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Predicted Clustering on the Surface of Sn Isotopes Explored in Nuclear Reactions

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Abstract. The influence of clusters as an important ingredient of an equation of state (EoS) of nuclear matter is evaluated. A recent prediction that the generalized relativistic mean-field model displays an isotope-dependant quenching of α-clustering on the surface of Sn as the nuclear system becomes more asymmetric with increasing isotopic mass number is compared with existing experimental results of an α-pickup reaction. The implications of this study for other proposed investigations of the clustering in Sn isotopes are discussed.

1 Introduction

The equation of state (EoS) of nuclear matter is of profound interest to a wide range of topics in nuclear physics, astrophysics and cosmology. In this context, the properties and behaviour of clusters in the nuclear medium, such as its formation and disassociation under varying conditions of density and temperature, are of crucial importance. Typel *et al.* [1] systematically investigated dynamical properties of light-ion clusters by means of two many-body theories, namely a microscopic quantum statistical (QS) approach as well as a generalized relativistic mean-field (RMF) model. The interplay between conditions of the nuclear medium and cluster response leads to an RMF prediction [1] of cluster formation on the surface of Sn isotopes, as well as the neutron skin thickness as a function of isotopic mass number.

This paper draws attention to a comparison [4] between results from existing experimental α-pickup data [3] and the prediction [1] for surface clustering on a range of Sn isotopes. The predicted α-clustering trend with isotopic mass number appears to be reproduced experimentally.

The virtue of a direct quasifree knockout measurement of the α-cluster trend of Sn isotopes is discussed in this work. In addition, it is proposed that an experimental study of the possible influence of the apparent quenching of α-cluster preformation yield with increasing isotopic mass number of Sn should provide valuable insight into the reaction mechanism of pre-equilibrium (p, α) reactions.

2 EoS and Neutron Skin Thickness of Nuclei

The equation of state (EoS), which gives the energy per particle $e(\rho, \delta)$ in asymmetric nuclear matter, is expressed as [5]

$$e(\rho, \delta) = e(\rho, 0) + S(\rho)\delta^2 + \mathcal{O}(\delta^4), \tag{1}$$

where the total nuclear density is $\rho = \rho_n + \rho_p$ in terms of the density of neutrons ρ_n and protons ρ_p. The quantity δ is given by

$$\delta = \frac{\rho_n - \rho_p}{\rho}, \tag{2}$$

and the symmetry energy $S(\rho)$ is defined as

$$S(\rho) = \frac{1}{2}\frac{\partial^2 e(\rho, \delta)}{\partial \delta^2}\bigg|_{\delta=0}. \tag{3}$$

Eq. 1 provides a deceptively simple expression for the EoS, which may be explored in a variety of theoretical approaches (See for example [6, 7]). A natural outcome of the formulation is a prediction of the neutron skin of nuclei, such as ^{208}Pb. This quantity has a direct relationship with properties of neutron stars, such as radii and cooling of their hot cores. It stands to reason that a crucial test of the predictions of a specific implementation of the EoS would consist of a comparison with an experimental value of the neutron skin thickness. Unfortunately, because most current experiments rely on hadron probes, such as protons, pions and antineutrons, the extracted values are highly model dependent. A novel approach [8] is to exploit the fact that the weak charge of the neutron is dominant (more than 99% of the proton value), which means that in

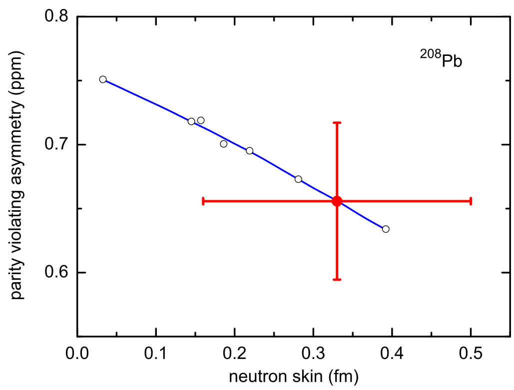

Figure 1. Neutron skin thickness of ^{208}Pb from an electron scattering parity-violating asymmetry measurement (Ref. [8], solid circle with error bars). Mean-field predictions are shown as open circles. Figure adapted from Ref. [8].

electron scattering a parity-violation asymmetry measurement for all practical purposes exclusively probes the neutron distribution of a nucleus. An initial result [8] from this type of experiment is displayed in Figure 1. Clearly the existing data suffers from poor statistics, leading to a disappointingly large uncertainty in the measured neutron skin thickness. Fortunately a follow-up experiment to improve on the deficiency in the experimental uncertainty has been approved at Jefferson Lab [9].

3 Comparison of an EoS α-Clustering Prediction with the Values from the Reaction Sn(d,^{6}Li)Cd

As was already mentioned, Typel *et al.* [1] introduce light-ion clusters in a generalized relativistic mean-field model in their development of an EoS. This leads to an interplay between conditions of the nuclear medium and cluster response to predict [1] a trend with isotopic mass number of α-cluster formation on the surface of Sn isotopes. This is shown in Figure 2. As would be expected, α-cluster densities are much smaller than those of the nucleons. The neutron distributions extend to larger nuclei radii with increasing neutron excess of larger atomic mass number of Sn isotopes. Correlated with this trend, the position of the maximum

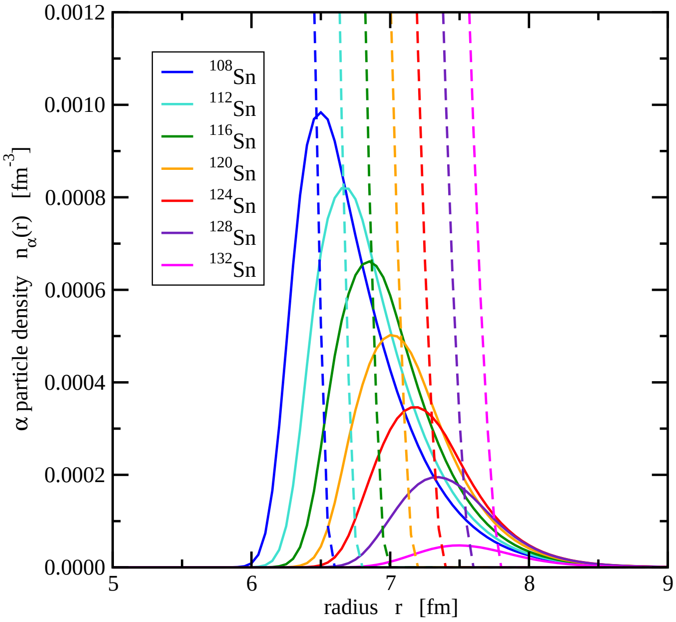

Figure 2. Radial density distribution of α-clusters (continuous curves) and neutrons (dashed curves) for isotopes of Sn. Figure from Ref. [2] reproduced with permission of S. Typel. ©2014 American Physical Society.

in the α-cluster density also moves to larger radii, and simultaneously the maximum height of its distribution decreases significantly. The integrated yield of α-clusters at and near the surface therefore becomes smaller with increasing Sn mass. Consequently a comparison of the predicted trend with experimental α-cluster pickup is of interest.

For α-particle pickup in the Sn(d,^{6}Li)Cd reaction, the relationship between the experimental cross section $d\sigma_{\rm exp}(\theta)/d\Omega$ and the quantity $d\sigma^{DW}(\theta)/d\Omega$ calculated in a zero-range DWBA [10] at a scattering angle θ is expressed as

$$\frac{d\sigma_{\rm exp}(\theta)}{d\Omega} = \mathcal{N}\frac{S_\alpha}{2J+1}\frac{d\sigma^{DW}(\theta)}{d\Omega}, \tag{4}$$

where S_α is the spectroscopic factor, which is a measure of the cluster preformation in the target nucleus. The quantity $\mathcal{N}$ is a normalization factor, which is both model dependent as well as affected by the implementation of the DWBA theory. This problem is illustrated in Figure 3, where two different analyses of the same (d,^{6}Li) experimental data are found to give relative spectroscopic factors which differ by a factor of 4 for all target nuclei. As indicated, the trends with target mass are not affected appreciably - only the absolute magnitudes differ seriously.

Jänecke *et al.* [3] extract absolute spectroscopic factors by exploiting the fact that α-decay and pickup from ^{148}Sm need to provide a consistent clustering probability. The relevant expressions which relate the spectroscopic factor S_α in the ^{148}Sm(d,^{6}Li)^{144}Nd reaction and the decay reduced width $\gamma^2_\alpha(s)$ calculated

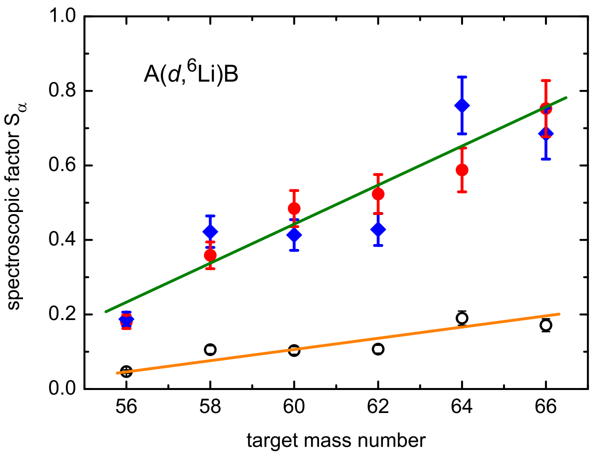

Figure 3. Comparison between relative spectroscopic factors extracted by Fullbright *et al.* [11] (open circles) and Anantaraman *et al.* [12] (closed circles). These values are extracted from a figure by Carey *et al.* [15]. The results from Ref. [11] multiplied by a factor of 4 are shown as diamond symbols. Lines serve to guide the eye.

for the same cluster bound state are

$$\gamma_\alpha^2(s) = \frac{\hbar^2 s}{2\mu} S_\alpha \left|R^{DW}(s)\right|^2, \quad (5)$$

$$\theta_\alpha^2(s) = \gamma_\alpha^2(s)/\gamma_W^2(s) = \frac{1}{3} S_\alpha s^3 \left|R^{DW}(s)\right|^2, \quad (6)$$

$$\gamma_W^2(s) = \frac{3\hbar^2}{2\mu s^2}, \quad (7)$$

where $\theta_\alpha^2(s)$ is a dimensionless reduced width , and $\gamma_W^2(s)$ is the Wigner limit (see Ref. [3]). The channel radius s is chosen by Jänecke *et al.* [3] as the position at which the α-cluster is preferentially picked up, namely $s = 1.7A^{1/3}$ in their application. The quantity μ is the α-particle reduced mass and $R^{DW}(r)$ is the radial part of the normalized α-cluster bound-state wave function.

The procedure employed by Jänecke *et al.* suggests that we may use any of their extracted quantities, S_α, $\gamma_\alpha^2(s)$ or $\theta_\alpha^2(s)$, as a measure of the of the amount of predicted [2] α-particle correlations in Sn target nuclei.

In Figure 4 theoretical predictions of the EoS for the number of α-clusters (solid curve; left Y-axis scale) as a function of atomic mass number of Sn are compared with the dimensionless reduced widths (symbols, right Y-axis scale) extracted by Jänecke *et al.* [3] for the pickup reaction $^{112-124}$Sn(d,^{6}Li)$^{108-120}$Cd. The relationship of scale for the experimental reduced widths, as plotted relative to the number of clusters on the left-hand scale, implies an arbitrarily normal-

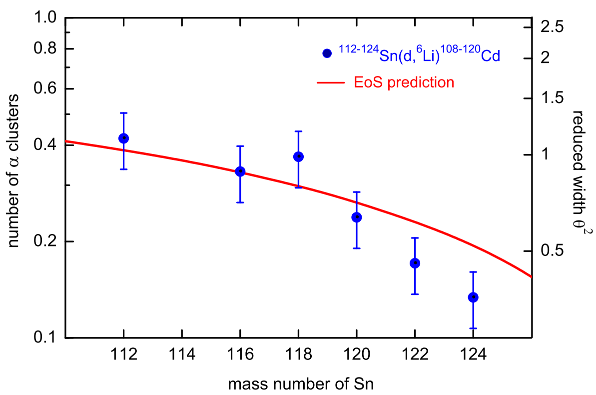

Figure 4. Dependence of the number of α-clusters (solid curve; left Y-axis scale) as a function of atomic mass number of Sn as predicted by Typel [2]. The symbols are dimensionless reduced widths (right Y-axis scale) extracted by Jänecke *et al.* [3] for the reaction $^{112-124}$Sn(d,^{6}Li)$^{108-120}$Cd at an incident energy of 33 MeV. The experimental reduced widths are arbitrarily normalized to the theoretical prediction at mass number 116 for comparison of the mass-dependent variation by displacing the scales on the left and right Y-axes appropriately. Note the logarithmic scale on both Y-axes. (Figure reproduced from Cowley [4]).

ization to the theoretical value at mass number 116. Other minor details of the comparison are discussed in Ref. [4]. Note that apart from a very slight mass dependence, for our purpose in which we introduce an arbitrary renormalisation of the pickup yield, an evaluation of the constant $\mathcal{N}$ in Eq. 4 is not strictly required.

Clearly the trend of the experimental cluster probability is reproduced well by the EoS prediction.

4 Quasifree Proton-Induced α-Cluster Knockout from Sn Isotopes

The quasifree (p,$p\alpha$ reaction to the ground state of the residual nucleus offers a convenient technique to measure the ground-state cluster preformation of Sn isotopes. It provides superior results compared to a pickup reaction, because issues such as the structures of the projectile and the composite outgoing particle in transfer reactions are not a concern. Furthermore, three-body in kinematics in knockout can be adjusted to achieve momentum matching between the incident and exit channels. This removes much of the model dependence experienced in transfer reactions, and consequently absolute spectroscopic values can be extracted.

The notation of Chant and Roos [13] formulates a knockout reaction as A(a, cd)B. For a $(p, p\alpha)$ reaction we have $a = c$ and the bound knocked-out cluster $b = d$. The general structure of the cross section becomes evident in a plane wave impulse approximation (PWIA) as

$$\frac{d^3\sigma}{d\Omega_c d\Omega_d dE_c} = S_b F_k \frac{d\sigma^{p\alpha}}{d\Omega} \left|\psi\right|^2 \ , \tag{8}$$

where $d\Omega_c$ and $d\Omega_d$ are the solid angles of observation of the light ejectiles. E_c is a kinetic energy, S_b is a spectroscopic factor, and F_k is a kinematic factor. The quantity $d\sigma^{p\alpha}/d\Omega$ is a half-shell two-body cross section that describes the scattering of the projectile from the bound α–cluster, and ψ is the Fourier transform of the radial wave function of the bound particle.

The PWIA ignores not only the interaction of the projectile with the core of the target system, but also final state interactions between the outgoing light products with the residual nucleus. This is clearly unrealistic.

In a proper DWIA formulation, the cross section [14] is expressed as

$$\begin{aligned} &\frac{d^3\sigma}{d\Omega_c d\Omega_d dE_c} \\ &= \quad S_b F_k \sum_{\rho_c' L\Lambda} \left| \sum_{\substack{\rho_a \sigma_a \\ \sigma_c \sigma_c'}} D^{s_a}_{\rho_a \rho_a'}(R_{ap}) \times D^{s_a^*}_{\sigma_c \sigma_c'}(R_{ac}) T^{L\Lambda}_{\substack{\sigma_a \sigma_c' \\ \rho_a \rho_c'}} \langle \sigma_c | t | \sigma_a \rangle \right|^2 \ , \end{aligned} \tag{9}$$

where the D's are rotation matrices and the t-matrix for the two-body scattering is denoted by $\langle \sigma_c \left| t \right| \sigma_a \rangle$. The quantity $T^{L\Lambda}_{\sigma_a \sigma_c' \rho_p \rho_c'}$ contains the overlap of the various distorted waves with the target structure.

As may be seen from Eq. 9 the cross section does not factorize, but if spin-orbit forces in the distorted waves are ignored, an expression [14] which resembles the factorized form of Eq. 8 is obtained. With this simplification the quantity $|\psi|^2$ is a convolution of the distorted waves with the bound-state wave function [13]. The resulting approximate DWIA cross section expression serves as guidance for the design of a knockout experiment.

The experiment of Carey *et al.* [15] of quasifree (p,$p\alpha$) knockout over a large range of target masses at an incident energy of 100 MeV serves as a useful benchmark to study groud state clustering in the Sn isotopes. Carey *et al.* selected a geometry of coplanar quasifree angle pairs to minimise processes of inelastic scattering which lead to α-particle emission. Such so-called sequential decay events intefere with extraction of reliable spectroscopic factors.

5 Influence of Clusters in Pre-Equilibrium Reactions

The reaction mechanism of pre-equilibrium emission of α-particles into the continuum induced by medium energy protons is well understood as a statistical multistep process consisting of a few nucleon-nucleon collisions preceding the final emission of an α-particle (see for example Ref. [16]. These pre-equilibrium (p,α) studies indicate that the terminating process is a combination of knockout and three-nucleon pickup [17]. Although the multistep character of the pre-equilibrium reaction observed in angular distributions is successfully reproduced by statistical theories such as that of Feshbach, Kerman and Koonin (FKK) [18], a reasonable explanation for the interplay between the competing knockout and pickup processes in the final stage of the reaction has only recently been presented [19].

Cowley *et al.* [19] conclude that the contribution of the knockout mechanism to the total cross section of the ^{93}Nb(p,α) reaction at incident energies between 65 MeV and 160 MeV is strongly related to the large momentum mismatch between the entrance and exit channels. Assuming only small to zero orbital angular momenta available for knockout, its cross section drops off nearly exponentially with increasing incident energy. The three-nucleon pickup mechanism, on the other hand, dominates at the intermediate incident energy of $\sim$ 100 MeV where the momentum mismatch, as a result of optimum angular momentum transfer, is effectively zero. The combined effect is an enhanced contribution of the knockout mechanism at the lower (65 MeV) and higher (160 MeV) projectile energies and an almost equal mixture of knockout and pickup in between.

The measurement of the analysing power as function of scattering angle is also a powerful tool to examine the interplay between the two terminating processes. Bonetti *et al.* [20], for example, show that for ^{58}Ni(p,α) at an incident energy of 72 MeV the sign of the analysing power for knockout in the final step is opposite to that of knockout, whereas the cross sections do not show a distinguishable difference.

It has generally been accepted until recently that cross section angular distributions of pre-equilibrium (p,α) reactions on are not very sensitive to the choice of target, apart from a gradual increase in integrated yield with target mass. However, recent insight, as discussed above, suggests otherwise. For example, the differing amounts of α-clustering, as predicted by Typel [2] for individual Sn isotopes should influence the magnitudes of the pre-equilibrium cross sections for the $^{112-126}$Sn$(p,\alpha)^{108-120}$Cd reactions.

In Figure 5 the experimental yield [21], as a function of target mass number is shown for the (p,α) pre-equilibrium reaction at a forward scattering angle of 25° and at an incident energy of 150 MeV. There is a general increase of yield with target mass, but there is also a hint of differences in isotopic yield, for example for Sn. Superimposed on the experimental trend, the prediction of normalised numbers of Typel for α are also presented. Of course, with competition between knockout and pickup in a pre-equilibrium reaction, the effect change of α-clustering in the target will not be directly reflected in a decrease of cross section with target mass. Nevertheless, Figure 5 gives a general idea of the expected qualitative relationship.

An experiment to study the influence of clustering on the surface of Sn isotopes in pre-equilibrium reactions has recently been approved at iThemba LABS [22].

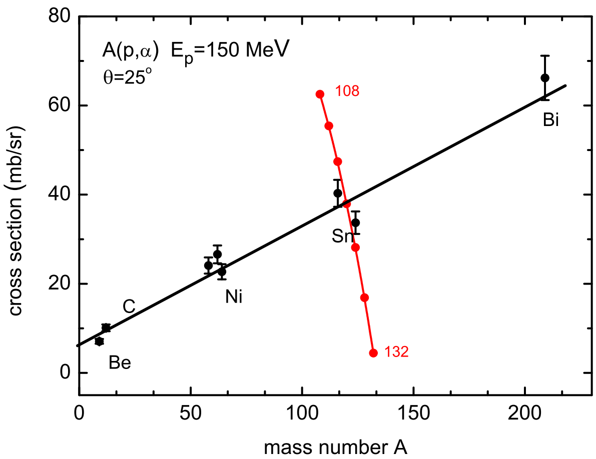

Figure 5. Emission-energy integrated experimental data [21], as a function of target mass number, for the (p,α) pre-equilibrium reaction at a scattering angle of 25° and at an incident energy of 150 MeV (solid circles with error bars). The numbers of α-clusters for Sn isotopes predicted by Typel [2], appropriately normalised to the experimental trend, are shown as smaller solid circles (symbols without error bars). The range of Sn isotopes is the same as in Figure 2, but only the extreme mass numbers are indicated in the figure. Lines serve to guide the eye.

6 Summary and Conclusion

The implications of an equation of state (EoS) formulation with inclusion of light ion clusters and prediction of ground-state clustering in a large range of Sn isotopes was assessed. It was shown how the EoS isotopic trend for the number of α-clusters on the surface of Sn accurately follows the values extracted from existing data of the (d,^{6}Li) pickup reaction at an incident energy of 33 MeV.

Quasifree knockout to confirm the pickup numbers, as well as details of how the EoS prediction should manifest its influence in pre-equilibrium (p,α) reaction on Sn isotopes were reviewed.

In view of the importance of EoS formulations to current topics in nuclear physics, astrophysics and cosmology, it is clearly very useful to investigate experimentally explicit cluster predictions of the theory.

Acknowledgements

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