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Decay Half-Life of Nuclei-Proton, Alpha, Cluster Decays and Spontaneous Fissions

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Abstract. Many years ago the possibility of classical (without tunneling of Gamow) calculation of the nuclei decay half-life's has been demonstrated. Such possibility was based on the diffusion mechanism of Langevin-Kramers' decay process, the classical interpretation of the Bohm-Chetaev mechanics, the hypothesis for the presence of the Chetaev's dissipative forces generated from the Gryzinsky translational precession of the charged particles spin. In this paper an unified model of proton, alpha decays, cluster radioactivity and spontaneous fission half-lives is present as an explicit function, which depends on the total decay and kinetic energies, the number of protons and neutrons of the mother and daughter nuclei and from a set of digital parameters. The half-lives of the 573 nuclei taken from NuDat database together with the recent experimental data from Oganessian paper provide a basis for discovering the explicit form of the solution of Kramers of the Langevin type equation in a framework of inverse problems of the Alexandrov dynamic auto-regularization method (FORTRAN program REGN-Dubna). The procedure LCH in program REGN permitted to reduce the number of digital parameters from 137 to 79. The model describes 424 decays quantities with deviation of order one in years power scale.

1 Introduction

The beginning of the twentieth century brought surprising non-classical phenomena. Max Planck's explanation of the black body radiation [1] the work of Albert Einstein on the photoelectric effect [2], Niels Bohr's model of the electron orbits around the nuclei [3], the existence of protons and neutrons in the atomic nuclei [4, 5] established what is now known as quantum theory.

1.1 Bohmian mechanics

The ignored by the scientific community till present Bohmian quantum mechanics was first proposed by Louis de Broglie [6] and rediscovered by David Bohm [7], many years later [8]. In 1926 Schrödinger published his equation for the wave function (field) $\psi(r, t)$ [9]; in 1932 von Neumann put quantum theory on rigorous mathematical basis [10]. The main result was a state that quantum-mechanical probabilities cannot be understand in terms of any conceivable distribution over hidden parameters. In 1935 Einstein, Podolsky and Rosen [11], based on the hypothesis for the absence of action at a distance, argued that the quantum theory is either nonlocal or incomplete. In 1952 David Bohm [7] demonstrated that von Neumann theorem [9] has limited validity. In 1964, inspired by the paper [11] and Bohm's works on nonlocal hidden variables [7], Bell elaborated a theorem establishing clear mathematical inequalities, now known as Bell inequalities, for experimental results that would be fulfilled by local theories but would be violated by nonlocal ones [12]. In 1987 Bell explained that the orthodox (Copenhagen) interpretation of the Quantum mechanics is not adequate to the processes in the Nature, nevertheless that the calculations describe the experimental data [13]. Contemporary clear and full presentation of Bohmian quantum mechanics, including its chemical many particles applications, which can be seen in papers [14, 15]

1.2 Chetaev's stable model

In 50s of the 20th century, Nikolai Gurevich Chetaev [16] used the Lyapunov theorem for arbitrary small perturbation forces, which can do the motion unstable, formulated his famous theorem for "stable trajectories in dynamics". The reason for such a stability Chetaev explained with existence of small dissipative forces with full dissipation, which always exist in the nature. Analyzing holonomic mechanical systems, Chetaev demonstrated that its solutions give us a picture of quantum phenomena, because of analogy with Schrödinger type equation. The origin of dissipative forces for the stable movement (orbits) of the electrons in the atoms can be the precession of the proton and electron spin. Such statement was proposed by Michal Gryzinsky [17], who demonstrated the description of Hydrogen atom Ballmer's series in a framework of classical Kepler problem with phenomenological vector potential. The source of the Gryzinsky potential is the Coulomb interaction and the oscillating electromagnetic field of photon and electron caused by translational precession of the spin.

2 The Langevin-Kramers Description of Nucleus Decays

In papers [18], the Chetaev theorem on stable trajectories in dynamics was generalized to the case, when the Hamiltonian of a system is explicitly time-dependent. In a case of particle with mass m in the field of conservative forces presented by U, which depends on the time, the result was that the Chetaev's motion stability condition has the form of Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + U\psi\,. \tag{1}$$

The substitution $\psi = A\exp(\frac{i}{\hbar}S)$, where S is the classical action, in Eq. (1) born an equivalent 2-system of equations, known as Bohm-Madelung system of equations [14, 19]

$$\frac{\partial A}{\partial t} = \frac{1}{2m}(A\Delta S + 2\nabla A \nabla S)\,, \tag{2}$$

$$\frac{\partial S}{\partial t} = \left[\frac{(\nabla S)^2}{2m} + U - \frac{\hbar^2}{2m}\frac{\Delta A}{A}\right]. \tag{3}$$

It is important to note that the last term in Eq. (2),

$$Q = -\frac{\hbar^2}{2m}\frac{\Delta A}{A}\,, \tag{4}$$

is the quantum potential of the Bohm ψ-field. After the substitution $P(q,t) = \psi\psi^* = A(q,t)^2$ in Eq. (2), we have the forms

$$\frac{\partial P}{\partial t} = -\frac{1}{m}\nabla(P\nabla S) \tag{5}$$

and

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + U - \frac{\hbar^2}{2m}\left[\frac{\Delta P}{p} - \frac{1}{2}\frac{(\Delta P)^2}{p^2}\right] = 0.$$

The solution of Eq. (3) is the probability density $P(q,t)$ to find the particle from Eq. (1) in a certain point in the space-time. According to Eq. (4) the vector variable $v = \nabla S/m$ has meaning of velocity.

In paper [20], for the description the half-life data of the alpha-, cluster decays and spontaneous fissions was used the Kramers diffusion mechanism over potential barrier [21]. The theoretical argument was that the probability density $P(q,t)$ moves according to the laws of classical mechanics with a classical velocity $v = \nabla S/m$ [18].

The explicit form of half-life formulae was derived in the framework of Langevin's theory of Brownian motion [22], diffusion mechanism of Kramers over potential barrier [21] and Fermi-gas model for connection between thermodynamic temperature and internal excitation energy of many particles system [23]

$$\lg T_{1/2} = -\lg\frac{W_{Kramers}}{2\pi} + \lg\exp\left(\frac{A}{8\mu}\right)^{1/2}\frac{V_{Coul} - E_k}{\sqrt{E_k}}\,,$$

where the charge Coulomb potential is

$$V_{\text{Coul}} = \frac{(Z - Z_{\text{cl}})}{R_{\text{Coul}}}Z_{\text{cl}}\,, \qquad R_{\text{Coul}} = R_{A-A_{\text{cl}},Z-Z_{\text{cl}}} + R_{A_{\text{cl}},Z_{\text{cl}}} + R_{\text{NuclF}}\,,$$

A and Z are the mass number and charge of parent nucleus, ZZ_{cl} is the charge of the daughter nucleus and R_{Coul} [fm] is the minimal Coulomb radius, and the variable E_k [MeV] is the kinetic energy of the cluster.

It is important to note that from numerical point of view (the number of freedom in the sense of Weierstrass approximation theorem), the half-life according to "Kramers' over potential barrier picture" and "Gamow's tunneling quantum mechanism" [24], are equivalent to the Geiger-Nuttall formulae [25].

The aim of this paper is to apply the results of paper [20] for the description the half-life data for of proton (22 decays), alpha (497 decays), cluster (28 decays) decays and spontaneous fissions (26 decays) from NUDAT data-base [26] and paper [27], total 573 data.

3 Formulation of the Inverse Problem

If we rewrite Eq. (5) in the form

$$T^{Th}_{1/2}(Z,N.a) = 10^{W(Z,N,a)+U(Z,N,a)}, \tag{6}$$

where $W(Z,N,a)$ and $U(Z,N,a)$ are unknown functions of the variables Z, N, Z_{cl}, N_{cl}, kinetic energy E_k, total energy Q_t, isotopic spin characteristics of parent nuclei and a set $a = \{a_i\} = a_i$, $i = 1,\ldots,n$ unknown digital parameters, the solution of over determined nonlinear system of M algebraic equations for n real unknown parameters

$$T^{\text{Expt}}_{(1/2)}(Z_j,N_j) = T^{\text{Th}}_{(1/2)}(Z_j,N_j,((a_i),i=1,\ldots,n)),\; j = 1\ldots,M\,, \tag{7}$$

where $M \geq n$, and the superscripts "Expt" and "Th" mean the experimental and model values of the half-life, correspondingly.

3.1 Solution of the over determined system of equations and discovering the explicit form of functions

For solution of ill-posed problem (7) (see [28]) we use the Alexandrov dynamic autoregularization method (FORTRAN code REGN-Dubna [29–31]. The use of procedure LCH in REGN permits to discover the explicit form of unknown functions because one can chose uniquely the better one from two functions which give the same hi-squared [32].

4 Unified Description of the Proton, Alpha, Cluster Decays and Spontaneously Fissions Half-Life

In NuDat database [26], the 519 half-life data for proton, alpha, cluster decays and spontaneous fissions are published. Also 54 data for the alpha decay are are presented in [27]. So, in our database we will use 573 half-life data.

4.1 The choose of the variables

For solving such type of inverse problems it is very convenient to choose the variables to be in the interval $[-1,+1]$, as well as the variables to be linearly independent.

By using simple nonlinear substitutions from 8 variables

$$A = Z + N, Z, N, N - Z, A_{\text{cl}} = Z_{\text{cl}} + N_{\text{cl}}, Z_{\text{cl}}, N_{\text{cl}}, N_{\text{cl}} - Z_{\text{cl}}$$

we choose the variables

$$v_1 = \frac{Z}{A}, \quad v_2 = \frac{N}{A}, \quad v_3 = \frac{N-Z}{A}, \quad v_4 = \frac{Z}{A_{\text{cl}}},$$
$$v_5 = \frac{N_{\text{cl}}}{A_{\text{cl}}}, \quad v_6 = \frac{N_{\text{cl}} - Z_{\text{cl}}}{A_{\text{cl}}}, \quad v_7 = \frac{E_k}{Q_t}, \quad v_8 = \frac{Z_{\text{cl}}(Z - Z_{\text{cl}})}{Z Z_{\text{cl}}}.$$

The isotopic spin dependence is included in the analysis with using the variables $v_9 = v_{10} = v_{11} = 0$, if A, Z, N are even and if A, Z, N are odd $v_9 = v_{10} = v_{11} = 1$.

4.2 Explicit form of the half-life as function of Z, N and parameters $\boldsymbol{a}$

The explicit from of the function $W(Z, N, a)$ is:

$$\begin{aligned}
W(Z,N,a) &= -\exp(a_{n-2}) R(Z,N,a) + MagNum_c(Z,N,a), \\
MagNum_c(Z,N,a) &= A_Z(Z,N,a) \frac{\exp(Z - Z_{MN})^2}{(Z - Z_{MN})^2 + w_Z^2)} \\
&\quad + A_N(Z,N,a) \frac{\exp(N - N_{MN})^2}{(N - N_{MN})^2 + w_N^2)}, \\
A_Z(Z,N,a) &= \exp\Big(a_{Np2+4} + \Sigma_{i=1}^{Np1}(a_{i+9Np1} v_i \\
&\quad + a_{i+10Np1} v_i^2 + a_{i+11Np1} v_i^3)\Big), \\
A_N(Z,N,a) &= \exp\Big(a_{Np2+5} + \Sigma_{i=1}^{Np1}(a_{i+12Np1} v_i \\
&\quad + a_{i+13Np1} v_i^2 + a_{i+14Np1} v_i^3)) ,
\end{aligned}$$

where Z_{MN}, N_{MN} are the nearest to Z and N magic numbers and w_Z, w_N are equal to the half of difference between magic numbers in which interval belong Z and N correspondingly. For the radius $R(Z, N, A)$ [20] with influence of isotopic spin correction we have

$$\begin{aligned}
R(Z,N,a) &= (Be(Z,N,a)((A - A_{cl})^{1/3} + A_{cl}^{1/3}) - 1) Ce(Z,N,a), \\
Be(Z,N,a) &= \exp(a_{Np2+2}) \exp\Big(\sum_{i=1}^{Np1}(a_{i+3Np1} v_i + a_{i+4Np1} v_i^2 \\
&\quad + a_{i+5Np1} v_i^3) + B_c(Z,N,a)\Big) \qquad (8) \\
B_c(Z,N,a) &= \sum_{i=1}^{3} a_{Ns+3+i} v_{8+i}
\end{aligned}$$

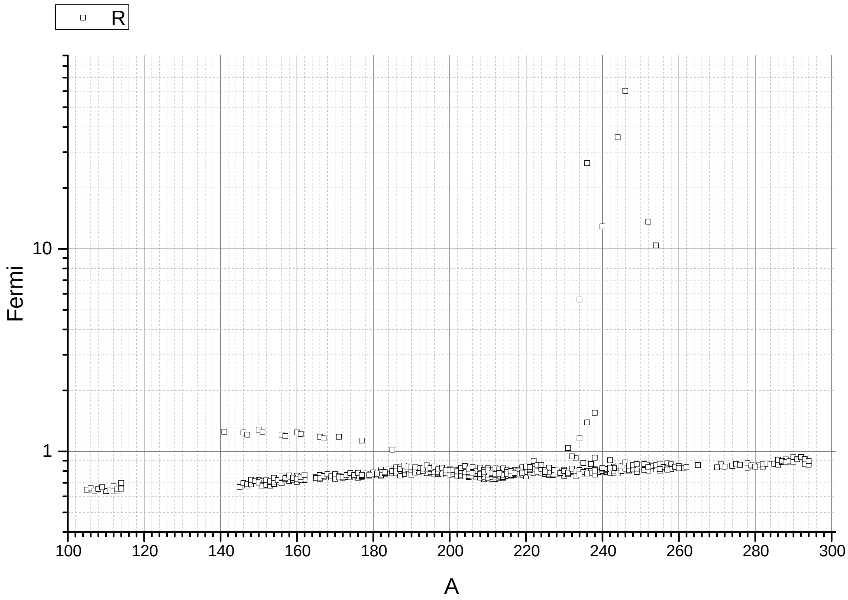

Figure 1. The minimal Coulomb radius R_{Coul} $[fm]$

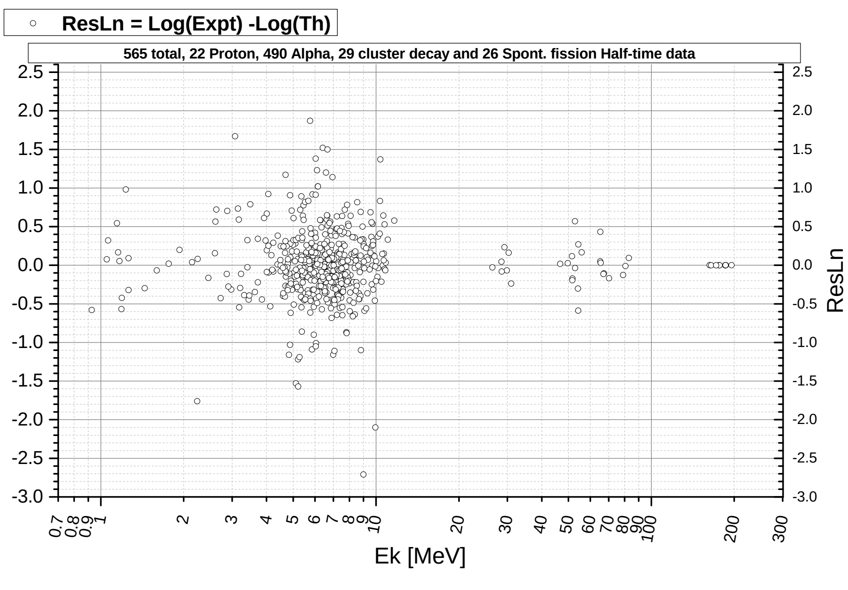

Figure 2. The difference between of the Expt and the Th of $lgT_{1/2}$.

$$
\begin{aligned}
Ce(Z,N,a) &= \exp\left(a_{Np2+3}\right)\exp\Big(\sum_{i=1}^{Np1}(a_{i+6Np1}v_i + a_{i+7Np1}v_i^2 \\
&\qquad + a_{i+8Np1}v_i^3) + C_c(Z,N,a)\Big) \qquad (9) \\
C_c(Z,N,a) &= \sum_{i=1}^{3} a_{Ns+6+i}v_{8+i}
\end{aligned}
$$

For the term $U(Z,N,a)$ in Eq. (6), we have [21]

$$
U(Z,N,a) = \sqrt{\frac{A}{Mu(Z,N,a)}}\frac{\frac{Z_{\mathrm{cl}}(Z-Z_{\mathrm{cl}})}{R(Z,N,a)}E_k}{\sqrt{E_k}}
$$

$$
\begin{aligned}
Mu(Z,N,a) &= a_{n-1}(1+\exp-(a_{Np2+1}+sMu(Z,N,a))^2)\,, \\
sMu(Z,N,a) &= \sum_{i=1}^{Np1}(a_i v_i + a_{i+Np1}v_i^2 + a_{i+2Np1}v_i^3) + Mu_c(Z,N,A)\,, \\
Mu_c(Z,N,a) &= \sum_{i=1}^{3} a_{Ns+6+i}v_{8+i}\,.
\end{aligned}
$$

The integers in the above formulae have a values $Np1 = 8$, $Nc = 3$, $Np2 = 15$, $Np1 = 120$, $Ns = Np2+5 = 125$, $n = Np2+5+3Nc+3 = 137$. By using a Fortran code, the initial number of unknown parameters $n = 137$ is reduced to 79. The estimation of description accuracy $\chi^2 = 346$ is calculated as in the following way:

$$
\chi^2 = \Sigma_{k=1}^{421}\left(\frac{Expt(Z_k,N_k) - Th(Z_k,N_k,a)}{\sigma(z_k,N_k)}\right)^2
$$

where

$$
\sigma(N_k,Z_k,a) = \sigma_{stat}(N_k,Z_k) + Expt(Z_k,N_k)
$$

$$
\chi^2 = \sqrt{\frac{\chi^2}{346-79}} = 1.02.
$$

5 Conclusion

The presented model of proton, alpha, cluster decays and spontaneous fissions half-life describes with accuracy less than of one order in year's power scale, 9 proton, 368 alpha, 21 cluster decays and 25 spontaneous fissions, total 424 data from 573 initial experimental values, as explicit function of the total decay energy Q_t and kinetic E_k energy, the number of protons Z_{cl} and neutrons N_{cl} of

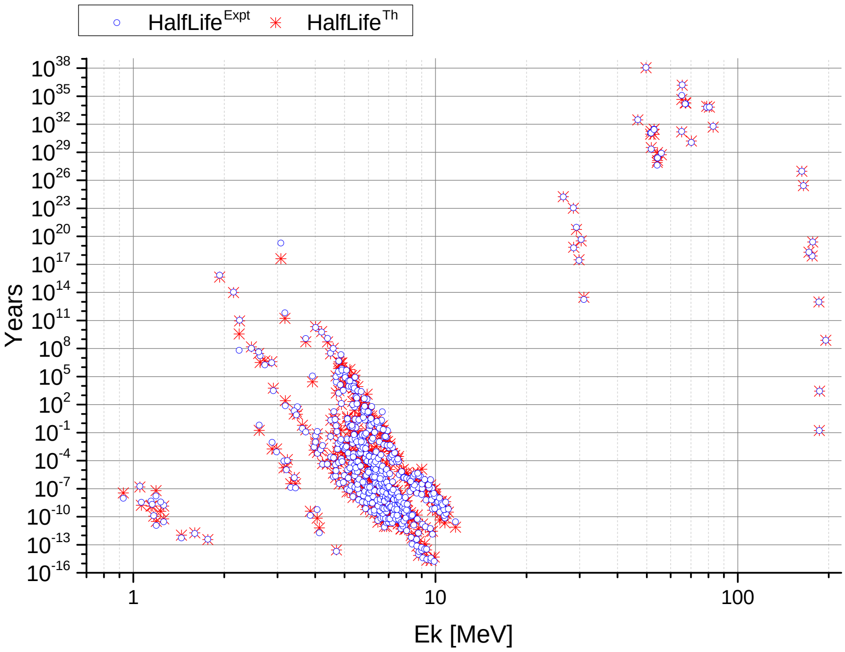

Figure 3. The experimental and theoretical half-life of the nuclei as a function of the kinetic energy.

daughter product, the number of protons Z and neutrons of mother nuclei and from a set of $a = (a_i, \;\; i = 1, \ldots, 79)$ digital parameters:

$$Ht_{1/2}(Z, N, Z_{\text{cl}}, N_{\text{cl}}, E_k, Q_t, a).$$

The result of this paper can be applied in the theoretical research of stability islands problem (see for example the paper [28]) because the kinetic and total energy of decay E_k, Q_t can be calculated using the phenomenological model for nuclei masses [33, 34].

The bad accuracy in the description of 149 half-life data from [26] and [27] data base is, probably, related to the fact that in our model the information about the eccentricity of the nuclei was not used. In the next paper this problem will be analyzed.

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