NUCLEAR THEORY, Vol. 37 (2018)
eds. M. Gaidarov, N. Minkov, Heron Press, Sofia

$T=1$ Pairing and Number-Projection Effects on the Spectroscopic Factor of One-Proton Stripping Reactions Within the Picket-Fence Model

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Abstract. Isovector ($T = 1$) neutron-proton (np) pairing effects on the spectroscopic factor (SF) of one-proton stripping reactions are studied within the generalized BCS approach. As the main shortcoming of this approach is the non-conservation of the particle-number, a number-projection is necessary. In the present contribution, we use the Sharp-BCS (SBCS) method. As a first step, expressions of the SF are derived, either before or after the projection. As a second step, calculations are performed using the schematic picket-fence model.

1 Introduction

Spectroscopic factors (SF) have been introduced as a means of comparison between experiment and the prediction of nuclear models [1]. On the other hand, the pairing correlations are of great importance in nuclear structure. They must be included in the determination of the SF, since the latter depends essentially on the wave-function. The pairing correlations may exist between like-particles and between neutron and proton (np). In the latter case, there exist two different types, i.e. isovector ($T = 1$) and isoscalar ($T = 0$) pairing. In the present contribution, we will consider only the $T = 1$ np pairing which is often studied within the framework of the BCS theory [2]. However, it is well known that the main shortcoming of this method is the non-conservation of the particle number [3]. One of the most used approaches to overcome the defect of the BCS wave-function is the particle-number projection. In the present work, we use the Sharp-BCS (SBCS) projection method [4, 5]. The particle-number projection effect on the SF corresponding to one-pair of like-nucleon transfer reactions has been recently studied, when including [6] or not [7] the isovector pairing correlations. The goal of the present work is to study the same effect on the SF of one-proton stripping reactions in $N \simeq Z$ systems. With this aim, we will use the single-particle energies of the schematic picket-fence model [8].

2 Formalism

Let us consider a system of N neutrons and Z protons in which the neutrons and the protons are assumed to occupy the same levels. In the isovector pairing case, it is described by the Hamiltonian

$$H = \sum_{\nu>0,t} \varepsilon_{\nu t}(a^+_{\nu t}a_{\nu t} + a^+_{\widetilde{\nu}t}a_{\widetilde{\nu}t}) - \frac{1}{2}\sum_{tt'} G_{tt'} \sum_{\nu,\mu>0} \left(a^+_{\nu t}a^+_{\widetilde{\nu}t'}a_{\widetilde{\mu}t'}a_{\mu t} + a^+_{\nu t}a^+_{\widetilde{\nu}t'}a_{\widetilde{\mu}t}a_{\mu t'}\right). \quad (1)$$

In this expression, t is the isospin component ($t = n, p$), $a^+_{\nu t}$ and $a_{\nu t}$ respectively represent the creation and annihilation operators of a particle in the $|\nu t\rangle$ state, of energy $\varepsilon_{\nu t}$. The time-reversed of the state $|\nu t\rangle$ is denoted $|\widetilde{\nu} t\rangle$. The pairing-strength $G_{tt'}$ $(t, t' = n, p)$ is assumed to be constant and such as $G_{pn} = G_{np}$.

The corresponding ground-state is given by [5]

$$|\psi\rangle = \prod_{j>0} |\psi_j\rangle \quad , \quad (2)$$

with the notations

$$|\psi_j\rangle = \left[B^j_1 A^+_{jp}A^+_{jn} + B^j_p A^+_{jp} + B^j_n A^+_{jn} + B^j_4(a^+_{\widetilde{j}p}a^+_{jn} + a^+_{\widetilde{j}n}a^+_{jp}) + B^j_5\right]|0\rangle. \quad (3)$$

A^+_{jt} is the creation operator of a pair of particles, i.e., $A^+_{jt} = a^+_{\widetilde{j}t}a^+_{jt}$, $t = n, p$. The coefficients B^j_i are not given here for lack of place.

Nevertheless, the state $|\psi\rangle$ does not conserve the particle number. It must then be projected on the good proton and neutron numbers. Using the SBCS method, the ground-state is given by [5]

$$|\psi_{mm'}\rangle = C_{mm'} \left\{ \sum_{k=0}^{m+1}\sum_{k'=0}^{m'+1} \xi_k \xi_{k'} z_k^{-P_n} z_{k'}^{-P_p} |\psi(z_k, z_{k'})\rangle + \mathcal{CC} \right\}, \quad (4)$$

where $|\psi(z_k, z_{k'})\rangle = \prod_{j>0} |\psi_j(z_k, z_{k'})\rangle$, with the notations

$$|\psi_j(z_k, z_{k'})\rangle = \Big[z_k z_{k'} B^j_1 A^+_{jp}A^+_{jn} + z_k B^j_n A^+_{jn} + z_{k'} B^j_p A^+_{jp} + \sqrt{z_k z_{k'}} B^j_4 \left(a^+_{\widetilde{j}p}a^+_{jn} + a^+_{\widetilde{j}n}a^+_{jp}\right) + B^j_5\Big] |0\rangle \quad (5)$$

and

$$\xi_k = \begin{cases} \frac{1}{2} & \text{if } k = 0 \text{ or } k = m+1 \\ 1 & \text{if } 0 < k < m+1 \end{cases} , \qquad z_k = \exp\left(\frac{ik\pi}{m+1}\right).$$

Here, m, m' are non-zero integers, $C_{mm'}$ is the normalization constant and $\mathcal{CC}$ means the summation over the same terms where $(z_k, z_{k'})$ is replaced by $(\overline{z_k}, z_{k'})$, then by $(z_k, \overline{z_{k'}})$ and finally by $(\overline{z_k}, \overline{z_{k'}})$.

However, the states given by Eqs. (2) and (4) can only describe even-even systems (i.e. such as $N = 2P_n$ and $Z = 2P_p$). When the particle-number is odd, the BCS wave-function is derived using the blocked-level technique. In the following, it will be assumed that the blocked particle is a proton in the ν state (i.e. $N = 2P_n$ and $Z = 2P_p + 1$). The BCS ground-state is then given by [9]

$$|\nu P\rangle = a^+_{\nu p}\left(B^\nu_n A^+_{\nu n} + B^\nu_5\right) \prod_{j>0, j\neq\nu} |\psi_j\rangle\,, \tag{6}$$

$|\psi_j\rangle$ being defined by Eq. (3).

Let us note that the dependence of the B^j_i $(i = 1, p, n, 4, 5)$ versus ν has been omitted for the simplicity of notations. Furthermore, one just has to invert the p and n indexes in Eq. (6) to deduce the state where the blocked particle is a neutron. The corresponding projected state is given by [9]

$$\begin{aligned}|(\nu P)_{mm'}\rangle &= C^{\nu P}_{mm'} \sum_{k=0}^{m+1} \sum_{k'=0}^{m'+1} \xi_k \xi_{k'} \\ &\times \left\{ z_k^{-P_n} z_{k'}^{-P_p} a^+_{\nu p}\left(B^\nu_n z_k A^+_{\nu n} + B^\nu_5\right) \prod_{j>0, j\neq\nu} |\psi_j(z_k, z_{k'})\rangle + \mathcal{CC} \right\}, \end{aligned} \tag{7}$$

$C^{\nu P}_{mm'}$ being the normalization constant.

In the case of one-particle stripping reactions, the SF is deduced from the relation

$$\sqrt{S^{STR}_t} = \left\langle \psi^f(A+1) \left| \sum_{l>0} \left(a^+_{lt} + a^+_{\tilde{l}t}\right) \right| \psi^i(A) \right\rangle, t = n, p. \tag{8}$$

where $\left|\psi^i(A)\right\rangle$ and $\left|\psi^f(A\pm1)\right\rangle$ respectively refer to the wave-functions of the initial (i) and final (f) states of the considered nucleus. A corresponds to the total number of nucleons in the initial state.

Within the generalized BCS approach, the SF defined by Eq. (8) reads, in the case of the transfer of one proton from an even-even nucleus to an odd one [10],

$$\sqrt{S^{STR(1)}_p} = \left[B^{\nu i}_n B^{\nu f}_n(\nu) + B^{\nu i}_5 B^{\nu f}_5(\nu)\right] \prod_{j>0, j\neq\nu} D^{if}_j(\nu) \tag{9}$$

where we used the wave-functions defined by Eqs. (2) and (6).

In the reciprocal case, the SF is given by

$$\sqrt{S^{STR(2)}_p} = \left[B^{\nu i}_1 B^{\nu f}_n(\nu) + B^{\nu i}_p B^{\nu f}_5(\nu)\right] \prod_{j>0, j\neq\nu} D^{fi}_j(\nu) \tag{10}$$

with the notation

$$D_j^{if}(\nu) = B_1^{ji}B_1^{jf}(\nu) + B_p^{ji}B_p^{jf}(\nu) + B_n^{ji}B_n^{jf}(\nu) + 2B_4^{ji}B_4^{jf}(\nu) + B_5^{ji}B_5^{jf}(\nu)\,.$$

After the projection, the SF are derived using the states (4) and (7). One then has, in the case of the transfer of one proton from an even-even nucleus to an odd one (respectively from an odd nucleus to an even-even one) [10],

$$\sqrt{\left(S_p^{STR1(2)}\right)_{mm'}} = 4\,(m+1)\,(m'+1)\,C_{mm'}^{i(f)}C_{mm'}^{(\nu p)f(i)}\sum_{k=0}^{m+1}\sum_{k'=0}^{m'+1}\xi_k\xi_{k'} \times \left[z_k^{-P_n^f}z_{k'}^{-P_p^f}F_{n5(p)}^{if}(z_k)\prod_{j>0,j\neq\nu}D_j^{if(fi)}(z_k,z_{k'}) + \mathcal{CC}\right], \quad (11)$$

with the notations

$$D_j^{if}(z_k,z_{k'}) = z_kz_{k'}B_1^{ji}B_1^{jf}(\nu) + z_{k'}B_p^{ji}B_p^{jf}(\nu) + z_kB_n^{ji}B_n^{jf}(\nu) + 2\sqrt{z_kz_{k'}}B_4^{ji}B_4^{jf}(\nu) + B_5^{ji}B_5^{jf}(\nu)$$

$$F_{n5}^{if}(z_k) = z_k^2B_n^{\nu i}B_n^{\nu f}(\nu) + B_5^{\nu i}B_5^{\nu f}(\nu)$$

$$F_{np}^{if}(z_k,z_{k'}) = z_k^2z_{k'}B_1^{\nu i}B_n^{\nu f}(\nu) + z_{k'}B_p^{\nu i}B_5^{\nu f}(\nu).$$

3 Numerical Results. Discussion

The previously described formalism has been applied using the schematic picket-fence model. In the latter, the levels are such $\varepsilon_\nu = \nu$, $\nu = 1, 2, ..., \Omega$, where Ω is the total number of levels. The values of the pairing gap parameters $\Delta_{tt'}$ $(t, t' = n, p)$ are chosen arbitrarily. The $G_{tt'}$ $(t, t' = n, p)$ values are then chosen such as to reproduce these values. We considered a one-proton stripping reaction in the case $Z^i = N^i = 16$ (Z^i and N^i being the proton and neutron numbers in the initial state), taken as an example for even-even systems, and $Z^i = 15$, $N^i = 16$, taken as an example for odd systems. We have studied separately the np pairing and projection effects.

The np pairing effect, before and after the projection, is evaluated using the relative discrepancies

$$\delta S_{np} = \frac{S_{BCS} - S_{BCS-np}}{S_{BCS}} \quad \text{and} \quad \delta S_{np-proj} = \frac{S_{SBCS} - S_{SBCS-np}}{S_{SBCS}}.$$

They have been calculated as a function of the np pairing gap parameter in the initial state Δ_{np}^i, for fixed values of the other gap parameters. The used values are $\Delta_{pp}^i = 1.6$, $\Delta_{nn}^i = 1.0$, $\Delta_{pp}^f = 1.4$, $\Delta_{nn}^f = 1.3$, $\Delta_{np}^f = 0.2$ (MeV) and $\Omega = 18$. The corresponding results are displayed in Figure 1. The behavior of δS is obviously different when the initial state is even-even and when it is odd.

One may conclude from this figure that the np pairing effect on the SF is important for this kind of reaction since $|\delta S_{np}|$ may reach up to 65%, whereas $|\delta S_{np-proj}|$ may reach up to 30%. On the other hand, the np pairing effect in absolute value is clearly more important on average after the projection than before it .

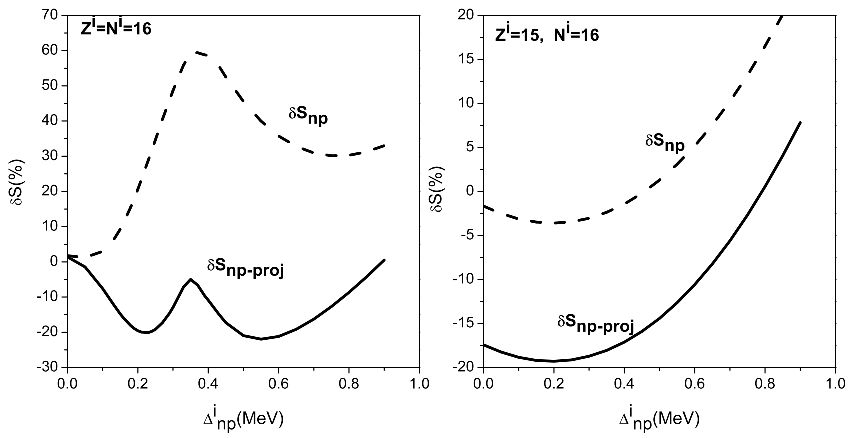

Figure 1. Variations of the relative discrepancies of the SF (see the text for notations), in the case of the systems $Z^i = N^i = 16$ (left-hand part) and $Z^i = 15, N^i = 16$ (right-hand part), as a function of Δ^i_{np}. Dashed (respectively solid) lines show values obtained before (respectively after) the projection.

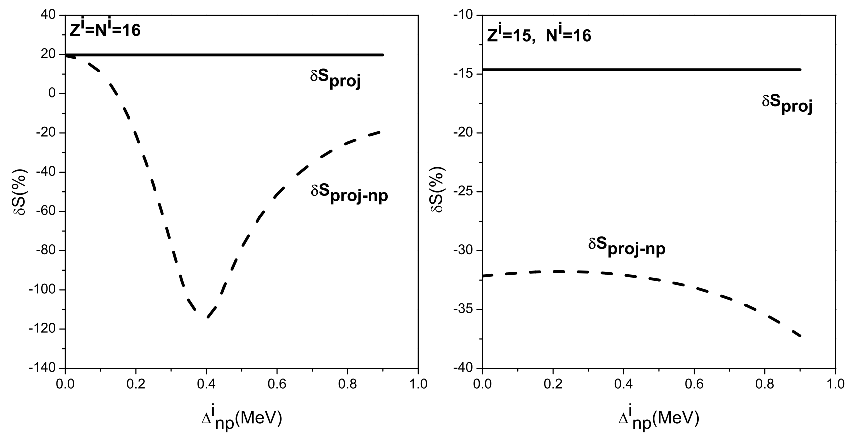

Figure 2. Variations of the relative discrepancies of the SF (see the text for notations), in the case of the systems $Z^i = N^i = 16$ (left-hand part) and $Z^i = 15, N^i = 16$ (right-hand part), as a function Δ^i_{np}. Solid lines correspond to the like-particle pairing, dashed lines correspond the np pairing.

The projection effect, in the pairing between like-particles, as well as in the np pairing, is evaluated using the relative discrepancies

$$\delta S_{proj} = \frac{S_{BCS} - S_{SBCS}}{S_{BCS}} \quad \text{and} \quad \delta S_{proj-np} = \frac{S_{BCS-np} - S_{SBCS-np}}{S_{BCS-np}}.$$

They have been calculated as a function of Δ^i_{np} with the same parameters as in the previous section. The corresponding results are displayed in Figure 2. When only the pairing between like-particles is considered, the projection effect is obviously constant as a function of Δ^i_{np}. Either when the initial state is even-even or odd, the projection effect is clearly more important in the np pairing case than in the pairing between like-particle case. However, it is far from negligible in the latter case since δS_{proj} is of the order of 20% and 15%, in absolute value, respectively.

From Figures 1 and 2, one may conclude that both the np pairing and projection effects on the SF are important and must be taken into account. These effects strongly depend on the pairing gap parameter values (and thus the pairing constant values) which must then be carefully chosen.

4 Conclusion

Expressions of the SF corresponding to one-proton stripping reactions have been established by taking into account the isovector np pairing correlations and a particle-number projection in the framework of the Sharp-BCS method. The formalism has been applied using the single-particle energies of the schematic picket-fence model. It was shown that the np pairing and particle-number projection effects on the SF are important and they strongly depend on the pairing gap parameter values. Furthermore, these effects are very different when the parent nucleus is even-even and when it is odd.

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