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Investigation of Binding Energies of Lambda Hypernuclei

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Abstract. We consider statistical disintegration of hypernuclear systems at high energy nuclear reactions and the connection of fragment production with the binding energies of hyperons. We have demonstrated that the hyperon binding energies can be evaluated from the yields of different isotopes of Lambda hypernuclei via the double ratio method. In this method, the universal behaviour can be demonstrated by taking into account many different isotopes. It can also be applied for multi-strange nuclei, for which binding energies were very difficult to measure in previous hypernuclear experiments. Hopefully, our calculations would be the pioneering for the extraction and analysis of future hyperon experiments such as at GSI/FAIR facilites.

1 Introduction

In this presentation,we discuss the binding energy of hypernuclei by taking into account the deep-inelastic reaction processes in relativistic projectiles leading to abundant production of hyperons in primary and secondary hadron collisions. It is considered that the capture of these hyperons by the matter is verified with producing hyper-matter in chemical equilibrium. Fragmentation and multifragmentation reactions can be assumed to be initiated in relativistic peripheral nucleus-nucleus collisions and in high-energy hadron/lepton reactions on large targets. Many various hypernuclei which may be produced in the same reaction bring new opportunities for their investigation in comparison with previous methods. It is clear that complex multi-hypernuclear systems incorporating more than two hyperons can be created in the energetic nucleus-nucleus collisions [1, 2]. In this way one may go beyond double hypernuclei, and obtain new experimental information on properties of multi-hyperon systems. In this paper we focus on the theoretical investigation of this kind of reactions and propose a new double ratio method demonstrating how the important knowledge on the hyperon binding energies, including in multi-strange nuclei, can be extracted from analysis of the relative yields of hypernuclei.

2 Statistical Model Description for Lambda Hypernuclei

In high-energy nucleus-nucleus and hadron-nucleus collisions formation of excited nuclear residues were studied by the analyses of fragmentation and multifragmentation processes. In the existing experimental and theoretical works [2, 3], masses and excitation energies of the residues are known. Multifragmentation process is one of the decay modes and it appears at high excitation energies [4–6]. The hyperon interactions in a nucleus are similar to normal nuclear ones, and its potential is around 2/3 of the nucleon one. Therefore, the general picture of disintegration reactions with a large energy deposition in a big piece of nuclear matter does not change in the presence of few hyperons. It is known that, multifragmentation is a relatively fast process, with a characteristic time around 100 fm/c, where a high degree of statistical equilibration is reached. This is a consequence of the strong interactions between baryons located in the vicinity of each other at the freeze-out volume. Since the statistical models have very good agreement with fragmentation and multifragmentation data [3,4,6,7], we aim to extend the statistical approach for hypernuclear systems on the same basis. The statistical multifragmentation model (SMM), which was very successfully applied for description of standard multifragmentation processes, was generalized for hypernuclei in Ref. [8]. A transition from the compound hypernucleus to the multifragmentation regime was also under investigation [8, 9]. In this approach, the break-up channels are generated according to their statistical weight. The Grand Canonical approximation leads to the following average yields of individual fragments with the mass (baryon) number A, charge Z, and the Λ-hyperon number H:

$$\begin{aligned} Y_{\rm A,Z,H} &= g_{\rm A,Z,H} \cdot V_f \frac{A^{3/2}}{\lambda_T^3} \exp\left[-\frac{1}{T} \left(F_{A,Z,H} - \mu_{AZH} \right) \right], \\ \mu_{AZH} &= A\mu + Z\nu + H\xi. \end{aligned} \tag{1}$$

Here T is the temperature, $F_{A,Z,H}$ is the internal free energies of these fragments, V_f is the free volume available for the translation motion of the fragments. $g_{\rm A,Z,H}$ is the spin degeneracy factor of species (A, Z, H) and $\lambda_T = \left(2\pi\hbar^2/m_N T\right)^{1/2}$ is the baryon thermal wavelength, m_N is the average baryon mass. The chemical potentials μ, ν, and ξ are responsible for the mass (baryon) number, charge, and strangeness conservation in the system, and they can be numerically found from the corresponding conservation laws accounting for the total baryon number A_0, the total charge Z_0, and the total hyperon number H_0 in the system. In this model the statistical ensemble includes all break-up channels composed of baryons and excited fragments. The primary fragments are formed in the freeze-out volume V. We use the excluded volume approximation $V = V_0 + V_f$, where $V_0 = A_0/\rho_0$ ($\rho_0 \approx 0.15$ fm^{-3} is the normal nuclear density), and parametrize the free volume $V_f = \kappa V_0$, with $\kappa \approx 2$, as taken in description of experiments in Refs. [3, 6, 7].

The model prescriptions depend on the physical processes which are the most adequate to the analyzed reactions. In many cases nuclear clusters in the freeze-out volume can be described in the liquid-drop approximation where the light fragments with mass number $A < 4$ are treated as elementary particles with corresponding spins and translation degrees of freedom ("nuclear gas"). Their binding energies were taken from experimental data [4, 10, 11]. The fragments with $A = 4$ are also treated as gas particles with table masses, however, some excitation energy is allowed $E_x = AT^2/\varepsilon_0$ ($\varepsilon_0 \approx 16$ MeV is the inverse volume level density parameter [4]), that reflects a presence of excited states in ^{4}He, $^4_\Lambda$H, and $^4_\Lambda$He nuclei. Fragments with $A > 4$ are treated as heated liquid drops. In this way we can study the nuclear liquid-gas coexistence of hypermatter in the freeze-out volume. The internal free energies of these fragments are parametrized as the sum of the bulk (F^B_A), the surface (F^S_A), the symmetry ($F^{\rm sym}_{AZH}$), the Coulomb (F^C_{AZ}), and the hyper energy ($F^{\rm hyp}_{AH}$):

$$F_{A,Z,H} = F^B_A + F^S_A + F^{\rm sym}_{AZH} + F^C_{AZ} + F^{\rm hyp}_{AH} \ . \tag{2}$$

Here, the first three terms are written in the standard liquid-drop form [4]:

$$F^B_A = \left(-w_0 - \frac{T^2}{\varepsilon_0}\right) A \ , \tag{3}$$

$$F^S_A = \beta_0 \left(\frac{T_c^2 - T^2}{T_c^2 + T^2}\right)^{5/4} A^{2/3} \ , \tag{4}$$

$$F^{\rm sym}_{AZH} = \gamma \frac{(A - H - 2Z)^2}{A - H} \ , \tag{5}$$

where $w_0 = 16$ MeV, $\beta_0 = 18$ MeV, $T_c = 18$ MeV and $\gamma = 25$ MeV are the model parameters which are extracted from nuclear phenomenology and provide a good description of multifragmentation data [3, 4, 6, 7]. The Coulomb interaction of fragments is described within the Wigner-Seitz approximation, and F^C_{AZ} is taken as in the ref. [4, 8]:

$$F^C_{AZ}(V) = \frac{3}{5}\left[1 - \left(\frac{V_0}{V}\right)^{1/3}\right] \frac{(eZ)^2}{r_0 A^{1/3}} \ . \tag{6}$$

where $r_0 = 1.2$ fm and e denotes the electron charge.

The free hyper-energy term $F^{\rm hyp}_{AH}$ determines the binding energy of Lambda hyper-fragments. Presently, only few ten masses of single hypernuclei (mostly light ones) are experimentally established [10, 11], and only few single-event measurements of double hypernuclei exist. Still, there are theoretical estimations of their masses including hyperon binding energies based on this limited amount of available data. In Ref. [8] we have suggested a liquid drop hyper–term:

$$F^{\rm hyp}_{AH} = (H/A) \cdot (-10.68A + 21.27A^{2/3}) \ \text{MeV}. \tag{7}$$

Such a term is proportional to the share of hyperons in matter (H/A). The second part is the volume contribution minus the surface one, which is a normal liquid-drop parametrization assuming saturation of the nuclear interactions. The linear dependence at small H/A is in agreement with theoretical predictions [12] for hyper-matter. As was demonstrated in Refs. [8, 9] this parametrization of the hyperon free energy describes available experimental data on the hyperon separation energy quite reasonable. It is important that two boundary physical effects are correctly reproduced: The binding energies of light hypernuclei (if a hyperon substitutes a neutron) can be lower than those of normal nuclei, since the hyperon-nucleon potential is smaller than that of the nucleon-nucleon one. However, since the hyperon can take the lowest s-state, it can increase the nuclear binding energies, specially for large nuclei. There were also suggested other phenomenological hyper-formulae (e.g., [13]), and their sensitivity for fragment production was under investigations [8].

3 Double Ratio Method for Lambda Hypernuclei

One can write the binding energy $E_A^{\rm bh}$ of one hyperon at the temperature T inside a hypernucleus with (A, Z, H) :

$$E_A^{\rm bh} = F_{A,Z,H} - F_{A-1,Z,H-1} \ . \tag{8}$$

Since Λ-hyperon is usually bound, this value is negative. Then the yield of hypernuclei with an additional Λ hyperon can be recursively written by using the former yields:

$$Y_{\rm A,Z,H} = Y_{\rm A-1,Z,H-1} \cdot C_{\rm A,Z,H} \cdot \exp\left[-\frac{1}{T}\left(E_A^{\rm bh} - \mu - \xi\right)\right], \tag{9}$$

where $C_{\rm A,Z,H} = (g_{\rm A,Z,H}/g_{\rm A-1,Z,H-1}) \cdot (A^{3/2}/(A-1)^{3/2})$ depends mainly on the ratio of the spin factors of (A, Z, H) and $(A-1, Z, H-1)$ nuclei, and very weakly on A. Since in the liquid-drop approximation we assume that the fragments with $A > 4$ are excited and do populate many states above the ground ones according to the given temperature dependence of the free energy, then we take $g_{\rm A,Z,H} = 1$. In this way we can connect the relative yields of hypernuclei to the hyperon binding energies via SMM using other parametrizations to describe nuclei in the freeze-out. We believe that this statistical approach is quite universal, and only small corrections, like the table-known spins and energies, may be required for more extensive consideration.

We propose the following recipe for obtaining information on the binding energies of hyperons inside nuclei from the hypernuclei yields [14]. Let us take two hyper-nuclei with different masses, (A_1, Z_1, H) and (A_2, Z_2, H), together with nuclei which differ from them only by one Λ hyperon. We consider the double ratio (DR) of $Y_{\rm A_1,Z_1,H}/Y_{\rm A_1-1,Z_1,H-1}$ to $Y_{\rm A_2,Z_2,H}/Y_{\rm A_2-1,Z_2,H-1}$. One can obtain from the above formulae that

$$DR_{A_1A_2} = \frac{Y_{\mathrm{A_1,Z_1,H}}/Y_{\mathrm{A_1-1,Z_1,H-1}}}{Y_{\mathrm{A_2,Z_2,H}}/Y_{\mathrm{A_2-1,Z_2,H-1}}} = \alpha_{A_1A_2}\mathrm{exp}\left[-\frac{1}{T}\left(\Delta E^{\mathrm{bh}}_{A_1A_2}\right)\right], \quad (10)$$

where

$$\Delta E^{\mathrm{bh}}_{A_1A_2} = E^{\mathrm{bh}}_{A_1} - E^{\mathrm{bh}}_{A_2}, \quad (11)$$

and the ratio of the C-coefficients we denote as

$$\alpha_{A_1A_2} = C_{\mathrm{A_1,Z_1,H}}/C_{\mathrm{A_2,Z_2,H}} \, . \quad (12)$$

We see that the double ratio depends only on the temperature of the system and the difference between the hyperon separation energies of the fragments.

One can simply deduce from eq.(10) that the logarithm of the double ratio is directly proportional to the difference of the hyperon binding energies in A_1 and A_2 hypernuclei, $\Delta E^{\mathrm{bh}}_{A_1A_2}$, divided by temperature. Therefore, we can finally rewrite the relation between the hypernuclei yields and the hyperon binding energies as

$$\Delta E^{\mathrm{bh}}_{A_1A_2} = T \cdot \left[ln(\alpha_{A_1A_2}) - ln(DR_{A_1A_2})\right]. \quad (13)$$

Sometimes we expect a large difference in hyperon binding energies of both involved nuclei. For example, according to the liquid-drop approach (see eq. (2)), it can happen when the difference between A_1 and A_2 is considerable (i.e., the mass number A_2 is much larger than A_1). The influence of the pre-exponential $\alpha_{A_1A_2}$ coefficients is small and can be directly evaluated, as depending on the selected hypernuclei. This opens a possibility for the explicit determination of the binding energy difference from the yields measured in experiments. Within this method, it is necessary to measure a certain number of the hypernuclei in one reaction and select the corresponding pairs of hypernuclei. One has to identify such hypernuclei, for example, by the correlations and vertex technique. However, there is no need to measure very precisely the momenta of all particles produced in the reaction (including after the week decay of hypernuclei) to obtain their binding energy, as it must be done if one use direct processes of the hyperon capture in the ground and slightly excited states of the target nuclei (e.g., in missing mass experiments [11, 15]).

4 Conclusion

We conclude that the suggested double ratio method can be related to deep inelastic reactions producing all kind of hypernuclei with sufficiently large cross-sections in the multifragmentation processes at relativistic ion-ion and hadron-ion collisions. Only the identification of hypernuclei is required and, as demonstrated in recent ion experiments, there are effective ways to perform it. The experimental extraction of the difference in the hyperon binding energies between hypernuclei ($\Delta E^{\mathrm{bh}}_{A_1A_2}$) via their yields is a novel and practical way to pursue hypernuclear studies. With the double ratio method novel conclusions can be obtained for neutron-rich and neutron-poor hypernuclei. The isospin influence on the hyperon interaction in matter will be possible to extract directly from experimental data. Especially multi-strange nuclear systems would be interesting, since they can give information on evolution of the hyperon-hyperon interactions depending on strangeness. These measurements are important for many astrophysical sites, for example, for understanding the neutron star structure [16, 17]. Such kind of research may be possible at the new generation of ion accelerators of intermediate energies, such as FAIR (Darmstadt), NICA (Dubna), and others. It is promising that new advanced experimental installations for the fragment detection will be available soon [18, 19].

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