NUCLEAR THEORY, Vol. 37 (2018)
eds. M. Gaidarov, N. Minkov, Heron Press, Sofia

# Nucleon Properties & Nuclear Equation of State

**J. Rożynek**[1]

[1]National Center for Nuclear Research, Hoża 69, 00-681 Warsaw, Poland

**Abstract.** We propose to benefit from a concept of the enthalpy in order to include volume corrections to a nucleon rest energy, which are proportional to pressure and absent in a standard Relativistic Mean Field (RMF) with point-like nucleons. It is shown, how the EOS depends from nucleon sizes inside NM. The course of the EoS in our RMF model agrees with a semi-empirical estimate and is close to results obtained from extensive DBHF calculations with a Bonn A potential, which produce the EoS stiff enough to describe neutron star properties (mass–radius constraint), especially the masses of "PSR J16142230" and "PSR J0348+0432", most massive ($\sim 2M_{\odot}$) known neutron stars. The presented model has proper saturation properties, including good values of a compressibility.

Taking into account thermodynamic effects of pressure in finite volumes, we will describe how an energy per nucleon $\varepsilon_A = M_A/A$ and pressure evolves with NM density $\varrho$ in an RMF approach [1–5]. The original Walecka version [1] of the linear RMF in introduces two potentials: a negative scalar $g_S U_S$ and a positive vector $U_V = g_V(U_V^0, \boldsymbol{0})$ fitted to a nuclear binding energy at the equilibrium density $\varrho = \varrho_0$. The EoS for this linear, scalar-vector $(\sigma, \omega)$ RMF model [1, 2] match a saturation point with too large compressibility $K^{-1} = \varrho^2 \frac{d^2}{d\varrho^2}\varepsilon_A \sim 550$ MeV and is very stiff for higher densities, where the repulsive vector potential starts to predominate the attractive scalar part. Nevertheless RMF models produce, after the Foldy-Wouthuysen reduction, the good value of a spin-orbit strength at the saturation density [1]. The dynamics of the potentials in the RMF approach are discussed e.g. in four specific mean-field models [1–4]. In the ZM model [3] a fermion wave function is re-scaled and interprets a new, density dependent nucleon mass. It starts to decrease from $\varrho = 0$ and at the saturation point $\varrho = \varrho_0$ reaches 85% of a nucleon mass $M_N$. But the nucleon mass replaced at the saturation point by a smaller value would change the nucleon deeply inelastic Parton Distribution Function (PDF) [6], shifting the Björken $x \propto (1/M_N)$. Such a shift means that nucleons will carry 15% less of the Longitudinal Momentum (LM), what should be compensated by the enhanced contribution from a meson cloud for small $x < 0.3$ to describe the EMC effect [7,8] in the RMF. There is no evidence for a such huge enhancement

[9] in the EMC effect for small x. Also the nuclear Drell-Yan experiments [10], which measure the sea quark enhancement, we described [11] with a small 1% admixture of nuclear pions and the $M_N$ unchanged. Thus the deep inelastic phenomenology indicates that a change of the nucleon mass at the saturation density is rather negligible. A nonlinear extension of the RMF model [4, 12] assumes self-interaction of the $\sigma$-field with the help of two additional parameters fitted to $K^{-1} \sim 250$ MeV and an effective mass $M_N^* = M_N + g_S U_S$. These modifications of a scalar potential give a softening of EOS with a good value of compressibility. Modern RMF calculations [12, 13] have adjusted the EOS, fitting more mesons fields ($\rho$ for an isospin dependence) and including the octet of baryons. The excluded volume correction were already discuss in [14] with constant nucleon mass and radius.

We have proposed [15] to improve nuclear RMF models in a different way, namely by taking into account volume contributions to a nucleon rest energy instead of a constant nucleon mass and radius, used so far in standard RMF models. Any extended object inside a compressed medium (like a submerged submarine) needs an extra energy to preserve its volume. Thus from the "deep" point of view, finite pressure correction should be taken into account in RMF calculations with point-like nucleons, but also in the Quark-Meson Coupling (QMC) model [17]. To describe that dependence of a nucleon rest energy in a compressed medium we will adopt a bag model. Considering a role of finite nucleon sizes in compressed NM, the simplest, original $(\sigma, \omega)$ model [1, 2] with point-like nucleons, which is too stiff, will be extended to get clear conclusions.

For fixed pressure and a zero temperature it is easy to show (see a first paragraph in a next section), that definitions of a chemical potential $\mu$ or a Fermi energy, have the same energy balance as an average, single particle enthalpy. An enthalpy contains in a homogenous medium an interesting term, a work of a nuclear pressure $p_H$ in a nuclear/nucleon volume, which will be investigated. It is the argument for our choice of a Gibbs free energy with independent pressure $p_H$ in favor of a Helmholtz free one (here an internal energy) with the volume, as an independent variable. Our results are independent [18] of that choice; like expressions on a chemical potential $\mu$ in (2).

We will neglect nuclear pion contributions above the saturation point. Dirac-Brueckner calculations show that a pion effective cross section, in the reaction of two nucleons $N + N = N + N + \pi$, is strongly reduced at higher nuclear densities above the threshold , also with RPA insertions to a self energy of $N$ and $\Delta$ [19]. We restrict our degrees of freedom to interacting nucleons.

## 1 Nuclear Enthalpy

At the beginning, let us consider effects generated by a volume of compressed NM. Start with $A$ nucleons which occupy a volume $V_A = A/\varrho$. They have to perform a necessary work $W_A = p_H V_A$ to keep a space $V_A$ inside compressed NM against nuclear pressure $p_H \doteq -(\partial M_A/\partial V_A)$. Thus interacting nucleons should provide not only the nuclear mass $M_A$, but rather the nuclear enthalpy

$$H_A \doteq M_A + W_A = M_A + A\frac{p_H}{\varrho} \tag{1}$$

which contains, besides the nuclear mass as an internal energy, the necessary work. Taking appropriate thermodynamical derivatives with respect to $A$, we get following relations between chemical potential $\mu$ and the enthalpy,

$$\mu \doteq (\partial M_A/\partial A)_{V_A} \equiv (\partial H_A/\partial A)_{p_H} = \varepsilon_A + \frac{p_H}{\varrho} = H_A/A \tag{2}$$

for $A \to \infty$. Please note that the same relation with pressure fulfills a nucleon Fermi energy

$$E_F \doteq P_N^0(P_F) = (\partial M_A/\partial A)_{V_A} = \varepsilon_A + p_H/\varrho = \mu \tag{3}$$

of a nucleon with a Fermi momentum $P_F$; well-known as the Hugenholtz-van Hove (HvH) relation [18], also proven in the self-consistent RMF approach [20].

The relativistic nuclear dynamics of nucleons in a nucleus, described by "light cone" momenta $(P_N^+, P_N^-, \boldsymbol{P}_N^{\perp})$, can be formulated [6, 7, 21] in the target rest frame, where $\boldsymbol{P}_A = 0$. In order to specify a total nuclear energy $P_A^0$ in compressed NM in a single particle approach, let us discuss a longitudinal Momentum Sum Rules (MSR). Let's focus our attention on the LM components $P_N^+ = P_N^0 + P_N^Z$ of $A$ nucleons. The question is: do they add up to the internal energy $M_A$ or rather to the $H_A$, greater then $M_A$ for positive pressure? To proceed our question let us look at a LM distribution

$$f_N(y) = \int \frac{d^4P_N}{(2\pi)^4}\delta\left(y - \frac{AP_N^+}{P_A^+}\right) Tr\left[\gamma^+ G(P_N, P_A)\right], \tag{4}$$

with $y = AP_N^+/P_A^+$, which gives a Lorentz invariant fraction of a nucleon LM $P_N^+$ in the NM with a LM $P_A^+ = P_A^0$. This distribution is manifestly covariant and is expressed by a single nucleon Green's function [6] $G(P_N, P_A)$ in the nuclear medium, given e.g. in [1, 7]. The trace is taken over the Dirac and isospin indices and finally [7, 22]

$$f_N \quad ( \quad y) = \frac{4}{\varrho}\int \frac{S_N(P_N)d^4P_N}{(2\pi)^3}\alpha\, \delta(y - AP_N^+/P_A^0); \tag{5}$$

where a nucleon spectral function

$$S_N = n(|\,\boldsymbol{P}_N\,|)\delta(P_N^0 - \sqrt{{M_N^*}^2 + {\boldsymbol{P}_N}^2} - g_V U_V^0)$$

is given in the impulse approximation and $n$ is the Fermi distribution. Such a LM distribution [6], derived from matrix elements containing lower components of a hadron wave function, includes a flux factor $\alpha = (1 + P_N^3/E_N^*)$ and thanks to this is properly normalized to the number of nucleons [21]. After integration (5) the result is:

$$f(y) = 3/4)[P_A^0/(AP_F)]^3[(AP_F/P_A^0)^2 - (y - AE_F/P_A^0)^2],$$

where $y$ takes the values determined by the inequality $(E_F - P_F)/P_A^0 < (y/A) < (E_F + P_F)/P_A^0$. Integrating the LM fraction $y$ in NM

$$\int dyyf_N(y) = \frac{AE_F}{P_A^0} = A\frac{\varepsilon_A + p_H/\varrho}{P_A^0} = 1, \tag{6}$$

and using HvH relation (3) in a middle step we get the longitudinal MSR (6) which gives a fraction of the nuclear LM taken by all nucleons [7,21]; therefore equal 1.

Let us check it with the usual "on mass shell" choice: $P_A^0 = M_A = A\varepsilon_A$. Then the MSR (6) is satisfied only at the saturation point where $p_H = 0$ [7]. However, in the beginning we advocate to choose the enthalpy $P_A^0 = H_A = A\varepsilon_A + p_H V_A$ as a total nuclear energy. Taking (2) $H_A = A\mu$ we get

$$\int dyyf_N(y) = \frac{AE_F}{P_A^0} = \frac{AE_F}{H_A} = \frac{E_F}{\mu} = 1.$$

Now the MSR (6) is always satisfied (3) thanks to the finite volume contribution $p_H V_A$ to the nuclear energy. Thus we will use enthalpies, as compact forms for total rest energies of nuclear or nucleon (parton) system.

## 2 Nucleon Enthalpy

We will discuss in a bag model, whether the nucleon mass $M_N$ or rather a nucleon enthalpy $H_N$ should be, eventually, constant - independent from the density inside the compressed medium. Such a question is absent in the standard RMF, where nucleons are point-like with the constant mass $M_N$ independent of pressure inside NM. In a compressed nucleon, partons (quarks and gluons) have to do a work $W_N = p_H V_N$ to keep a space $V_N$ for a nucleon "bag". It will involve functional corrections to a nucleon rest energy, dependent from external pressure with a physical parameter - a nucleon radius $R$. we introduce a nucleon enthalpy $H_N$ with the nucleon mass $M_{pr}$ modified in the compressed medium

$$H_N(\varrho) \doteq M_{pr}(\varrho) + p_H V_N \quad \text{with} \quad H_N(\varrho_0) = M_N, \tag{7}$$

as a "useful" expression for the total rest energy of a nucleon "bag". Please note, that "external" pressure $p_H$ used in (7) is, of course, identical with nuclear pressure appearing in (1,2). Our volume corrections will change a nucleon rest energy but also will diminish effectively a free space between nucleons for the given nuclear density, what modifies an available space $V_{A-} = (V_A - AV_N)$ and so nuclear pressure. Now $p_H \doteq -(\partial M_A/\partial V_{A-})_A$. A total enthalpy $H_A^T =$

$H_{A-} + A(H_N - M_N)$ and using (1,2,7) we arrive to the HvH relation with extended nucleons.

$$H_A^T/A = \varepsilon_A - (\partial M_A/\partial V_{A-})_A/\varrho = \varepsilon_A + p_H/\varrho = E_F; \tag{8}$$

### 2.1 The nucleon mass in the bag model in NM

Describing nucleons as bags, pressure will influence their surfaces [17,23–26]. Finite pressure corrections to a mass can not be described clearly by a perturbative QCD [27]. Let us discuss the relation (7) in the simple bag model where the nucleon in the lowest state of three quarks is a sphere of a volume $V_N$. Its energy $E_{Bag}$ is a function of the radius $R_0$ with phenomenological constants - $\omega_0$, $Z_0$ [17] and a density dependent bag "constant" $B(\varrho)$ with $B_0 = B(\varrho_0)$. We have [28]

$$E^0_{Bag}(R_0) \;=\; \frac{3\omega_0 - Z_0}{R_0} + \frac{4\pi}{3}B(\varrho_0)R_0^3 \propto\; 1/R_0, \tag{9}$$

The condition

$$p_B = -\left(\partial E^0_{Bag}/\partial V_N\right)_{surface} = 0 \tag{10}$$

for pressure inside a bag in equilibrium, measured on a surface, gives the relation between $R_0$ and $B$, used in the end of (9). $E^0_{Bag}$ fits to the mass $M_N$ at equilibrium $p_H = p_B = 0$. ($E^0_{Bag}$ differs from the $M_N$ by the c.m. correction [24]). In a compressed medium, pressure generated by free quarks inside the bag [28] is balanced at the bag surface not only by intrinsic confining "pressure" $B(\varrho)$ but also by nuclear pressure $p_H$; generated e.q. by elastic collisions with other hadron [23,25] bags, also derived in QMC model in a medium [17]. In equilibrium internal parton pressure $p_B$ (10) inside the bag is equal (cf. [17]), on a bag surface, nuclear pressure

$$p_H = p_B \;=\; \frac{3\omega_0 - Z_0}{4\pi R^4} - B(\varrho) \;\;\rightarrow\;\; (B(\varrho) + p_H)R^4 = const$$

and we get the radius depending from $B + p_H$:

$$R(\varrho) \;=\; \left[\frac{3\omega_0 - Z_0}{4\pi(B(\varrho) + p_H(\varrho))}\right]^{1/4}. \tag{11}$$

Thus, the pressure $p_H(\varrho)$ between the hadrons acts on the bag surface similarly to the bag "constant" $B(\varrho)$. A mass $M_{pr}$ for finite $p_H(\varrho)$ can be obtained from (9,11):

$$M_{pr}(\varrho) = \frac{4}{3}\pi R^3\left[4(B + p_H) - p_H\right] = E^0_{Bag}\frac{R_0}{R} - p_H V_N. \tag{12}$$

The scaling factor $R_0/R$ comes from a well-known model dependence (9) ($E^0_{bag} \propto 1/R_0$) in the spherical bag [28]. This simple radial dependence is now lost in (12) and responsible for that is the pressure dependent correction to the mass of a nucleon given by the product $p_H V_N$. This term is identical with the work $W_N$ in (7) and disappear for the nucleon enthalpy

$$H_N(\varrho) = E^0_{Bag}\frac{R_0}{R(\varrho)} \propto 1/R(\varrho). \tag{13}$$

The nucleon radius $R(\varrho)$ reflects a scale of a confinement of partons. Generally, for increasing $R(\varrho)$, $H_N(\varrho)$ (13) decreasing, thus part of the nucleon rest energy is transferred from a confined region $V_N$ to an remaining space $V_{A-}$ (8). For decreasing $R$, the $H_N$ increasing; this allows the constant or increasing mass $M_{pr}$ (12). The constant $R$ (13) require the work $W_N$ to keep the constant volume at the expense of the nucleon mass $M_{pr}$ (12). The internal pressure $B(\varrho)$, just as the external pressure $p_H(\varrho)$ (generated by an effective meson exchanges), has the same origin [29] from an interaction of quarks. Therefore, increasing $p_H(\varrho)$ we can expect the corresponding decrease in $B(\varrho)$. The sum $B(\varrho) + p_H(\varrho)$ weakly depends on density in GCM [26] or QMC models [17,24] with a reasonable stiff EOS, thus the bag radius remains about constant (11). It justify the choice of the total nucleon rest energy $H_N$, unchanged by an increasing NN repulsion. Just opposite to the case with the constant nucleon mass $M_{pr} = M_N$, which requires the increasing total energy $H_N$ (12,13) and a decrease of the nucleon size.

## 3 Results and Discussion

We will compare two scenarios: (A) constant nucleon mass with decreasing radius [16] and (B) constant nucleon radius with decreasing mass [15]. We applied therefore following formulas (7,8) for nucleon mass $M_{pr}$ inside NM in scenario (B) decreasing with pressure:

$$\begin{aligned} &M_{pr}(\varrho) = M_N - p_H(\varrho)V_N, \qquad \varrho \geq \varrho_0 \\ &p_H(\varrho) = \varrho^2 \varepsilon'_A(\varrho)/(1 - \varrho V_N). \end{aligned} \tag{14}$$

In scenario (A) we have instead of decreasing mass the compression of the nucleon volume accompanying with energy transfer $\Delta E$ from meson fiels, in order to keep the nucleon mass constant.

To carry out calculations in scenario (B) we combine the $M_{pr}$ dependence (14) of pressure $p_H$ at the constant nucleon radius $R = R_0$, with the following standard $(\sigma - \omega)$ RMF equations [15, 16] for the energy $\varepsilon_A$ in terms of the effective mass $M^*_{pr}$ and energy transfer $\Delta E$ present only in scenario (A):

$$\varepsilon^q_N(\varrho) = \frac{g_v^2}{2m_v^2}\varrho + \frac{m_s^2}{g_s^2\varrho}(M_{pr} - M^*_{pr})^2 + \frac{\gamma}{\varrho}\int_0^{P_F}\frac{d^3\boldsymbol{P}_N}{(2\pi)^3}\sqrt{\boldsymbol{P}_N^2 + M^{*2}_{pr}} - \Delta E \tag{15}$$

$$\text{with} \qquad M_{pr}^* = M_{pr} - \frac{\gamma g_s^2}{2m_s^2} \int_0^{P_F} \frac{d^3\boldsymbol{P}_N}{(2\pi)^3} \frac{M_{pr}^*}{\sqrt{\boldsymbol{P}_N^2 + M_{pr}^{*2}}}$$

$$\text{where} \qquad \Delta E(\varrho) = p_H V_N = \frac{\varrho^2 \varepsilon_N^{q\,\prime}(\varrho) V_N(\varrho)}{(1 - \varrho V_N(\varrho))}$$

$$\text{with} \qquad R_0/R(\varrho) = 1 + \Delta E(\varrho)/M_{pr}(\varrho)$$

$\gamma$ denotes a level degeneracy and there are two (coupling) constants: a vector $g_v^2$ and a scalar $g_s^2$, which were fitted [1, 2] at two different saturation points ($\varrho_0 = 0.16, 0.19$ fm$^{-3}$ – see a figure caption) in NM.

The EoS (pressure versus density) present in scenario (A) is shown in Figure 3 of [16], the EoS of scenario (B) in displayed in Figure 2 of [15]. In both cases the area indicated by the "flow constraint" taken from [30] determines in the plots the allowed course of the EoS, using an analysis which extracts information from the matter flow in heavy ion collisions from the high pressure obtained there. Walecka [1] and DBHF calculations [31] with a Bonn $A$ interaction are shown for references. Results for pressure in both scenarios are similar, however critical densities are very different. This difference illustrates Figure 1, where the nuclear energy density $\varrho\varepsilon_N^q(\varrho)$ grows with density while the nucleon energy density $M_{pr}(\varrho)/V_N(\varrho)$ in scenario (**B**) declines and finally both energy densities for $\varrho \sim 0.5$ fm$^{-3}$ are equal. For that density, nucleon bags with constant $R_0 \sim 0.7$ starts to overlap in case (**B**) and multi-quark bags would be possibly formed. The alignment density depends strongly on the nucleon radius, in turn the points where $B(\varrho)$=0 depend mainly from the starting value $B(\varrho_0)$. For example, for $R_0 = 0.75$ fm the alignment density $\varrho_{al} = 0.44$ fm$^{-3}$, shown in Figure 1, almost coincides [16] a vanishing bag constant $B(\varrho_0) = 100$ MeV fm$^{-3}$. Therefore, scenario (**B**) with constant nucleon radius and the gradual alignment of the energy densities inside and outside the bag suggests the

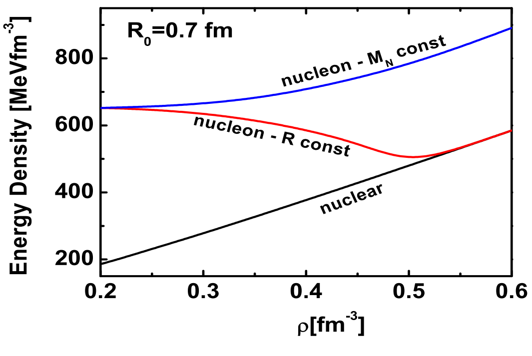


Figure 1. Energy density inside nucleons as a function of the nuclear density for $R_0 = 0.7$ fm in two cases: (A) const. nucleon mass (red line) and (B) const. radius (blue line). The density of nuclear energy (black line) is shown for reference.

crossover transition below $\varrho = 0.45$ fm$^{-3}$. However, such a transition around $\varrho \simeq 0.4$ fm$^{-3}$ is not observed in heavy ion experiments. Also in neutron stars [34], for that density of star core we would expected for the quark core to decrease the radius of the star, but such a decrease is not expected in comparison to lighter stars with a standard neutron core. The scenario (**A**) with constant nucleon mass [16] is more realistic then scenario (**B**) [15] without energy transfer. The energy transfer, equal to the volume energy $\Delta E = p_H V_N$, provides the constant nucleon mass, the good values of the compressibility $K_q^{-1} \sim (250 - 350)$ MeV and the symmetry energy $E_s = 31$ MeV. The energy transfer at the saturation region above the equilibrium density, reduces also the value of the slope of the symmetry energy from $L \simeq 108$ MeV to $L \simeq 61$ MeV [35]. Also for constant nucleon mass in scenario (**A**), a nucleon volume decreases with $\varrho$, therefore nucleon bags do not overlap for large density and the energy density of the nucleon increases due to the energy transfer into nucleon bags.

## 4 Conclusions

We have shown, how nucleon volumes in compressed NM affect the nuclear compressibility at equilibrium, reducing the nucleon mass or volume in the EoS. It effectively corresponds to nonlinear, pressure dependent modifications of a scalar potential. Not accidentally, in the widely used standard [12, 13] RMF model with point-like nucleons the good compressibility is fit by nonlinear modifications of a scalar mean field with the help of two additional parameters. Thus, our results suggests to reconsider these mean field parameters. The presented model of compressed nucleons in dense NM is suitable for studying heavy ion collisions and neutron star properties (mass–radius constraint); especially the most massive known neutron stars [33] recently discover and we plan to calculate the symmetry energy and include the octet of baryon, including strangeness, in a next work.

## References

[1] B.D. Serot and J.D. Walecka, *Adv. Nucl. Phys.* **16** (Plenum, N. Y. 1986).

[2] R.J. Furnstahl and B.D. Serot, *Phys. Rev. C* **41** (1990) 262.

[3] J. Zimanyi and S.A. Moszkowski, *Phys. Rev. C* **42** (1990) 1416.

[4] J. Boguta and A.R. Bodmer, *Nucl. Phys. A* **292** (1977) 413; J. Boguta, Stocker, *Phys. Lett. B* **120** (1983) 289.

[5] N.K. Glendenning, S.A. Moszkowski, *Phys. Rev. Lett.* **67** (1991) 2414; N.K. Glendenning, F. Weber, S.A. Moszkowski, *Phys. Rev. C* **45** (1992) 844 (1992).

[6] R.L. Jaffe, Los Alamos School on Nuclear Physics, CTP 1261, Los Alamos, July 1985.

[7] J.R. Smith and G.A. Miller, *Phys. Rev. C* **65** (2002) 015211, 055206.

[8] J. Rożynek, G. Wilk, *Phys. Rev. C* **71** (2005) 068202.

[9] J. Arrington, R. Ent, C.E. Keppel, J. Mammei, I. Niculescu, *Phys. Rev. C* **73** (2006) 035205; L.B. Weinstein et al., *Phys. Rev. Lett*. **106** (2011) 052301.

[10] D.M. Alde et al., *Phys. Rev. Lett*. **64** (1990) 2479.

[11] J. Rożynek, *Nucl. Phys. A* **755** (2004) 357.

[12] N.K. Glendenning, "*Compact Stars*" (Springer-Verlag, New York, 2000); P. Haensel, A.Y. Pothekin, D.G. Yakovlev, "*Neutron Stars 1*" (Springer, 2007).

[13] J. Schaffner-Bielich, M. Hanauske, H. Stöcker, W. Greiner, *Phys. Rev. Lett.* **89** (2002) 171101.

[14] D.H. Rishke, M.I. Gorenstein, H. Stöcker, W. Greiner, *Z. Phys.* **51** (1991) 485.

[15] J. Rożynek, *J. Phys. G* **42** (2015) 045109.

[16] J. Rożynek, *IJMPE* **27** (2018) 1850030.

[17] Guo Hua, *J. Phys. G* **25** (1999) 1701; P.A. Guichon, *Phys. Rev. Lett. B* **200** (1988) 235.

[18] N.M. Hugenholtz and L.M. van Hove, *Physica* **24** (1958) 363.

[19] E. Oset, L.L. Salcedo, *Nucl. Phys.* **468** (1987) 631; "*The Nuclear Methods and the Nuclear Equation of State*", ed. M. Baldo, (World Scientific 1999).

[20] J. Boguta, *Phys. Lett.* **106B** (19881) 255.

[21] L.L. Frankfurt and M.I. Strikman, *Phys. Rep*. **160** (1988) 235.

[22] M. Birse, *Phys. Lett. B* **299** (1993) 188; L.L. Frankfurt and M.I. Strikman, *Nucl. Phys. B* **316** (1989) 340.

[23] L. Ferroni and V. Koch, *Phys. Rev. C* **79** (2009) 034905.

[24] X. Jin and B. K. Jennings, *Phys. Rev. C* **54** (1996) 1427; H. Müller, B.K. Jennings, *Nucl. Phys. A* **626** (1997) 966.

[25] J.I. Kapusta and Ch. Gale, "*Finite Teperatures Field Theory*" (Cambridge University Press, New York 2006).

[26] Y. Liu, D. Gao, H. Guo, *Nucl. Phys.* **A695** (2001) 353; R.T. Cahill, C.D. Roberts J. Praschifka, *Ann. Phys. (NY)* **188** (1988) 20.

[27] G.E. Brown, M. Rho, *Phys. Rev. Lett*. **66** (1991) 2720.

[28] K. Johnson, *Acta Phys. Pol.* **B6** (1975) 865; A. Chodos et al., *Phys. Rev. D* **9** (1974) 3471.

[29] Buballa M., *Nucl. Phys. A* **611** (1996) 393.

[30] P.Danielewicz, R. Lacey, W. G. Lynch, *Science* **298** (2002) 1592.

[31] T. Gross-Boelting, C. Fuchs, A. Faessler, *Nucl. Phys. A* **648** (1999) 105 (1999); E.N.E. van Dalen, C. Fuchs, A. Faessler, *Phys. Rev. Lett*. **95** (2005) 022302; C. Fuchs, *J. Phys. G* **35** (2008) 014049; D.P. Menezes et al., *Phys. Rev. C* **76** (2007) 064902.

[32] I. Bombaci et al. Phys. Rep. 280 (1997) ???.

[33] P.B. Demorest et al., *Nature* **467** (2010) 7319; Antoniadis et al., *Science* **340** (2013) 6131.

[34] M.A. Kayum Jafry et al. *Astrophys. Space Sci.* **362** (2017) 188.

[35] to be published.