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Study of Pear-Shape Effects in the Spectra of Even-Even Nuclei

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Abstract. The experimental confirmations of stable octupole shape in certain nuclei, such as ^{224}Ra and ^{144,146}Ba, achieved not long ago, motivated new applications and further development of a collective model approach capable to describe alternating-parity spectra corresponding to the manifestation of axial quadrupole-octupole (QO) (pear-shape) deformations in even-even nuclei. With its two versions (limits) describing the so-called "rigid" and "soft" QO modes the approach allows one to explore wide ranges of nuclei including regions with less expected manifestation of QO collectivity. Such is the region of light Neodymium isotopes $^{130-136}$Nd considered in the present work. It is shown that in the four isotopes the experimental data provide alternating-parity band (APB) structures developed enough to allow model fits. For each nucleus the calculations are performed in the two model limits. The obtained results and the analysis of data suggest possible stabilization of the octupole shape in ^{136}Nd and presence of soft QO mode in $^{130-134}$Nd, with possible transition behaviour of ^{134}Nd between the two limits of QO collectivity.

1 Introduction

Recent experimental confirmation of stable octupole shape in the nucleus ^{224}Ra obtained in REX-ISOLDE [1] and in the nuclei ^{144,146}Ba obtained in ANL [2] has attracted a new interest in the structure and properties of the so-called pear-shaped atomic nuclei. These results motivate new applications and further development of a collective model approach capable to describe alternating-parity spectra inherent for the nuclei with pear-shape deformations [3,4]. In its "rigid" version, called QO rotation model (QORM), the model describes rotations of a stable quadrupole-octupole (QO) shape with low-energy oscillations in a double-well octupole potential [3]. In the "soft" realization, called coherent QO model (CQOM), it describes non-adiabatic coherent QO vibrations and rotation [4]. The overall approach allows us to study, from one side, the angular momentum dependence of the QO mode in given spectrum and, from another side, the evolution of alternating-parity spectra in given nuclear region between the manifestation of soft and stable octupole deformations. On this basis it provides a detailed test for the presence of octupole collectivity and can be of special use in the investigation of less studied nuclei.

The purpose of this study is to examine the yrast positive- and negative-parity levels of the isotopes $^{130-136}$Nd and clarify to what extent and in what form the octupole deformation mode manifests in the corresponding alternating-parity band (APB) structures. For this reason the above mentioned two versions of the collective QO model are applied. Detailed explanations of the model formalism, numerical realizations, parameters etc. for both models are given in [3,4] as well as in the subsequent works [5] for QORM and [6] for CQOM. Here we only remark that in each of the considered nuclei both versions are applied independently on the same footing allowing us to compare their relevance in each case and to assess on this basis the degree of stability of the corresponding QO mode.

2 Alternating-Parity Bands in $^{130\text{-}136}$Nd: Data Analysis and Model Descriptions

The evaluation of QO collectivity on the basis of energy spectra requires a thorough examination of the fine APB structure. Therefore, here in addition to the direct examination of the energy levels we also apply the so-called "staggering" analysis, in which the following five-point finite-differences expression is applied to the APB energy E as a function of the angular momentum I:

$$\mathrm{Stg}(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2)\,, \quad (1)$$

where $\Delta E(I) = E(I+1) - E(I)$. (See e.g. [3,4] for details.) By applying a similar expression in [7] it has been shown that in the higher angular-momentum regions of the octupole bands in light actinide nuclei (Rn, Ra, Th), where stabilized QO shapes are considered, the spectrum exhibits the so-called "beat" staggering structure. This effect has been explained through the high-order angular momentum terms in the QORM Hamiltonian appearing as the result of a specific point-symmetry based property of the rotating QO shape [3]. In the same way the fine structure of the octupole bands in several Ba and Ce nuclei was explained [5]. It was found that the axial QO (pear-shape) deformation plays the major role in the rotation dynamics of these nuclei. An important feature of the quantity $\mathrm{Stg}(I)$ is that in the case of stabilizing octupole deformation it decreases with the angular momentum, reaching zero at some value of I, whereas in the case of soft QO mode it only slightly decreases without reaching zero. In this way the staggering analysis provides a rather fine test for assessing the QO deformation stiffness. Below it will be seen that by making use of the staggering expression (1) we can evaluate the relevance of the model descriptions obtained by each of both model versions, QORM and CQOM, for the considered Nd nuclei.

The data for all isotopes $^{130-136}$Nd are taken from the ENSDF [8]. For $^{130-134}$Nd they allow us to construct APBs up to $I = 30$ having in mind that in all isotopes the lowest $I = 1^-$ and $I = 3^-$ states are missing in the experiment. Here mainly levels up to $I = 20$ were included in the fits, with only in one case,

^{134}Nd, a test of the models being made up to $I = 30$. In ^{136}Nd the APB is constructed up to $I = 14$ with all levels being included in the fit. (See band A (up to $I = 8$) and F ($I = 10-14$) for the positive-parity levels and band C for the negative-parity levels of ^{136}Nd given in [8]).

The results of the data analysis and the performed model calculations are given in Figures 1–6. For the model descriptions the root-mean-square (RMS) deviations between the theory and experiment are given in keV. The parameters of the fits in both models are not given for simplicity and we can only say that the obtained values are in the ranges typical for the other applications of both model versions (see [3,4]).

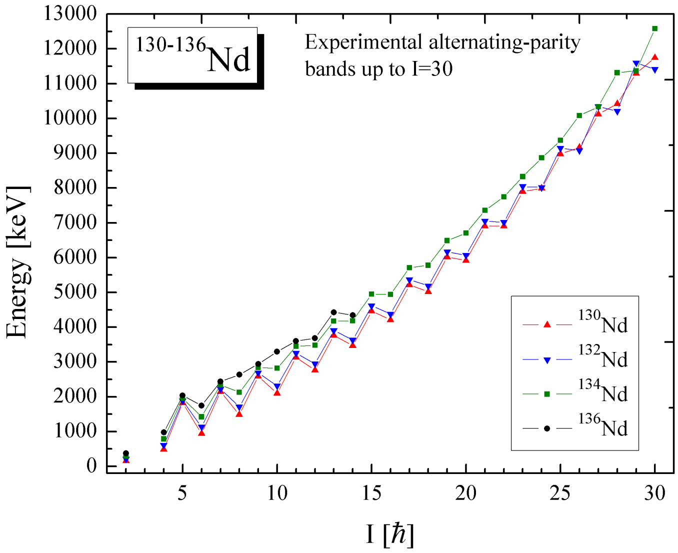

Figure 1. Experimental APBs in $^{130-136}$Nd up to $I = 30$. Data from ENSDF [8].

In Figure 1 the full APBs (according to available data) in the four Nd isotopes are compared. It is seen that the parity-shift considerably decreases in the bands with increasing neutron number from ^{130}Nd to ^{136}Nd. Moreover, in ^{136}Nd it is seen that between $I = 7$ and 11 the energy shift practically disappears and the levels appear ordered as in the octupole bands inherent for the nuclei with stable octupole deformation. Similar ordering is observed also in ^{134}Nd but at the much larger angular momenta $I = 22-25$.

In Figure 2 the QORM and CQOM descriptions of the APBs in the four isotopes $^{130-136}$Nd are compared with the experimental data. For ^{136}Nd it is seen, and also confirmed by the obtained RMS factors, 94 keV and 144 keV, respectively, that the rigid QORM limit of the model suggests more reasonable interpretation of this APB. In Figure 3 this observation is supported by the more detailed staggering diagrams which show that, despite of the small amount of

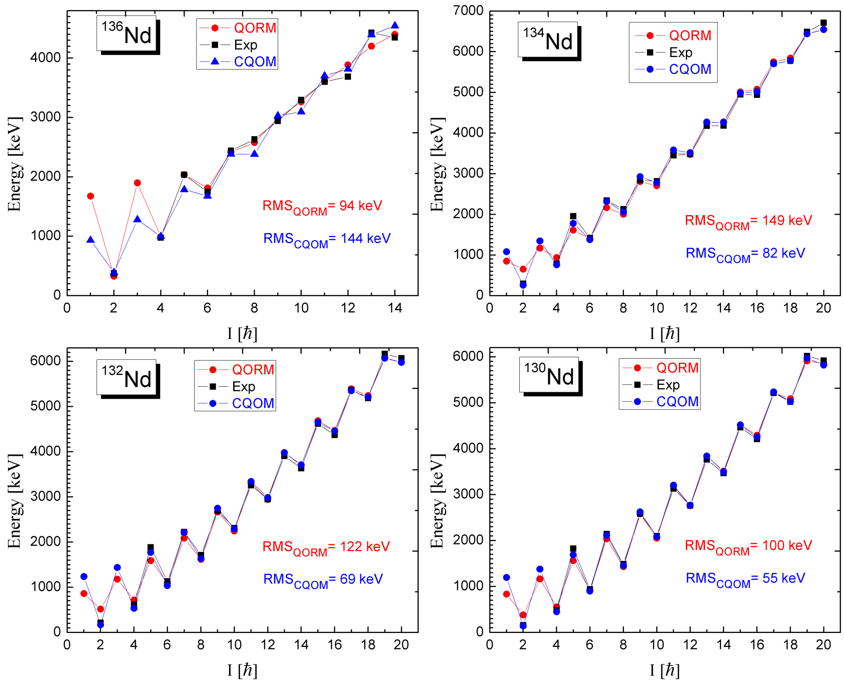

Figure 2. QORM and CQOM descriptions of $^{130-136}$Nd APBs. Data from ENSDF [8].

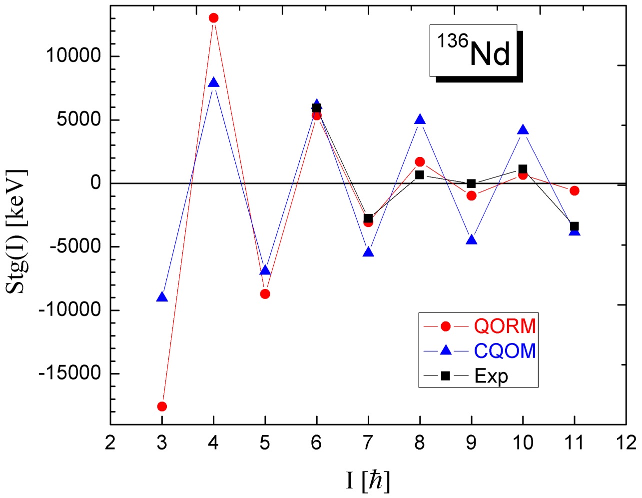

Figure 3. Experimental and theoretical staggering patterns for ^{136}Nd APB.

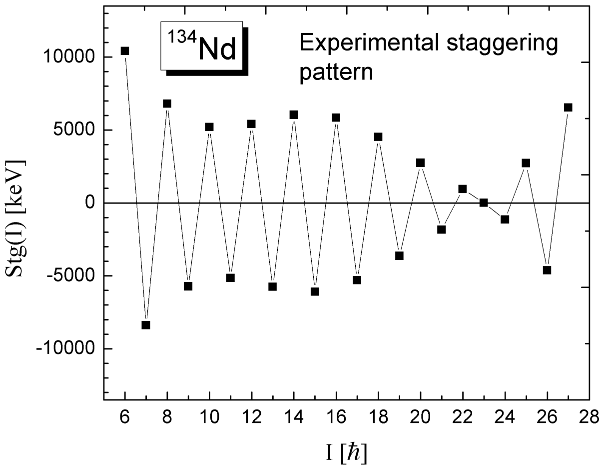

Figure 4. Experimental staggering pattern for ^{134}Nd APB with energy levels taken up to $I = 30$.

data, the fine structure of the APB in ^{136}Nd behaves similarly to the ones in nuclei with recognized octupole deformations [3,5,7].

The QORM and CQOM descriptions for the isotopes ^{134}Nd, ^{132}Nd and ^{130}Nd illustrated in Figure 2 are obtained up to $I = 20$. Now it is seen that in all considered levels of the three APBs there is a pronounced parity-shift effect. Also, we see that the soft CQOM limit of the model already provides better description of the experimental data compared to the rigid QORM. This result suggests that the APBs of these three nuclei can be generally referred to as a manifestation of the soft QO collective mode.

Now we recall that for ^{134}Nd in the region of $I = 22 - 25$ the APB starts to behave like an octupole band though further, towards $I = 30$ this spectrum again obtains a staggering structure (see Figure 1). Therefore, in Figure 4 we show the evolution of the ^{134}Nd APB up to $I = 30$ revealed in terms of the staggering function (1). Thus, we clearly identify the presence of a region around $I = 23$, where the staggering reaches zero amplitude and further reappears with inverse phase in the form of a secondary "beat" oscillation, similarly to the bands with stable octupole deformation (e.g. see [7]). It is, therefore, interesting to see how this structure can be classified according to the overall model interpretation of the rigid and soft octupole modes. For this reason crude QORM and CQOM fits were made for the extended ^{134}Nd APB up to $I = 30$, with RMS factors of 329 keV and 294 keV, respectively. The result is illustrated in Figure 5. In Figure 6 the corresponding theoretical staggering diagrams are given together with the experimental one. It is seen that as overall the experimental pattern is bracketed between the "soft" CQOM and the "rigid" QORM staggering points. Although the both descriptions are obtained beyond the angular momentum lim-

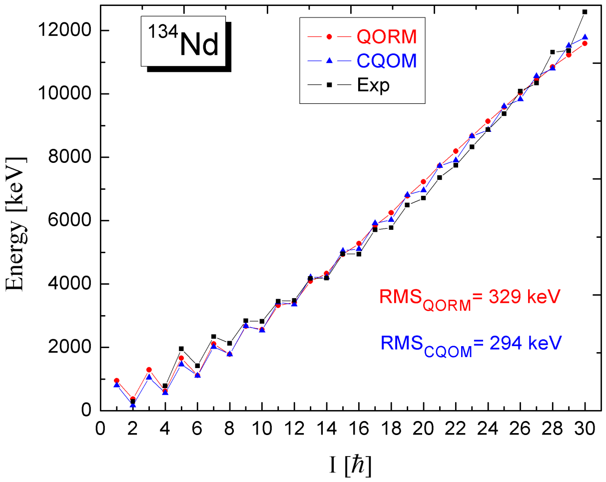

Figure 5. QORM and CQOM descriptions of ^{134}Nd APB up to $I = 30$. Data from ENSDF [8].

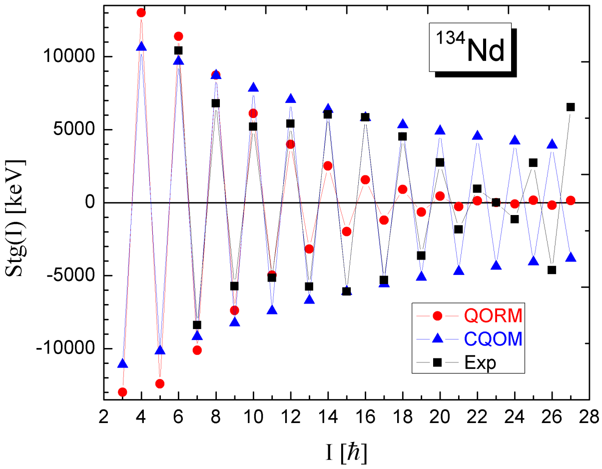

Figure 6. Experimental and theoretical staggering patterns for ^{134}Nd APB.

its of model applicability, they qualitatively suggest that ^{134}Nd may exhibit an intermediate behaviour with respect to the soft and stable QO modes. We remark that in this case there is no sense to compare or assess both model limits on the RMS basis.

3 Concluding Remarks

Summarizing the above considerations, it can be concluded that the analysis of data and the performed model calculations generally suggest a possibility for the presence of a soft QO mode in $^{130-134}$Nd and give an indication for a stabilization of the octupole shape in ^{136}Nd at relatively low angular momenta. At the same time the analysis made at higher angular momenta reveals possible transition behaviour of ^{134}Nd between these two limits. The obtained results clearly point out the need for further detailed study of the shape dynamics and related spectroscopic characteristics of the nuclei in this region.

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