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Coupled-Channels Analyses on ^{40}Ar + ^{176,178,180}Hf Heavy-Ion Fusion Reactions

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Abstract. Heavy-ion collisions are typically characterized by the presence of many open reaction channels. In the energies around the Coulomb barrier, the main processes are elastic scattering, inelastic excitations of low-lying modes and fusion operations of one or two nuclei. The fusion process is generally defined as the effect of one-dimensional barrier penetration model, taking scattering potential as the sum of Coulomb and proximity potential. We have performed heavy-ion fusion reactions with coupled-channels (CC) calculations. Coupled- channels formalism is carried out under barrier energy in heavy- ion fusion reactions. In this work fusion cross sections have been calculated and analysed in detail for the three systems ^{40}Ar + ^{176,178,180}Hf in the framework of coupled-channels approach, using the codes CCFULL, CCFUS and CCDEF and compared with experimental data. CCFULL and CCDEF explains the fusion reactions of heavy-ions very well, while using the scattering potential as WOODS- SAXON volume potential with Akyuz-Winther parameters. It was observed that AWpotential parameters are able to reproduce the experimentally observed fusion cross sections reasonably well for these systems. There is a good agreement between the calculated results and the experimental results [19].

1 Introduction

Heavy-ion collisions in low energy range at, above and below the Coulomb get attracts both experimentalists and theorists. It has been a very rich variety of phenomena for many years in Nuclear Physics. The fusion of two nuclei at very low energy is an example of tunnelling phenomena in nuclear physics. These reactions are not only of central important for stellar energy production and nucleosynthesis, but they also provide new insights into reaction dynamics and nuclear structure. To analyze heavy-ion fusion cross sections above the Coulomb barrier , the inter-nuclear interaction, a combination of the repulsive, long range Coulomb, centrifugal potentials and the attractive , short range nuclear potential plays a major role. The total potential attains a maximum value at a distance beyond the touching configuration where the repulsive Coulomb force and the attractive nuclear forces balance each other and when the energy of relative motion overcomes this potential barrier, the nuclei gets captured and fused.

Many different phenomena take place in heavy-ion collisions depending on the bombarding energy, the impact parameter, the mass number of the target and projectile. We discuss here fusion reactions at energies near and below the Coulomb barrier. The fusion cross section in heavy-ion collisions at energies somewhat above the Coulomb barrier can be well accounted for by a simple potential model, which explicitly deals with only the relative distance between the projectile and target or a model supplemented by a friction [1]. After comprehensive theoretical as well as experimental studies, it is pretty good now set that the large enhancement of the fusion cross section is caused by the coupling of the relative motion between the colliding nuclei to other degrees of freedom, [10,11]. They are called channel-coupling effects [1].

The aim of the present work to investigate the effectt of the coupled-channels caalculations using codes CCFULL, CCFUS, CCDEF. The fusion excitation functions for the fusion of ^{40}Ar + ^{176,178,180}Hf have been calculated using onde dimensional barrier penetration model, taking scattering potential as the sum of Coulomb and proximity potential and the calculated values of ^{40}Ar + ^{176,178,180}Hf are compared with experimental data from ‘nrv [19]. Reduced reaction cross sections for the fusion have been also described.

2 Coupled Channels Formalism for Heavy-Ion Fusion Reactions

The coupled-channels calculatons are not only important for theoretical point of view, it is also necessary for obtaining new data. The starting point to figure fusion reactions at energies below and slightly above the barrier where the coupling effects are the strongest and the number of channels coupled are workable. At energies high above the barrier so many channels become involved that it is no longer possible to treat them individually. For simplicity we consider only coupling to inelastic channels, ignoring rearrangement processes such as transfer. For a brief description of the coupled-channels formalism, the total Hamiltonian of the system can be written as:

$$H = \frac{\hbar^2}{2\mu}\nabla^2 + h\left(\xi\right) + V_0\left(r\right) + V_{\text{coup}}(r,\xi)\,, \tag{1}$$

where $h()$ is the internal Hamiltonian for the target nucleus here stands for the internal dynamical variables. $V_{\text{coup}}(r,)$ is the coupling term, introduces the coupling betweeen the relative motion and the internal degrees of freedom. $V_0(r)$ is the bare potential in the relative distance r. For the total wave function $(r,)$ the Schrödinger equation

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + h\left(\xi\right) + V_0(r) + V_{\text{coup}}(r,\xi)\right)\psi(r,\xi) = E\psi(r,\xi)\,. \tag{2}$$

The internal eigenstates $\phi_b()$ with eigenvalues can be presented as

$$(\xi)\,\phi_b\left(\xi\right) = {}_b\phi_b\left(\xi\right) \tag{3}$$

and also matrix elements are

$$V_{bc}(r) = \int d\xi \phi_b^*(\xi) V_{\text{coup}}(r,\xi)\phi_c(\xi)\,. \tag{4}$$

The coupling matrix elements V_{bc} will consist of Coulomb and nuclear components. In terms of the internal eigenstates the total wave function $(r,)$ is

$$\psi(r,\xi) = \sum_b \varphi_b(r)\phi_b(\xi)\,. \tag{5}$$

And finally for the channels wave function coupled-channels equations can be defined from the Schrödinger equation

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + V_{bb}(r) + \varepsilon_b + E\right)\varphi_b(r) = \sum_{c(c\neq b)} V_{bc}(r)\varphi_c(r)\,. \tag{6}$$

This system of coupled Schrödinger equations should be modified according to the boundary condition since the solution of the event channel consists of an input and an output wave, while all other channels contain only outgoing waves. The outgoing wave's coefficients determine the cross-sections of the various reactions. In practical applications imaginary potentials are introduced into the coupled-channels equations to account for the bulk of the flux which is lost from the direct channels to the compound reaction. Such a model is useful for calculating direct reactions. The fusion cross section can also be calculated from the difference between the total flux lost from the entrance channel and the flux which appears in the direct reaction channels [6,7]. However, this approach is not suited to study the channel coupling effects on barrier penetration since imaginary potentials suppress the coupling which acts as the nuclei inter-penetrate. The artificial limitations resulting from using strongly absorbing imaginary potentials in the coupled equations can be removed by imposing an ingoing boundary condition (IWBC) to account for the flux which leads to fusion. The IWBC form is given by Rawitscher [5] is

$$\varphi_b(r) \propto \frac{1}{\sqrt{k_b(r)}} \exp\left[-i\int_{R_b}^{r} k_b(r')dr'\right]. \tag{7}$$

k_b is the asymptotic wave number in the channel band, R_b is the starting point of the integration inside the Coulomb barrier.

The coupled equations solved under ingoing-wave boundary conditions provide a more realistic framework for describing fusion reactions. Within the coupled-channels formalism one determines the total reaction cross section and the total cross section in the excited channels. This difference is equal to the ingoing flux at R_b and is equated with the fusion cross section. The IWBC is applied at some point inside the barrier to obtain the S-matrices. The fusion cross section is then obtained as

$$\sigma = \frac{\pi}{k^2} \sum_l (2l+1) \left(1 - \sum_l |S_l(l)|^2 \right) . \quad (8)$$

Here the parameter l defines different scattering channels [21,22].

3 The Fusion Cross Section

With the view of the one-dimensional formula developed by C.Y. Wong, the partial wave fusion cross section can be given at energies not too much above the barrier and alsa at higher energies by

$$\sigma_f = \frac{\pi}{k^2} \sum_l (2l+1) T_l^f , \quad (9)$$

where $k = \sqrt{\frac{2\mu E}{\hbar^2}}$. Here T_l^f is the tunnelling probability and it is based on parabolic approximation as suggested by Hill and Wheeler;

$$T_l^{HW} = \frac{1}{\left[1 + \exp \frac{2\pi}{\hbar w_l (V_l - E)} \right]} , \quad (10)$$

where w_l is the curvature of the inverted parabola.

To obtain a simpler form for this parabolic approximation Wong used certain assumptions for barrier position, curvature and height and so obtained the final form the fusion cross section as given by the following formula:

$$\sigma_f = \frac{\hbar w}{2E} R_b^2 \ln \left[1 + \exp \frac{2\pi}{\hbar w_l} (E - V_b) \right] . \quad (11)$$

For more much values of E, the formula takes the form of well-known [8]

$$\sigma = \pi R_0^2 \left[1 - \frac{E_0}{E} \right] . \quad (12)$$

4 The Reduced Reaction Cross Section

The coupled-channels method is quite useful in treating collective excitations enhanced in nuclear scattering. To make a comparison between excitation functions which have differences in reaction mechanism; such as different projectiles on the same target nucleus; the procedure is to eliminate the geometrical factors concerning different systems by reducing the cross section and the centre of mass energy. The reducing process is mainly occurs with the division of the cross section by πR_0^2, here R_0 is the barrier radius and division of energy by Coulomb barrier E_0 [4,9].

5 Results and Discussions

The results of coupled-channels calculations are performed by using CCFULL, CCDEF, CCFUS ; and compared with the experimental data. In all coupled-channels codes calculations the lines represent calculations the vibrational couplings in projectile and target. The depth parameter V_0 and the surface diffuseness parameter a_0 (of the Wood-Saxon potentials), radius parameterr_0 have been computed and the values are shown in Table 1.

Table 1. List of depth parameter V_0 and surface diffuseness parameter a_0

Reaction	V_0	a_0
^{40}Ar + ^{176}Hf	79.015	0.684
^{40}Ar + ^{178}Hf	79.015	0.,684
^{40}Ar + ^{180}Hf	79.017	0.684

In codes we include the lowest states for all the reactions, that is the 2^+ (quadrupole) and 3^- (octupole) states. For ^{40}Ar + ^{176}Hf reaction $V_0 = 79.015$, $a_0 = 0.684$ fm and $r_0 = 1.1$ fm are the parameters. For ^{40}Ar + ^{178}Hf reaction $V_0 = 79.015$, $a_0 = 0.684$ fm and $r_0 = 1.1$ fm, and for the reaction ^{40}Ar + ^{180}Hf $V_0 = 79.017$, $a_0 = 0.684$ fm and $r_0 = 1.1$ fm.

We investigate here the roles of parameters and which parameters are more effective for the reactions using different codes. The results of coupled-channels calculations are compared with the experimental data. The computed cross sections with codes show good agreement with experimental data.

Table 2. Deformation parameters and excitation energies of 3^- states of ^{176}Hf, ^{178}Hf, ^{180}Hf projectiles

Target	Deformation parameter	Excitation energy
^{176}Hf	0.057	1.313
^{178}Hf	0.026	1.374
^{180}Hf	0.026	1.374

The results of coupled-channels calculations with codes are compared with the experimental data in Figure 1, Figure 2 and Figure 3 for three systems ^{40}Ar + ^{176,178,180}Hf. Figure 1 shows that all three codes well reproduce the experimental fusion cross section. As seen the data come from CCFULL code has the best agreement with experimental data at all energy scale. The code CCDEF is also reproduce the experimental data well at and above the middle energy range. However, the last code CCFUS is a bit out of the scale according to the other codes. We concluded that; below the barrier larger deformations corresponds to large sub barrier enhancement of fusion cross section. It can be added that the fusion process is as a tunnelling process below the barrier. It has

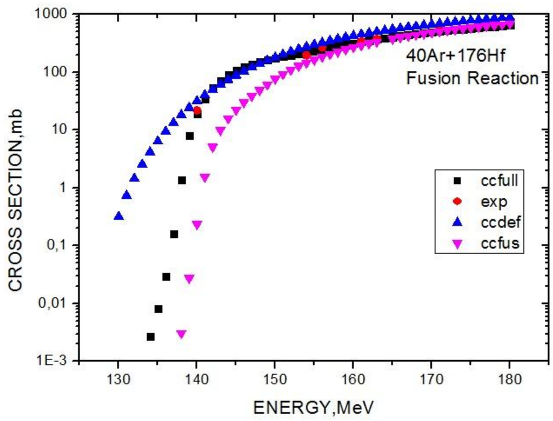

Figure 1. Fusion excitation functions for the reaction ^{40}Ar + ^{176}Hf with CCFUS, CCDEF, CCFULL codes, the comparison with experimantal data.

been found that the most utilizable code is CCFULL and CCDEF. CCFULL is a code thats for coupled-channels calculations with all order couplings for heavy-ion fusion reactions. It takes into account the effects of nonlinear couplings to all orders. And for CCDEF it can be noticed that, the difference from the codes using the coupled-channels method is that the projectile and the target nucleus are deformed. If parameters are meticulously calculated, the best fit to the experimental result is achieved with this codes. CCFUS needs less parameter according to others. So when we look at the graphics we see that it is out of the scale.

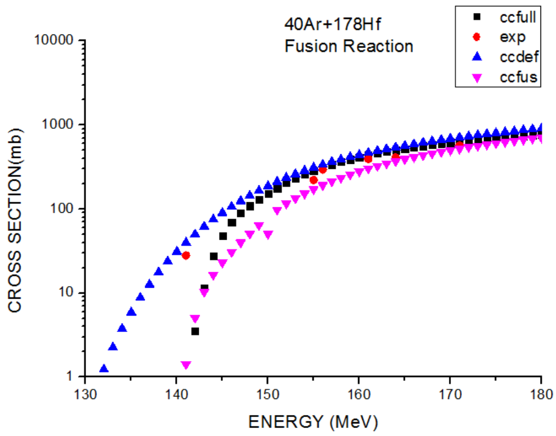

Figure 2. For the reaction ^{40}Ar + ^{178}Hf the comparison of the coupled-channels calculations with CCFUS, CCDEF, CCFULL codes, and experimental data.

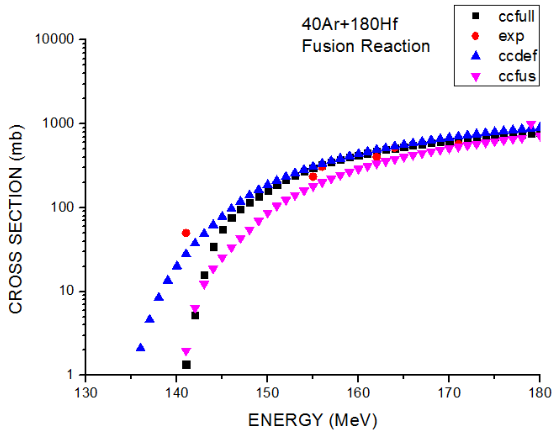

Figure 3. Fusion excitation functions with coupled-channels code CCFUS, CCDEF, CCFULL for the reaction ^{40}Ar + ^{1806}Hf figure shows the comparison of the code's calculations with experimental data.

6 Conclusions

We have investigated the effect of coupled-channels for heavy-ion fusion reactions of the ^{40}Ar + ^{176,178,180}Hf reactions using coupled-channels codes CCFULL, CCDEF and CCFUS. The codes use the scatterring potential as the sum of Coulomb and proximity potentials. Calculated results are compared with experimental data, including excitation of the projectile and target to the lowest $2+$ and 3^- states and with the data computed from CCDEF, CCFULL explains the fusion reactions of heavy-ions very well, while using the scattering potential as WOODS- SAXON volume potential with Akyuz-Winther parameters. It was observed that AW potential parameters are able to reproduce the experimentally observed fusion cross sections reasonably well for these systems. There is a good agreement between the calculated results with the experimental results.

We concluded that below the barrier larger deformations corresponds to large sub barrier enhancement of fusion cross section. It can be added that the fusion process is as a tunnelling process below the barrier.

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