Semi-Exact, Quasi-Exact and Conditionally-Exact Solutions of One-Particle Equations*

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Usually one-particle eigenvalue problems have to be solved numerically. Several exactly solvable cases (harmonic oscillator, hydrogen atom) play a fundamental role in quantum mechanics. Quasi-exactly solvable equations, where a finite number of solutions (in most cases just one) may be expressed analytically, have been intensively studied over the last two decades. The best known example is the *Hooke atom*, also known as *harmonium*. For equations referred to as *conditionally-exactly-solvable* analytic solutions may be obtained for specific values of the potential parameters. Finally, in the case of *semi-exactly-solvable* equations the wave functions may be expressed analytically, but the eigenvalues have to be derived numerically. The last two kinds of problems attracted much attention only recently [1,2].

One-particle potentials expressed as polynomials of the radial variable are known as the *power potentials*. In this presentation I demonstrate that the Schrödinger equations with power potentials are semi-exactly-solvable and their solutions may be expressed in terms of the Hessenberg determinants [2]. Conditions under which the equations with power potentials are either exactly or quasi-exactly solvable are also derived and examples of analytic solutions are presented.

In the relativistic case, Dirac and Klein-Gordon equations may be conditionally reduced to a form equivalent to the Schrödinger equation [3]. It is shown, that the relativistic equations in which potentials with electrostatic component of the vector potential are equal to the scalar part of the potential are always reducible to a Schrödinger equation with energy-dependent parameters. Several examples are briefly discussed.

References

[1] A.M. Ishkhanyan, A conditionally exactly solvable generalization of the inverse square root potential, *Phys. Lett. A* **380** (2016) 3786-3790.

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[3] J. Karwowski, Dirac operator and its properties in *Handbook of Relativistic Quantum Chemistry*, Ed. Wenjian Liu, Springer 2017, pp. 3-49.

*Dedicated to the memory of Dr. Rossen Pavlov