mass, atomic mass and mass excess, using step by step different models for unknown functions

$Vol(A,Z,a)$, $Sur(A,Z,a)$, $Cha(A,Z,a)$, $Sym(A,Z,a)$, $Wig(A,Z,a)$, $P_1(A,Z,a)$, $P_2(A,Z,a)$, $P_3(A,Z,a)$, $P_4(A,Z,a)$, and $K_{MN}(A,Z,a)$,

using the possibilities of FORTRAN code REGN [5–17] for to choose the "better" function (LCH procedure [21–26]), we receive their explicit forms as follow:

$$Vol(A,Z,a) = \exp(a_1) + P(v,a,I_s)) + C(v,a,N_0)\,, \tag{11}$$
$$Sur(A,Z,a) = \exp(a_2) + P(v,a,I_s+Np) + C(v,a,N_0+N_i)\,, \tag{12}$$
$$Cha(A,Z,a) = \exp(a_3) + P(v,a,I_s+2Np) + C(v,a,N_0+2N_i)\,, \tag{13}$$
$$Sym(A,Z,a) = \exp(a_4) + P(v,a,I_s+3Np) + C(v,a,N_0+3N_i)\,, \tag{14}$$
$$Wig(A,Z,a) = \exp(a_5) + P(v,a,I_s+4Np) + C(v,a,N_0+4N_i)\,. \tag{15}$$

The parametrized powers that were implemented in Eqs. (6) have been obtained using the same LHC procedure and defined as

$$P_1(A,Z,a) = \exp(a_6) + P(v,a,I_s+5Np) + C(v,a,N_0+5N_i)\,, \tag{16}$$
$$P_2(A,Z,a) = \exp(a_7) + P(v,a,I_s+6Np) + C(v,a,N_0+6N_i)\,, \tag{17}$$
$$P_3(A,Z,a) = \exp(a_8) + P(v,a,I_s+7Np) + C(v,a,N_0+7N_i)\,, \tag{18}$$
$$P_4(A,Z,a) = \exp(a_9) + P(v,a,I_s+8Np) + C(v,a,N_0+8N_i)\,, \tag{19}$$

where

$$P(v,a,i) = \exp\Big[-\Big(\sum_{j=1}^{3}\sum_{j=1}^{4} a_{i+l+4(j-1)}v_l^j + a_{i+13}v_6 + a_{i+14}v_5\Big)^2\Big] \tag{20}$$

and

$$C(v,a,i) = \exp\Big[(-\Big(a_{i+1}\frac{v_7}{A} + a_{i+2}\frac{v_8}{Z} + a_{i+3}\frac{v_9}{N+1} + a_{i+4}\Big)^2\Big]. \tag{21}$$

The different values of the proton and neutron magic numbers were considered in different inverce problems and the dependence on the magic numbers and the bounaries between them were determined.

For obtaining the dependencies from magic numbers and boundaries between them, which define their influence, there was formulated different inverse problems with different values of proton and neutron magic numbers.

The LCH analysis of solutions of different inverse problems gives the explicit form of function $K_{MN}(A,Z,a,i)$ for nine proton and ten neutron magic numbers (see Appendix) as follow:

$$K_{Mn}(A,Z,a) = \frac{1}{A^N}\left[w_Z(1+C(v,a,N_0+9N_i)) + G(v,a,N_1)\right]$$
$$\times \frac{\exp\dfrac{-(Z-Z_{MN})^2}{2G(v,a,N_1+2N_w)}}{(Z-Z_MN)^2 + G(v,a,N_1+2N_w)}$$
$$+ \frac{1}{A^N}\left[w_N(1+C(v,a,N_0+10N_i)) + G(v,a,N_1+N_w)\right]$$
$$\times \frac{\exp\dfrac{-(N-N_{MN})^2}{2G(v,a,N_1+3N_w)}}{(N-N_{MN})^2 + G(v,a,N_1+3N_w)}\,, \quad (22)$$

where Z_{MN} and N_{MN} are the narst to Z and $N = A - Z$ magic numbers, correspondingly, w_Z and w_N are the half sum of corresponding magic numbersbetween which are Z and N. The explicit form of the function $G(v,a,i)$ is

$$G(v,a,i) = \exp\Big(a_{i+15} - \Big(\sum_{j=1}^{3}\sum_{l=1}^{4} a_{i+l+4(j-1)} v_l^j + a_{i+13}v_6 + a_{i+14}v_5\Big)^2\Big). \quad (23)$$

The integer numbers in formulaes (12–23) have the values as follow:

$$I_s = 9, \quad Np = 15 = N_w, \quad N_i = 4, \quad N_0 = I_s(1+Np),$$
$$N_1 = N_0 + N_d, \quad N_d = 44, \quad N = N_0 + N_d + 4N_w + 1 = 249.$$

The investigation of parameter's errors and correlations between them will be presented in next work. Our estimation is that after using LCH procedure the number of parameters will be less in times with the same χ^2.

4 The Behavior of the Different Part of Generalized BW Mass Formulae like Functions of Variables A, Z and N

In paper [38] the interesting reader can see the behavior of the structures $Vol(A,Z,a)$, $Sur(A,Z,a)$, $Cha(A,Z,a)$, $Sym(A,Z,a)$ and $Wig(A,Z,a)$, see Eqs. (11–15), as functions of A, Z and N, the behavior of power factors $P_1(A,Z,a)$, $P_2(A,Z,a)$, $P_3(A,Z,a)$, $P_4(A,Z,a)$, see Eqs. (16–19), as functions of A, Z and N, see Eqs. (14) and Figure 1, the magic numbers correction energy function $K_{MN}(A,Z,a,i)$ of the magic numbers, see (14), as function of the proton Z, neutron N and atomic mass number A respectively, the behavior of structures

$$Vol(A,Z,a), \qquad Sur(A,Z,a)\frac{1}{A^(P_1(A,Z,a))},$$
$$Cha(A,Z,a)\frac{Z(Z-1)}{A^(P_2(A,Z,a))}, \qquad Sym(A,Z,a)\frac{(N-Z)^2}{A^(P_3(A,Z,a))}$$

as functions of A, Z and N, see Eqs. (3) and (22) and Figure 3.

5 Results

The challenge of low energy nuclear physics to describe the dependence of the binding energy, nuclear, atomic mass and mass excess as functions of the number of protons and neutrons is presented. This result was established by using the experimental data from AME2012 [2, 3] database, the inverse problem method for discovering the explicit form of unknown theoretical function (model) and the values, based on the REGN (L. Alekasandrov-Regularized Gauss-Newton iteration method) [5–17] for solving the over-determined non linear system of equations. One has to note that the LCH-weighting procedure [2, 21, 23–26] of the REGN program permits to choose the better function out of two functions with the same χ^2. The essential advantage of the Alexandrov method [5–17] from other similar methods is extremely eective ideology regularization of inverse problem solution, which on each iteration step controls not only the actual decision, but, very importantly, uncertainty of the solution. At the same time, the transition from the mathematical theory of the autoregularizated iterative processes, which is based on meaningful theorems of convergence L. Aleksandrov [6] to Fortran codes (REGN-Dubna [8], FXY-Sofia-Dubna [17] is very complicated, but technically clear work.

The explicit form for functions $E_B^{Th}(A, Z, a)$, $M_{Nucl}^{Th}(A, Z, a)$, $M_{At}^{Th}(A, Z, a)$, $M_{Exc}^{Th}(A, Z, a)$ and the solution for the values of digital parameters a describe 2564 nuclei, atomic masses, mass excess and binding energies starting from $A = 2$ ($Z = 1$, $N = 1$) with relative error ε_r

$$\varepsilon_r = \frac{1}{N} \sum_{i=1}^{N} \frac{Expt(A_i, Z_i) - Th(A_i, Z_i, a)}{Expt(A_i, Z_i)} . \tag{24}$$

The resuduals, which are the difference between experiment and model values

$$Residual = Expt - Th \tag{25}$$

belongs to the interval $(-2.0, 2.20)$ Mev for nuclear, atomic mass and mass excess and to $(-0.17, 0.15)$ for binding energy. Its distributions and Gauss fits are presented in the next two figures.

The χ^2 test (estimation of describing accuracy) has been done to determine what signicance there is with this value of χ^2 using the formula (see Eq. (6) in [28]):

$$\chi^2 = \sum_{k=1}^{M} \left(\frac{Expt(A_k, Z_k) - Th(A_k, Z_k, a)}{(A_k, Z_k)} \right)^2 \tag{26}$$

where

$$\sigma(A_k, Z_k) = C\sigma_{Stat}(A_k, Z_k) + PercentExpt(A_k, Z_k). \tag{27}$$

Here $\sigma_{Stat}(A_k, Z_k)$ is the uncertainty of a nuclei as it has been reported in AME2012, C and $Percent$ are the nuisance parameters, where $C = 1$ and

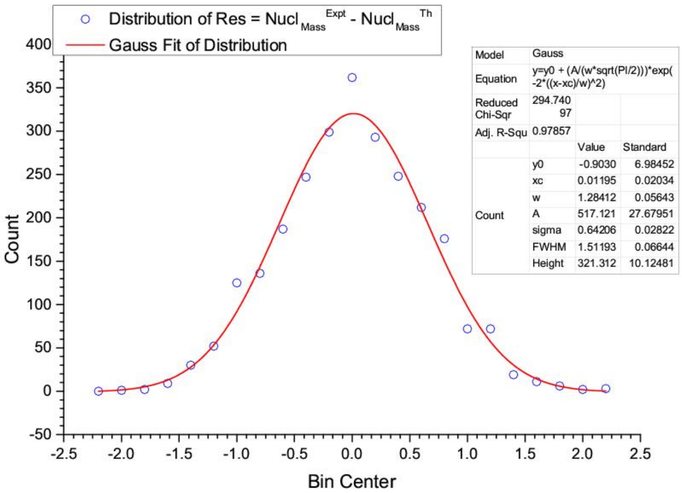

Figure 1. The distribution of nuclear mass residuals and its Gauss fit.

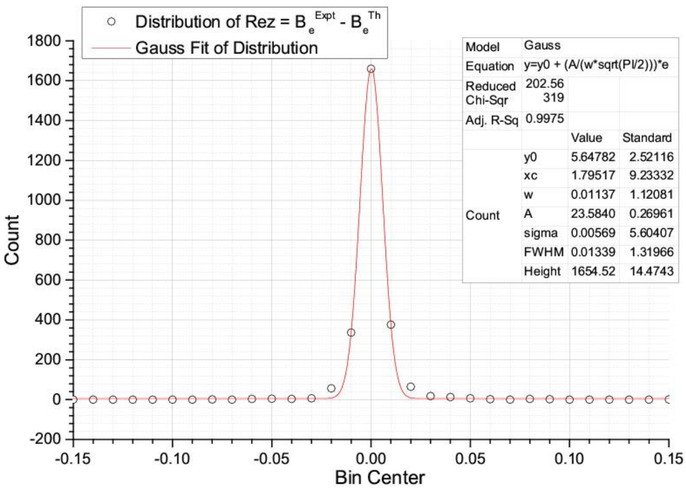

Figure 2. The distribution of Binding energy residuals and its Gauss fit.

$Percent$ is percentage of the given experimental value. Table 1 illustrates the quality of descriptions of the binding energy, the nuclear, atomic masses and the mass excess assuming different hypothesis for the $Percent$, ϵ_r, χ^2 and χ_n,

$$\chi_n = \sqrt{\frac{\chi^2}{M - N}} \quad (28)$$

where $M - N$ is the number of degrees of freedom. Note, that some masses of nuclei are measured with very high precision, which can noticed from mass

Table 1.

	$Percent$	ε_r	χ^2	χ_n
B_e	0.290E-02	0.417E+00	2564.	1.052
M_{Nucl}	0.118E-04	-0.672E-03	2468.	1.033
M_{At}	0.118E-04	-0.672E-03	2467.	1.032
M_{Exc}	0.217E+00	0.241E+02	2464.	1.03

excess column in AME2012, but due to artificial cut-off of the significant digits the uncertainties for these nuclei are given as zero uncertainty. Since we do not know the exact numbers we treat uncertainties for these nuclei as 1 percent of the given experimental value. The nuclear drip lines are the boundary delimiting the zone of Z, N in which atomic nuclei lose stability due to the transmutation of neutrons (down one) as well as because of Coulomb repulsion of protons (up). To nd where these drip lines are on the nuclear landscape we need to know the values of Z and N, where the separation energy is changed its sign. The coordinates, where separation energies change the sign can be calculated by using the explicit form for binding energy $E_B(A, Z, a)$ see (Eq. 3).

The formulae for two neutrons and two proton separation energies are

$$S_{2p}(Z,N,a) = (Z+N)E_B^{Th}(Z,N,a) - (Z-2+N)E_B^{Th}(Z-2,N,a) \quad (29)$$

and

$$S_{2n}(Z,N,a) = (Z+N)E_B^{Th}(Z,N,a) - (Z+N-2)E_B^{Th}(Z,N-2,a) \quad (30)$$

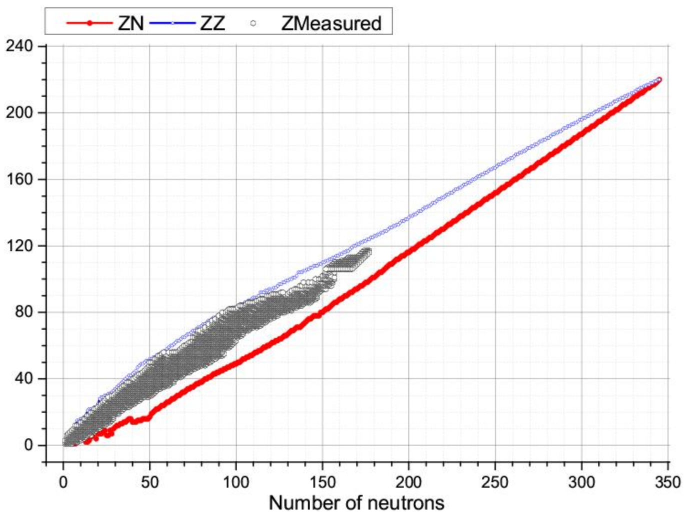

Figure 3. The asymptotoc behavior of calculated proton and neutron S2n drip-lines.

as well as simple algorithm for calculation of coordinates Z, N.

The next figure illustrates the behavior of calculated two proton and neutron drip-lines and their asymptotic.

In Table 2 the predictions for the values of binding energy, nuclear mass, atomic mass and mass excess for some heavy nuclei (paper [37]) are presented.

Table 2. The predictions of binding energy, nuclear mass, atomic mass and mass excess in MeV (paper [37])

NO	ELEMENT	A	Z	N	$N-Z$	BINDING ENERGY	NUCLMASS	ATOMICMASS	MASSEXCESS
1	104266	266	104	162	58	0.7346076E+01	0.2478347E+06	0.2478889E+06	0.1114747E+03
2	104267	267	104	163	59	0.7336931E+01	0.2487694E+06	0.2488236E+06	0.1146416E+03
3	104265	266	105	161	57	0.7335649E+01	0.2478362E+06	0.2478909E+06	0.1134652E+03
4	105270	270	105	165	60	0.7312202E+01	0.2515714E+06	0.2516261E+06	0.1227385E+03
5	105268	268	105	163	58	0.7324809E+01	0.2497035E+06	0.2497582E+06	0.1178416E+03
6	105267	267	105	162	57	0.7332857E+01	0.2487691E+06	0.2488239E+06	0.1149464E+03
7	106271	271	106	165	59	0.7306487E+01	0.2525039E+06	0.2525592E+06	0.1242633E+03
8	106269	269	106	163	57	0.7316995E+01	0.2506365E+06	0.2506918E+06	0.1199071E+03
9	107274	274	107	167	60	0.7281854E+01	0.2553061E+06	0.2553619E+06	0.1325240E+03
10	107272	272	107	165	58	0.7291935E+01	0.2534388E+06	0.2534946E+06	0.1282031E+03
11	107271	271	107	164	57	0.7298915E+01	0.2525046E+06	0.2525604E+06	0.1255323E+03
12	107270	270	107	163	56	0.7300343E+01	0.2515720E+06	0.2516278E+06	0.1243743E+03
13	108278	278	108	170	62	0.7259620E+01	0.2590401E+06	0.2590964E+06	0.1410800E+03
14	108277	277	108	169	61	0.7263104E+01	0.2581068E+06	0.2581632E+06	0.1393034E+03
15	108275	275	108	167	59	0.7272961E+01	0.2562395E+06	0.2562958E+06	0.1349762E+03
16	198273	273	108	165	57	0.7281152E+01	0.2543727E+06	0.2544290E+06	0.1311432E+03
17	110275	275	110	165	55	0.7245740E+01	0.2562443E+06	0.2563018E+06	0.1408958E+03
18	110282	282	110	172	62	0.7224057E+01	0.2627767E+06	0.2628341E+06	0.1527893E+03
19	112279	279	112	167	55	0.7201945E+01	0.2599832E+06	0.2600417E+06	0.1548508E+03
20	112286	286	112	174	62	0.7185912E+01	0.2665143E+06	0.2665728E+06	0.1655217E+03
21	113285	285	113	173	60	0.7168760E+01	0.2665179E+06	0.2665769E+06	0.1696442E+03
22	113284	284	113	172	59	0.7173337E+01	0.2655842E+06	0.2656432E+06	0.1674371E+03
23	113283	283	113	171	58	0.7173245E+01	0.2646518E+06	0.2647109E+06	0.1665652E+03
24	113282	282	113	170	57	0.7177131E+01	0.2637183E+06	0.2637774E+06	0.1645675E+03
25	113281	281	113	169	56	0.7176212E+01	0.2627862E+06	0.2628453E+06	0.1639325E+03
26	114283	283	114	169	55	0.7158593E+01	0.2637222E+06	0.2637819E+06	0.1690308E+03
27	114290	290	114	176	62	0.7148038E+01	0.2702522E+06	0.2703118E+06	0.1784806E+03
28	115290	290	115	175	60	0.7129986E+01	0.2702561E+06	0.2703162E+06	0.1829326E+03
29	115289	289	115	174	59	0.7133638E+01	0.2693226E+06	0.2693827E+06	0.1809359E+03
30	115288	288	115	173	58	0.7132842E+01	0.2683904E+06	0.2684505E+06	0.1802276E+03
31	115287	287	115	172	57	0.7135824E+01	0.2674571E+06	0.2675172E+06	0.1784331E+03
32	116294	294	116	178	62	0.7111165E+01	0.2739900E+06	0.2740507E+06	0.1914483E+03
33	116293	293	116	177	61	0.7111951E+01	0.2730573E+06	0.2731180E+06	0.1902580E+03
34	116292	292	116	176	60	0.7114608E+01	0.2721241E+06	0.2721848E+06	0.1885226E+03
35	116291	291	116	175	59	0.7114835E+01	0.2711916E+06	0.2712523E+06	0.1875000E+03
36	116290	290	116	174	58	0.7116715E+01	0.2702586E+06	0.2703193E+06	0.1859983E+03
37	118293	293	118	175	57	0.7078354E+01	0.2730645E+06	0.2731263E+06	0.1985357E+03
38	118294	294	118	176	58	0.7079403E+01	0.2739967E+06	0.2740585E+06	0.1992202E+03
39	118295	295	118	177	59	0.7078443E+01	0.2749295E+06	0.2749912E+06	0.2004954E+03
40	118296	296	118	178	60	0.7078998E+01	0.2758618E+06	0.2759236E+06	0.2013241E+03
41	118297	297	118	179	61	0.7077473E+01	0.2767947E+06	0.2768565E+06	0.2027690E+03
42	119295	295	119	176	57	0.7059040E+01	0.2749339E+06	0.2749962E+06	0.2054360E+03
45	120295	295	120	175	55	0.7038272E+01	0.2749387E+06	0.2750015E+06	0.2107797E+03
46	120296	296	120	176	56	0.7040502E+01	0.2758705E+06	0.2759334E+06	0.2111527E+03
47	120297	297	120	177	57	0.7041094E+01	0.2768029E+06	0.2768657E+06	0.2120076E+03
48	120298	298	120	178	58	0.7042875E+01	0.2777349E+06	0.2777977E+06	0.2125070E+03
49	120299	299	120	179	59	0.7042966E+01	0.2786674E+06	0.2787302E+06	0.2135083E+03

6 Conclusion

In this paper is presented an improved numerical generalization of Bethe-Weizsacker mass formulae which describes the values of measured 2654 nuclear mass in AME2012 nuclear database with residduals from (-2.00 to 2.20 MeV) for nuclear, atomic mass, mass excess and in interval (-0.17 to 0.15 MeV) for binding energy. The rediscovered in paper [1] well known proton and neutron magic numbers as well the discovered (108, 124 for protons and 152, 202 for neutrons) were confirmed. The first interesting application of the proposed explicit form of improved numerical generalization of Bethe-Weizsacker mass formulaeseems to be the calculation of two proton and neutron drip-lines and their intercept is approximately in $Z = 220$, $N = 345$ and $A = 565$. Proposed BW formulae can be used for calculation of not known nuclear mass, the total and kinetc energy of proton, alpha, cluster decays and spontaneously fission. The used in this paper approach for generalization of BW mass formulae can be applied for the actualization the half-life models (see for example [37] which describes the increasing volume for reasurching the problems in super heavy nuclei stability islands.

Acknowledgements

The author is thankful to Svetla Drenska nad Maksym Deliyergiyev for fruitful discussions.

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