

Temperature Dependence of the Symmetry Energy and Neutron Skins in Ni, Sn, and Pb Isotopic Chains

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Abstract. The symmetry energy and its temperature dependence for isotopic chains of even-even Ni, Sn, and Pb nuclei is studied in the framework of the local density approximation (LDA). The Skyrme energy density functional with two Skyrme-type effective interactions, SkM* and SLy4, is used in the calculations. The temperature-dependent local proton and neutron densities, as well as the kinetic energy densities, are calculated through the HFBTHO code that solves the nuclear Skyrme-Hartree-Fock-Bogoliubov problem by using the cylindrical transformed deformed harmonic-oscillator basis. In addition, two other density distributions of ²⁰⁸Pb, namely the Fermi-type density determined within the extended Thomas-Fermi (TF) method and symmetrized-Fermi local density obtained within the rigorous density functional approach, are used. Alternative ways to calculate the symmetry energy coefficient within the LDA are proposed. The results for the thermal evolution of the symmetry energy coefficient in the interval $T = 0-4$ MeV show that its values decrease with temperature. The temperature dependence of the neutron and proton root-mean-square radii and corresponding neutron skin thickness is also investigated. The results show that the larger temperatures lead to a substantial increase of the neutron radii and skins.

1 Introduction

The nuclear symmetry energy (NSE), as a fundamental quantity in nuclear physics and astrophysics, represents a measure of the energy gain in converting asymmetric nuclear matter to a symmetric system [1–3]. Its value depends on the density ρ and temperature T . Experimentally, the nuclear symmetry energy is not a directly measurable quantity and is extracted indirectly from observables that are related to it (e.g., [4, 5]). A sensitive probe of the nuclear symmetry

energy is the neutron-skin thickness of nuclei (see, for example, Ref. [6] and references therein).

In the present work, apart from the ρ -dependence investigated in our previous works [7–9], we study also the temperature dependence of the symmetry energy in finite nuclei within the local density approximation [10–13] with some modifications (see also our paper [14]). We explore the thermal evolution of the symmetry energy coefficient for Ni, Sn, and Pb isotopic chains in the interval $T=0$ –4 MeV using different model temperature-dependent local density distributions for these nuclei. The temperature-dependent densities of these nuclei are calculated within a self-consistent Skyrme-HFB method using the cylindrical transformed deformed harmonic-oscillator basis (HFBTHO densities) [15, 16]. The kinetic energy density is calculated either by the HFBTHO code or by the TF expression up to T^2 term [17]. We have used two parametrizations of the Skyrme force, namely, SLy4 and SkM*, which were able to give an appropriate description of bulk properties of spherical and deformed nuclei in the past. In addition, we present some results for the ^{208}Pb nucleus with densities obtained within the ETF method [18, 19] and the rigorous density functional approach (RDFA) [20]. The effect of temperature on the rms radii of protons and neutrons and the formation of neutron skin in hot nuclei is also analyzed and discussed.

2 Theoretical Formalism

For finite systems, different definitions of the symmetry energy coefficient and its temperature dependence are considered in the literature. In the present work we develop an approach to calculate the symmetry energy coefficient for a specific nucleus starting with the LDA expression given in [10, 11]:

$$e_{sym}(A, T) = \frac{1}{I^2 A} \int \rho(r) e_{sym}[\rho(r), T] \delta^2(r) d^3r. \quad (1)$$

In Eq. (1) $I = (N - Z)/A$, $e_{sym}[\rho(r), T]$ is the symmetry energy coefficient at temperature T of infinite nuclear matter at the value of the total local density $\rho(r) = \rho_n(r) + \rho_p(r)$, $\delta(r) = [\rho_n(r) - \rho_p(r)]/\rho(r)$ is the ratio between the isovector and the isoscalar parts of $\rho(r)$, with $\rho_n(r)$ and $\rho_p(r)$ being the neutron and proton local densities. The symmetry energy coefficient $e_{sym}(\rho, T)$ can be evaluated in different ways. Following Refs. [10, 13], we adopt in this work the definition

$$e_{sym}(\rho, T) = \frac{e(\rho, \delta, T) - e(\rho, \delta = 0, T)}{\delta^2}, \quad (2)$$

where $e(\rho, \delta, T)$ is the energy per nucleon in an asymmetric infinite matter, while $e(\rho, \delta = 0, T)$ is that one of symmetric nuclear matter. These quantities are expressed by $e = \mathcal{E}(r, T)/\rho$, where $\mathcal{E}(r, T)$ is the total energy density of the system. The Skyrme energy density functional (its part for infinite homogeneous

nuclear matter) that we use in our work has the form:

$$\begin{aligned} \mathcal{E}(r, T) = & \frac{\hbar^2}{2m_{n,k}}\tau_n + \frac{\hbar^2}{2m_{p,k}}\tau_p \\ & + \frac{1}{2}t_0 \left[\left(1 + \frac{1}{2}x_0\right)\rho^2 - \left(x_0 + \frac{1}{2}\right)(\rho_n^2 + \rho_p^2) \right] \\ & + \frac{1}{12}t_3\rho^\alpha \left[\left(1 + \frac{x_3}{2}\right)\rho^2 - \left(x_3 + \frac{1}{2}\right)(\rho_n^2 + \rho_p^2) \right]. \end{aligned} \quad (3)$$

In Eq. (3) $t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3$, and α are the Skyrme parameters. We use in this work the interactions SkM* [21] and SLy4 [22]. The nucleon effective mass $m_{q,k}$ is defined through

$$\begin{aligned} \frac{m}{m_{q,k}(r)} = & 1 + \frac{m}{2\hbar^2} \left\{ \left[t_1 \left(1 + \frac{x_1}{2}\right) + t_2 \left(1 + \frac{x_2}{2}\right) \right] \rho \right. \\ & \left. + \left[t_2 \left(x_2 + \frac{1}{2}\right) - t_1 \left(x_1 + \frac{1}{2}\right) \right] \rho_q \right\}, \end{aligned} \quad (4)$$

with $q = (n, p)$ referring to neutrons or protons. The dependence on temperature of $\mathcal{E}(r, T)$ [Eq. (3)] and $m/m_{q,k}(r)$ [Eq. (4)] comes from the T -dependence of the densities and kinetic energy densities.

A self-consistent approach based on the simultaneous treatment of temperature-dependent density distributions and kinetic energy density is related to the finite temperature formalism for the HFB method. In it the nuclear Skyrme-HFB problem is solved by using the cylindrical transformed deformed harmonic-oscillator basis [15]. The HFBTHO code based on the mentioned approach is used in our numerical calculations.

There exist various methods to obtain the kinetic energy density $\tau_q(r, T)$ entering the expression for $\mathcal{E}(r, T)$ [Eq. (3)]. One of them is, as mentioned above, to use the HFBTHO code. Another way is to use the TF approximation adopted in Ref. [10], or an extension of the TF expression up to T^2 terms valid for low temperatures [17]:

$$\tau_q(r, T) = \frac{2m}{\hbar^2}\varepsilon_{K_q} = \frac{3}{5}(3\pi^2)^{2/3} \left[\rho_q^{5/3} + \frac{5\pi^2 m_q^2}{3\hbar^4} \frac{1}{(3\pi^2)^{4/3}} \rho_q^{1/3} T^2 \right]. \quad (5)$$

In Eq. (5) the first term in square brackets is the degenerate limit at zero temperature and the T^2 term is the finite-temperature correction. We calculate the local density and the kinetic energy density using the self-consistent Skyrme-HFB method and the HFBTHO code. In addition, two other density distributions of ^{208}Pb [23], namely the Fermi-type density determined within the ETF method [18, 19] and the symmetrized-Fermi local density obtained within the rigorous density functional approach (RDFA) [20], are used.

In the present work we use the approach given by Eqs. (1) and (2), as well as the T -dependent Skyrme EDF [Eq. (3)] to calculate the symmetry energy coefficient. Here we note the specific problem that arises, namely how to calculate the term $e(\rho, \delta = 0, T)$ of Eq. (2) that is responsible for the contribution of the energy per particle of symmetric nuclear matter. Therefore, in our study aiming to investigate the temperature dependence of e_{sym} within a given isotopic chain, we introduced in [14] two other definitions of $e_{sym}(A, T)$ in LDA that, in our opinion, would be more appropriate in this case. They concern namely the above mentioned problem of calculating the term $e(\rho, \delta = 0, T)$ of Eq. (2) for symmetric nuclear matter. First, on the basis of Eqs. (1) and (2) with $e = \mathcal{E}(r)/\rho$, we present the integrand of the right-hand side of the following expression for $I^2 e_{sym}(A, T)$ as a difference of two terms with transparent physical meaning:

$$I^2 e_{sym}(A, T) = \int d\vec{r} \left[\frac{\mathcal{E}(\rho_A(r), \delta, T)}{A} - \frac{\mathcal{E}(\rho_{A1}(r), \delta = 0, T)}{A1} \right], \quad (6)$$

in which the first one corresponds to the energy per volume and particle of nuclear matter $\mathcal{E}(\rho_A(r), \delta, T)/A$ with a density $\rho_A(r)$ equal to that of the considered nucleus with A nucleons, Z protons and N neutrons from the given isotopic chain. The second term $\mathcal{E}(\rho_{A1}(r), \delta = 0, T)/A1$ is the analogous for the isotope with mass number $A1 = 2Z$ (with $N1$ neutrons and Z protons, $N1 = Z = A1/2$). For example, for the Ni isotopic chain the nucleus $A1$ is the double-closed shell nucleus ^{56}Ni ($Z = N1 = 28$), while for the Sn isotopic chain the nucleus $A1$ is the double-closed shell nucleus ^{100}Sn ($Z = N1 = 50$) and both ^{56}Ni and ^{100}Sn isotopes play a role of reference nuclei.

Our second new definition of $e_{sym}(A, T)$ using the LDA presents $e_{sym}(A, T)$ in the form:

$$I^2 e_{sym}(A, T) = \int \frac{d\vec{r}}{A} [\mathcal{E}(\rho_A(r), \delta, T) - \mathcal{E}(\rho_{\bar{A}}(r), N_1, Z_1, \delta = 0, T)], \quad (7)$$

in which the mass number $\bar{A} = A [A(Z, N)]$ is the same, but with different nucleon content, namely $\bar{A}(Z_1 = \bar{A}/2, N_1 = \bar{A}/2)$. This consideration requires the even-even nucleus with $N_1 = Z_1 = \bar{A}/2$ to be bound.

3 Results for Ni, Sn, and Pb Isotopic Chains and Discussion

Our analysis of the local density distributions $\rho(r)$ and their changes with respect to the temperature show that the densities decrease in the central part with the increase of the temperature. At the same time, it is observed that the nuclear surface becomes more diffuse with increasing T [14].

The results for the proton and neutron radii and their difference (neutron-skin thickness) as a function of the temperature T for selected ^{124}Sn , ^{132}Sn , and ^{152}Sn isotopes, are illustrated in Figure 1. The calculations are made by using SLy4 parametrization. In the temperature range $T=0-4$ MeV considered in the present work, we find a very slow increase of the proton radius compared to the

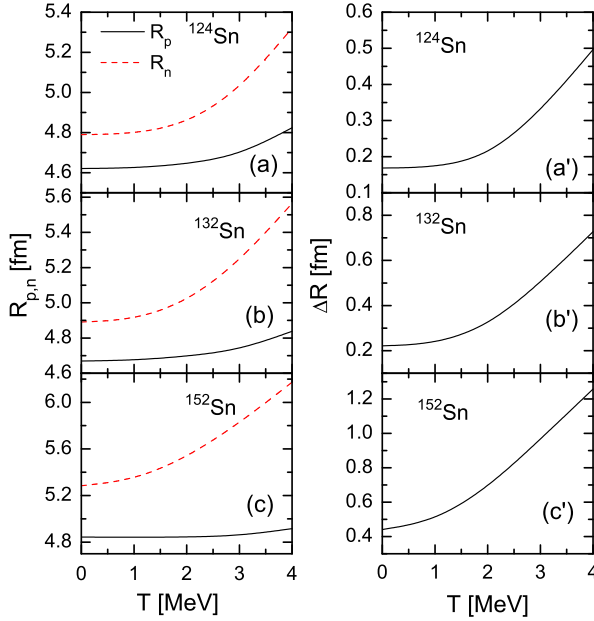


Figure 1. Left: Proton R_p (solid line) and neutron R_n (dashed line) radius of ^{124}Sn , ^{132}Sn , and ^{152}Sn isotopes with respect to the temperature T calculated with SLy4 interaction. Right: Neutron skin thickness ΔR for the same Sn isotopes as a function of T .

rapid increase of the neutron radius with the temperature. Here we would like to note that the use of SkM* interaction leads to results for the proton and neutron radii, as well as for the neutron skin thickness of the considered isotopes, very similar to those obtained by using of SLy4 Skyrme force.

In understanding the symmetry energy coefficient e_{sym} for finite nuclear systems and their thermal evolution, some ambiguities about their proper definition could be noted. First, we use the new definition of the symmetry energy coefficient e_{sym} given by Eq. (6) and the results for several nuclei from the three isotopic chains calculated with SkM* interaction are presented in Figure 2. They are obtained by simultaneous consistent treatment of both T -dependent nucleon densities and kinetic energy densities within the HFB method and computed by the HFBTHO code. As already noted, there exist difficulties in the calculations of the term $e(\rho, \delta = 0, T)$ of Eq. (2) for symmetric nuclear matter, namely, of using the reference case $\delta = 0$ when the nucleus with $Z = N1$ is unbound. Keeping this in mind, as an attempt, for Ni and Sn isotopes we take as reference nuclei ($A1$) the nuclei ^{56}Ni ($Z = N1 = 28$) and ^{100}Sn ($Z = N1 = 50$), respectively. The case of the Pb isotopic chain is even more difficult because the eventual nucleus of reference with $Z = N1 = 82$ is clearly unbound and there do not exist appropriate bound nuclei for the purpose. As a way to overcome

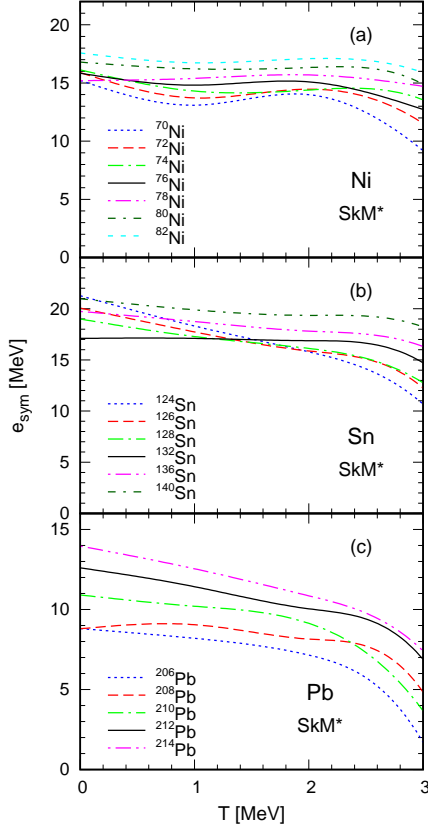


Figure 2. Temperature dependence of the symmetry energy coefficient e_{sym} obtained by using Eq. (6) for several nuclei from Ni ($A=70-82$) (a), Sn ($A=124-140$) (b), and Pb ($A=206-214$) (c) isotopic chains with SkM* force. The nucleon densities and kinetic energy densities used to calculate e_{sym} are consistently derived from HFBTHO code.

this difficulty, we try in this case to use again the ^{100}Sn as a reference nucleus with $Z = N_1 = 50$, normalized with $A_1 = 100$ in Eq. (6). The symmetry energy coefficient exhibits almost flat behavior for the double-magic ^{78}Ni and ^{132}Sn nuclei.

As a next step of our work, we give in Figure 3 (top panel) the results for the symmetry energy coefficient of five Ni isotopes obtained by using Eq. (7) and SkM* force. The same difficulties noted above at the discussion of the results presented in Figure 2 and obtained by using Eq. (6), appear in this case. We limited ourselves to these cases because the even-even nucleus with $N_1 = Z_1 = \bar{A}/2$ ($\bar{A} = A$) should be bound. This is possible only for Ni isotopes but not for Sn and Pb ones. For instance, in the case of Sn isotopes all the $N_1 = Z_1$ nuclei with \bar{A} ($\bar{A} = A$) starting at 124 ($N_1 = Z_1 = 62$) are unbound. So, we

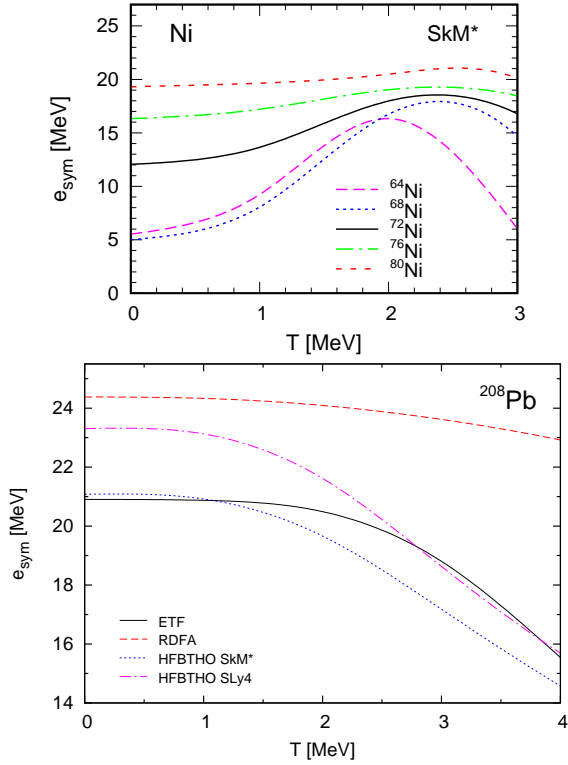


Figure 3. (Top) Temperature dependence of the symmetry energy coefficient e_{sym} obtained by using Eq. (7) for several nuclei from Ni ($A=64-80$) isotopic chain with SkM* force. The nucleon densities and kinetic energy densities used to calculate e_{sym} are consistently derived from HFBTHO code; (Bottom) Comparison of the results for the symmetry energy coefficient e_{sym} for ^{208}Pb calculated with ETF, RDFA, and HFB (with SkM* and SLy4 forces) densities. They are obtained by using Eqs. (1)-(4) and T^2 -approximation for the kinetic energy density [Eq. (5)].

consider the cases ^{64}Ni : $N_1 = Z_1 = 32$ (^{64}Ge), ^{68}Ni : $N_1 = Z_1 = 34$ (^{68}Se), ^{72}Ni : $N_1 = Z_1 = 36$ (^{72}Kr), ^{76}Ni : $N_1 = Z_1 = 38$ (^{76}Sr), ^{80}Ni : $N_1 = Z_1 = 40$ (^{80}Zr). In contrast to the results presented in Figure 2, the $e_{sym}(A, T)$ for the Ni isotopes calculated using Eq. (7) and shown in Figure 3 do not decrease smoothly and have a different behavior. The results obtained in both cases show a strong dependence of the symmetry energy coefficient for finite nuclei on the proper definition.

In the same Figure 3 (bottom panel) the results for ^{208}Pb obtained using Eqs. (1)-(5) with three different densities, namely those obtained within the ETF, RDFA, and HFB (with SkM* and SLy4 forces) methods are presented. The kinetic energy densities are obtained within TF method with T^2 term [Eq. (5)].

The results for the thermal evolution of the symmetry energy coefficient in the interval $T=0-4$ MeV show that its values decrease with temperature being larger in the case of symmetrized-Fermi density of ^{208}Pb obtained within the RDFA. As already discussed, the applications of different methods fail to give unique values for the symmetry energies for finite nuclei or their temperature dependence. Nevertheless, we would like to note that our results for e_{sym} are close to the result obtained within the LDA (in the version reported in Ref. [10]) and within the relativistic TF approximation in Ref. [24] for the same nucleus. The differences in the results can be referred to the different calculation ingredients (nucleon densities, kinetic energy density etc.) or the adopted procedure to obtain the symmetry energy coefficient.

For completeness, we performed a comparative analysis of e_{sym} for several isotopes from the same Ni, Sn, and Pb chains applying the LDA in the version based on Eqs. (1)-(4) (see Figure 4). The symmetric nuclear matter part of

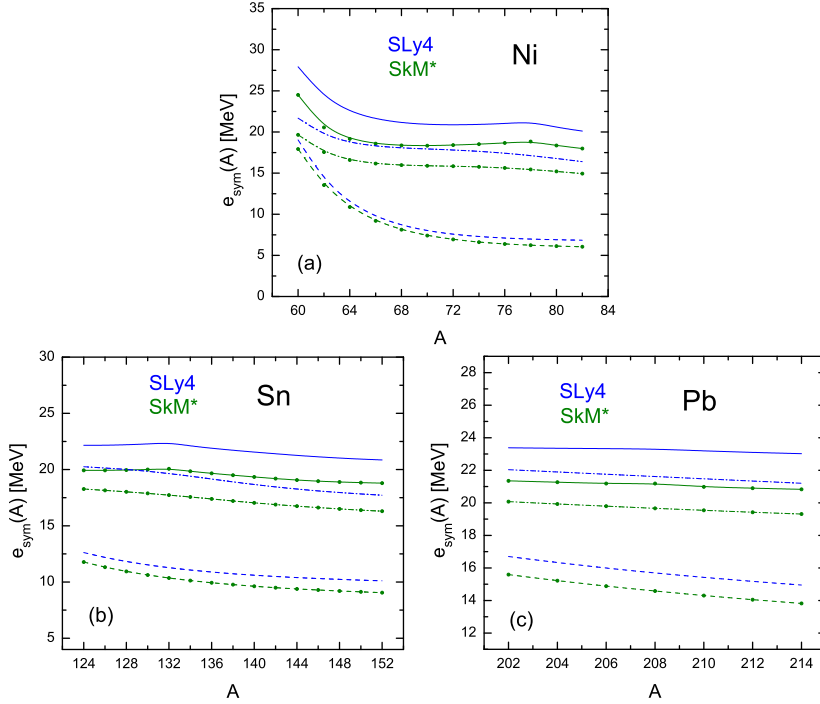


Figure 4. The mass dependence of the symmetry energy coefficient e_{sym} for Ni (a), Sn (b), and Pb (c) isotopic chains at temperatures $T = 0$ MeV (solid line), $T = 2$ MeV (dash-dotted line), and $T = 4$ MeV (dashed line) calculated with SLy4 (blue lines) and SkM* (green lines with points) Skyrme interactions. The results of e_{sym} are obtained by using Eqs. (1)-(4) with HFBTHO densities and T^2 -approximation for the kinetic energy density [Eq. (5)].

Eq. (2) $e(\rho, \delta = 0, T)$ is obtained approximately with densities $\rho_n = \rho_p = \rho/2$, where ρ is the total density calculated with the HFBTHO code. The kinetic energy density is from the TF method with T^2 term [17] in Eq. (5) calculated with the above densities. So, in this case $\tau_n \approx \tau_p$. Our results show an existence of a kink in the values of $e_{sym}(A)$ at zero temperature at the double-magic ^{78}Ni and ^{132}Sn nuclei as well as the lack of kinks in the Pb isotopic chain (see Figure 4 and Ref. [14]). These results confirm our previous observations when studying the density dependence of the symmetry energy for Ni, Sn, and Pb isotopes [7, 8]. We also note that in the cases of $e_{sym}(A)$ for Ni and Sn isotopic chains the kinks exist for $T = 0$ MeV, but not for $T = 2$ and $T = 4$ MeV. The reason is the well-known fact that the shell effects can be expected up to $T \leq 2$ MeV.

4 Conclusions

In this work (see also [14]), a theoretical approach has been used to study the temperature dependence of the symmetry energy coefficient in finite nuclei and other properties, such as the T -dependent nucleon densities and related rms radii, as well as the possibility of formation of neutron skins. The approach uses as a ground previous considerations within the local-density approximation (e.g., Refs. [10–13]) combining it with the self-consistent Skyrme-HFB method using the cylindrical transformed deformed harmonic-oscillator basis (HFBTHO code) [15, 16]. For infinite nuclear matter a Skyrme energy density functional with SkM* and SLy4 parametrizations is used. In our work we consider the isotopic chains of neutron-rich Ni, Sn, and Pb isotopes that represent an interest for future measurements with radioactive exotic beams. In addition to the HFBTHO densities of these isotopes, two other temperature-dependent densities of ^{208}Pb were used in the present paper: the local densities within the ETF method [18, 19] that reproduce the averaged THF results up to temperature $T=4$ MeV, and the symmetrized-Fermi local density distribution determined within the RDFA [20]. We restrict ourselves to this temperature range 0–4 MeV because, in accordance with several findings (e.g., in Ref. [25]), the limiting temperature (above which the nucleus cannot exist as a bound system) has been evaluated around 4 MeV for finite nuclei with mass number $A \geq 100$. In general, the density distributions decrease with the temperature in the center of the nucleus. Following the trend of the corresponding proton and neutron rms radii, the neutron-skin thickness grows significantly with the increase of T within a given isotopic chain. Second, we find that at zero temperature a formation of a neutron skin can be expected to start at $A > 78$ and $A > 132$ for Ni and Sn isotopes, respectively, thus confirming our previously obtained results in Refs. [6, 7].

As an attempt to analyze in a more appropriate way the symmetry energy coefficient $e_{sym}(A, T)$ of finite nuclei within a given chain, we introduce two new definitions of this quantity within the LDA [Eqs. (6) and (7)]. In general, the results of e_{sym} calculated for various isotopes in the present work are in good

agreement with theoretical predictions for some specific nuclei reported by other authors. At the same time, however, the difference between the results points out the dependence of the calculations of $e_{sym}(A, T)$ on various definitions of this quantity.

Studying the mass dependence of the symmetry energy coefficient, we note also the existence of a kink in the Ni and Sn isotopic chains at the double-magic ^{78}Ni and ^{132}Sn nuclei at $T = 0$ MeV, respectively, and a lack of kinks in the Pb chain. This observation confirms the result obtained previously in our works [7, 8] when studying the nuclear symmetry energy of spherical neutron-rich nuclei, particularly its isotopic evolution.

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