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Interplay of Break-Up and Transfer Processes in Reactions Involving Weakly-Bound Systems: A Simple One-Dimensional Model*

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Abstract. In this contribution I present a line of research which has been developed to describe the structure and the dynamics of weakly-bound systems with one or more active valence particles. A simple model is used in particular to clarify the reaction mechanism and the interplay of different reaction channels (elastic, inelastic, transfer, break-up) in heavy-ion collisions. The model involve two moving potential wells in one dimension with few active particles and, in spite of its simplicity, is supposed to maintain the main features, the properties and the physics of the full three-dimensional case. In reactions involving weakly-bound systems the attention is focussed on the role of the continuum states and the coupling to break-up states. Different approximation schemes (as first-order or coupled channels) as well as different continuum discretization procedures are tested. In the case of two active particles the reaction mechanism associated with two-particle transfer (successive or cluster transfer?) and the effect of pairing interaction are investigated.

In this contribution we discuss a line of research which has been developed to describe the structure and the dynamics of weakly-bound systems with one or more active valence particles [1]. To simplify the problem we are assuming particles moving in one dimension, a model that despite the drastic assumption encompasses many characteristics observed in experiments. A similar line of research, in one and three dimensions, has been also successfully carried out by Samarin and collaborators [2]. Additional applications of the model to one and two-particle systems can be found in Refs. [3].

We start with one or more particles moving in a fixed "target-like" potential and follow the evolution of the "exact" one- or many-body wave function under the action of a second moving "projectile-like" potential. According to the parameters of the two potentials and consequently the energies of the single-particle states, the initial velocity and the acceleration of the moving potential, and the distance of closest approach one can simulate a full variety of situations ranging from low to high bombarding energy, from cental to peripheral collisions, from weak to tight binding. Figure 1 shows an example of the final

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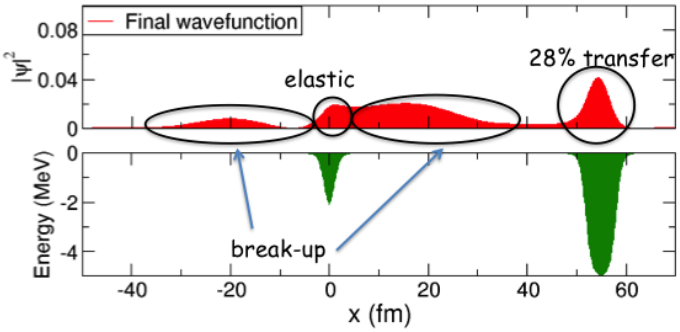


Figure 1. (lower frame) Position of the two wells at the end of the collision. (upper frame) Wave function (squared) of the single-particle state at the end of the collisions. A view of the full time evolution of wells and wave function in this specific case is given in Figure 3

situation of the two potentials (lower frame) and the corresponding wave function in the simplest case of a starting situation of just one weakly-bound particle (cf. Figure 3). By projecting out the final wave function on the bound and continuum states of the two potentials one can determine the probabilities of elastic, inelastic, transfer and break-up final processes.

As shown later, final wave functions can be compared with those obtained by using standard approximation schemes, such as coupled-channels or first-order approximation, and one can test different prescription for the treatment of continuum channels. In the case of two active particles one can study specifically the two-particle transfer channel and point out the role of the residual pairing interaction between the two particles, as well as the reaction mechanism (sequential vs cluster-like transfer).

We start then by first considering processes involving just one active particle, initially sitting on a single-particle level of a one-body potential and feeling the action of a second moving potential. Therefore, we have to solve the single-particle time-dependent equation

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \mathcal{H}(x,t)\Psi(x,t)$$

with

$$\mathcal{H}(x,t) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2} + V_T(x) + V_P(x - x_P(t))\,.$$

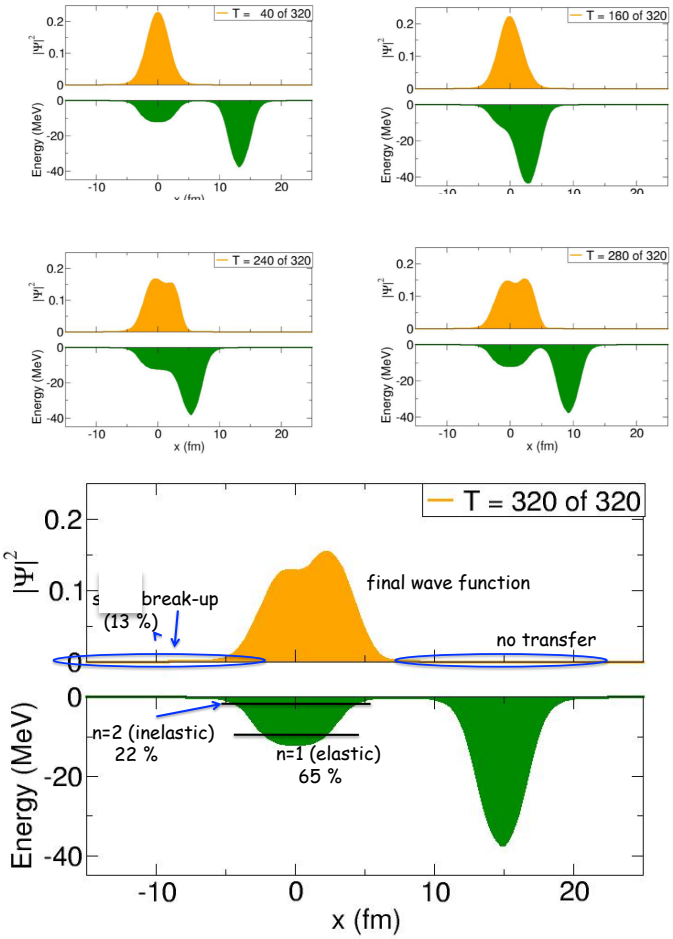
The choice of the parameters entering in the calculation will lead to different structural and kinematical conditions, corresponding to rather different physical situations and simulating different bombarding energy regimes, impact parameters, and Q-values for particle transfer: essentially one has to fix the parameters characterizing the two wells V_T and V_P (consequent energies of single-particle states in both potentials), initial condition (selecting one of the single particle state in target potential), and input for the projectile trajectory $x_P(t)$.

As a first example of the evolution of the wave function we choose a case in which both the target and the projectile potential wells admit two bound levels (with binding energies energies at 2.0 and 8.6 MeV for the fixed target and 27.0 and 8.4 MeV for the moving one). Initially, the particle occupies the lowest single-particle target state and the projectile trajectory is $x_p(t) = x_0 + vt + \frac{1}{2}at^2$; initial energy, reduced mass μ, distance of closest approach x_0 and acceleration* are given respectively as 1 MeV, 0.975 amu, 3 fm and $4500/\hbar^2$ fm/ps^2. The evolution of the wave function with time is illustrated in Figure 2. The different frames refer to different times (the total collision time is divided in 320 steps and the corresponding time is quoted in each frame), and in each frame the upper part gives the square of the one-particle wave function while the lower frame gives the actual position of the two potentials at the same time. Note that a part of the final wave function (although not easily detectable in the figure) appears outside the two wells, indicating a not negligible break-up probability ($\approx 13\%$). On the other side the condition of optimal Q-value associated with the single-particle energy difference and collision time seems to inhibit the transfer process.

The table at the bottom of Figure 2 gives, in comparison with the "exact" values, the values obtained in first-order approximation and in a coupled-channels scheme. This is done within the standard time-dependent coupled-channels formalism by constructing the non-diagonal transfer formfactors and expanding the wave function into the dual basis associated with the two wells (cf. Ref. [4]). The continuum is included via a set of pseudostates obtained by discretization procedure in a large box. Results obtained within other discretization procedures are very similar [1]. The importance of the full multistep couplings (including continuum, last column) is evidenced by the population of the continuum (break-up) states that, negligible in first order, can only be obtained via the excitation of the weakly-bound excited state.

As a second example we consider a physical situation in which the transfer and excitation to the continuum (breakup) play a more significant role. This is achieved considering as initial state a weakly-bound state of the target well ($E_b = -0.28$ MeV), as displayed in Figure 3a. In this case the projectile well follows the trajectory $x_p(t) = x_0 + \sqrt{\rho^2 + (vt)^2} - \rho$ proposed by Ref. [4]; this trajectory differs from the previous one in the fact that the projectile is changing its acceleration over the distance ρ thus simulating the nuclear interaction with the target, in fact at $t = \pm\infty$ the trajectory tends to a uniform motion with zero acceleration. Incident energy, reduced mass μ, ρ and distance of closest approach x_0 are given respectively as 5 MeV, 1.001 amu, 2 fm and 4 fm. The corresponding evolution of the wave function is shown in the different frames of Figure 3b. The weak-binding situation leads to an initial wave function with a longer tail than in the previous case. As a consequence, part of the wave function is already trasferred to the second well even before the overlap of the two wells (second frame). At the end of the process (cf. Figure 3c) there is a large transfer

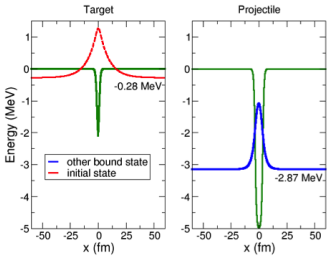
*For this case time is expressed in unit of $\hbar$.



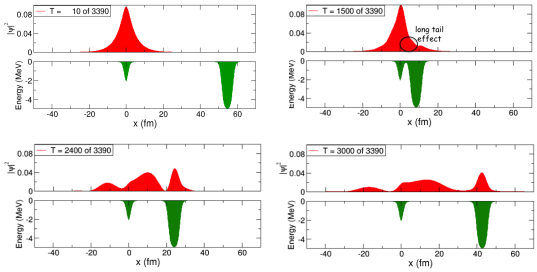
Final population	Exact	First-order	Coupled-channels (only bound states)	Coupled-channels (including Continuum)
n=1 elastic	65 %		73 %	65 %
n-2 inelastic	22 %	29 %	27 %	22 %
break-up	13 %	≈ 0		13 %

Figure 2. Evolution of the single-particle wave function (and corresponding evolution of the two wells) at different times. The final situation is better shown in the enlarged frame at the bottom of the figure. The parameters for this calculation are given in text. The table at the bottom refers to different models and approximation schemes.

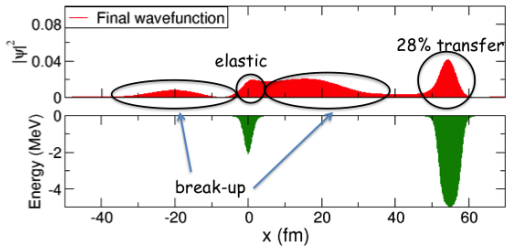
a)



b)



c)



d)

Final population	Exact	CC (with only bound states)	CC (with bound and continuum target states)	CC (with bound and continuum states in both target and projectile)
elastic	20.7 %	95 %	21.4 %	21 %
transfer	28 %	5 %		21 %
break-up	51 %		78.6	57 %

Figure 3. Cf. caption to Figure 2.

probability, but the weak-binding situation has also led to a large fraction of the wave function outside of the two wells, therefore associated to large break-up processes.

The table in Figure 3d compares the "exact" final probabilities with those obtained within the time-dependent coupled-channels approach. Three different model spaces are compared. In the first calculation we have included only target and projectile bound states. The second includes the target bound levels plus the first 50 continuum pseudostates obtained using a large box with a radius equal to

500 fm (this corresponds to an energy cutoff in the continuum of 0.5 MeV). The last (and more complete) calculation uses the target and projectile bound states plus the first 100 pseudostates of target and projectile continuum obtained using a large box with a radius of 40 fm (this corresponds to an energy cutoff in the continuum of 300 MeV). It is evident that, even in the simple one-dimensional case, the complexity induced by the different competing channels and the strong role of continuum in the case of weakly-bound systems implies the necessity of a very large model space and a proper choice of the parameters in order to obtain an acceptable agreement with the exact results.

We move now to the case of two-particle systems and two-particle transfer processes. We start by constructing the initial two-particle state in the target. In addition to the one-body target potential the Hamiltonian includes now a residual pairing-like interaction between the two particles, taken as a density-dependent delta interaction (i.e. acting only when the two particles are both inside the same well). The initial wave function has been obtained by diagonalizing the residual pairing interaction in the two-particle basis. In order to describe systems close to the drip line, we have chosen a one-body potential that does not admit any bound state and the two-particle basis consists therefore of continuum states obtained by a discretization procedure (cf. Ref. [5]). It is the effect of the residual interaction that produces the bound two-particle state.

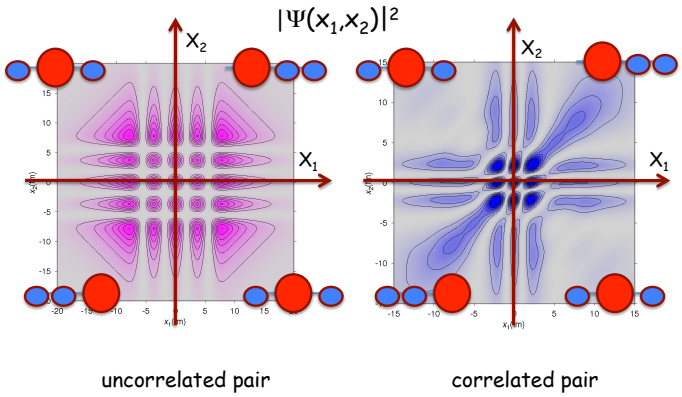


Figure 4. Square of the two-particle wave function shown as a contour plot as a function of x_1 and x_2. The two frames refer to the uncorrelated (left) and correlated case (right part).

The square of the resulting two-particle wave function $\Psi(x_1, x_2)$ is plotted as contour plot in Figure 4 (right frame). Note that due to the correlation the probability of finding both particles on the same side is clearly favored with respect to the situation in which the two particles are on opposite sides. As a comparison, we show in the left frame of the same figure the corresponding plot in the case of uncorrelated pair. In this latter case the depth of the one-body

potential has been increased to yield a bound single-particle state, such as to create an unperturbed two-particle state with the same energy as in the correlated case. In this uncorrelated cases all possible configuration are equally probable and the plot is symmetric in the four quadrants. Note that in the case reported in the figure the discretization of the continuum has been obtained by using a box with infinite walls at a boundary of 100 fm. Completely similar results have been obtained using either harmonic oscillators or transformed harmonic oscillator, but involving a rather large number of oscillator quanta (cf. Ref. [5]).

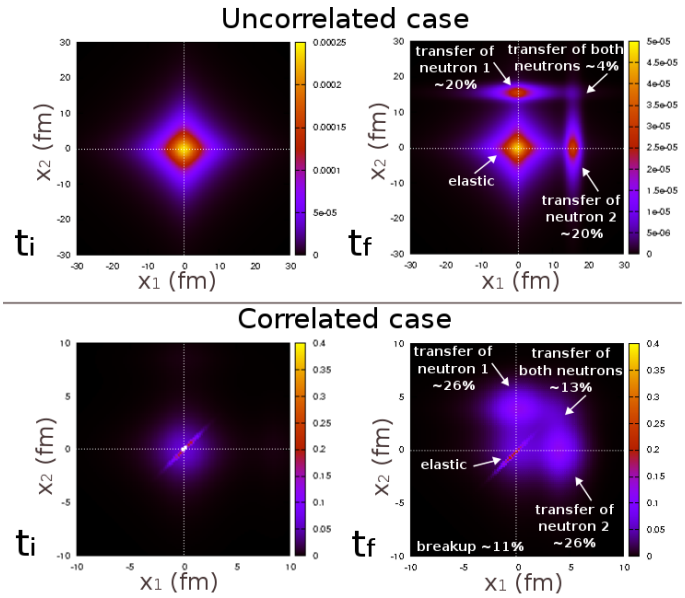


Figure 5. Square of the two-particle wave function shown as a contour plot as a function of x_1 and x_2. The four frames refer to the uncorrelated (upper part) and correlated case (lower part). Initial wave functions in the left column, final ones in the right column.

Starting from the initial two-particle state generated by the fixed well we can now follow the time evolution of the two-particle wave function due to the action of the moving one-body potential. Examples of the time evolution are given in Figure 5, where initial (at time $t = t_i$) and final (at $t = t_f$) wave functions are given as a contour plot as a function of x_1 and x_2. The upper frames refer to the case in which the pairing interaction is switched off (uncorrelated case) and the two particles sit in one single-particle state. From the final wave function we can separate different final states: elastic/inelastic, one-particle transfer, one-particle break-up, two-particle transfer and finally two-particle break-up. In this specific case break-up processes (both one and two-particle) are negligible. The total one-particle probability P_1 amounts to about 40%, while the two-particle transfer probability P_2 amounts to about 4%. Due to the absence of correlations

the transfer process is therefore produced by the successive transfer of single particles. In such a situation, in a perturbative approach, we expect a pair transfer probability $P_2 \approx (P_1)^2/4$, which is precisely the value obtained.

We switch now to the case with correlations (lower frames of Figure 5). The effect of the initial correlation will propagate during the scattering process and affect the final wave function (lower-right frame). At the end of the process one gets a total single-particle probability P_1 equal to 52 $\%$ and a pair transfer probability P_2 equal to 13$\%$. This latter value is a factor 2 larger than the uncorrelated estimate $P_2 \approx (P_1)^2/4$. This factor 2 rapresents therefore the enhancement factor due to the pairing correlation. From the point of view of the reaction mechanism, the processes results from the coherent contribution of successive one-particle transfers via the full set of levels in the intermediate one-particle system (continuum states included). Note that the signal of the dynamical effect of the collectivity induced by pairing correlations in the particle-particle channel is clear, but the quantitative effect is rather far from the one associated with the long-range interaction acting in the particle-hole channel. Note that pairing correlations will be dynamically effective (with enhancement of the two-step process with respect to the one-step) not only in two-particle transfer reactions, but also in two-particle break-up reactions, two-particle knock-out reactions and finally in two-particle decay processes.

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