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Shell Merging in SU(3)

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Abstract. In Shell model studies the islands of inversion, which appear aside with shape coexistence, derive due to shell merging [1]. A super shell is derived, due to the merging of a spin-orbit (s.o.) like shell [2] with a harmonic oscillator (h.o.) shell [3]. Shell merging can be realized in an SU(3) Model by coupling the SU(3) irreps: $(\lambda,\mu)_{s.o.} \times (\lambda,\mu)_{h.o.}$.

1 Introduction

Recently the islands of shape coexistence have been predicted [3,4] using Proxy SU(3) symmetry [5,6]. The nucleon numbers of nuclei with shape coexistence are predicted within the deformation:

$$\beta_{h.o.} \leq \beta_{s.o.} \tag{1}$$

The deformation parameter has been calculated through [7]:

$$\beta^2 = \frac{4\pi}{5(A\bar{r}^2)^2}(\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda+\mu)) \tag{2}$$

with the use of the highest weight SU(3) irreps for each set of magic numbers [9]. The condition (1) predicts the islands of shape coexistence on the nuclear chart [8].

2 The Mechanism for Shape Coexistence

We suggest, that the natural mechanism for shape coexistence involves the following steps:

1. The number of valence protons or neutrons is sufficient to create large QQ interaction.

2. The deformation compresses the single particle energy gaps at the spin-orbit magic numbers 14, 28, 50, 82.

3. Super shells from a h.o. magic number to a s.o. magic number: 2-14, 8-28, 20-50, 40-82, 70-126 are created, which come from the coupling of the two shells.

4. The super shells derive either the excited band, or the ground state band of nuclei with shape coexistence.

This mechanism is free from enhanced proton-neutron interaction due to the Federman-Pittel mechanism [10, 11] and does not involve proton excitations from a so like shell to the next so like shell. The mechanism emerges naturally due to the preference of the QQ interaction to the ho shells.

3 Coupling of the SU(3) irreps

The super shells are described by the coupled irreps:

$$(\lambda,\mu)_{s.o.} \times (\lambda,\mu)_{h.o.} \to (\lambda,\mu)_{coupl.} \tag{3}$$

The rules of the coupling of the SU(3) irreps are described in [12, 13]. In this article I will review the method of Sidney Coleman.

The first is to decompose the product $(\lambda,\mu)\times(\lambda',\mu')$ to a sum of reducible representations:

$$\begin{aligned}(\lambda,\mu)\times(\lambda',\mu') &= (\lambda,\lambda';\mu,\mu') \oplus (\lambda-1,\lambda';\mu,\mu'-1)\\ &\oplus (\lambda,\lambda'-1;\mu-1,\mu') \oplus (\lambda-1,\lambda'-1;\mu-1,\mu'-1) \oplus \ldots\end{aligned} \tag{4}$$

The procedure stops whenever a zero appears in the right side of the equation. The second step is to reduce the reducible representations

$$\begin{aligned}(\lambda,\lambda';\mu,\mu') =& (\lambda+\lambda',\mu+\mu')\\ &\oplus \sum_{i=1}^{\min(\lambda,\lambda')} (\lambda+\lambda'-2i,\mu+\mu'+i)\\ &\oplus \sum_{j=1}^{\min(\mu,\mu')} (\lambda+\lambda'+j,\mu+\mu'-2j).\end{aligned} \tag{5}$$

The code of [14] has the ability to export all the resulting irreps from the coupling. It has also the ability to derive the SU(3) Clebsch-Gordan (CG) coefficients, that arise from the coupling [15, 16].

4 The Energy

The simplest Hamiltonian in Elliott SU(3) [17, 18] is

$$H = H_0 - \frac{\chi}{2} QQ\,, \tag{6}$$

where $H_0 = \sum_{i=1}^{A}(\frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 r_i^2)$ and

$$QQ = 4C_2 - 3L^2 \tag{7}$$

with C_2 being the second order Casimir operator of SU(3):

$$C_2 = \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu). \tag{8}$$

If the excited 0_2^+ state of a nucleus with shape coexistence is derived by the coupled irrep and the ground state by the spin-orbit like irrep, then

$$0_2^+ = \frac{\chi}{2}(C_{2,s.o.} - C_{2,\text{coupl.}}) \,. \tag{9}$$

Using rational values for the strength χ there is at least one coupled irrep, which satisfies the data for the energy 0_2^+ of even-even nuclei with shape coexistence.

5 Conclusions

Shell merging is easy to be accomplished within a Fermionic SU(3) model. The right coupled irrep will satisfy the energy of the 0_2^+ state of even-even nuclei and will predict the right J^π of even-odd or odd-even nuclei with shape coexistence.

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