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Symmetry Energy in Dense Nuclear Matter

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Abstract. We present the new calculations of the symmetry energy in the frame of a Modified Relativistic Mean Field (RMF) model where we take into account excluded volume corrections to nuclear energy [1], proportional to nuclear pressure and absent in a standard RMF with point-like nucleons [2, 3]. In particular we show how to determine/correct the saturation density which now depends additionally from sizes of nucleon bags inside NM. The symmetry energy E_S in our model has the similar corrections emerging from finite nucleon volumes. It gives in our model the constraint between the E_S and its derivative L which is well satisfied by the empirical values of E_s and L [4]. Fitting the E_s and L at the initial saturation point we solve the differential equations and present the results of the energy and the symmetry energy with their derivatives for high density and pressure. Equation of state in our modified RMF model agrees with a semi-empirical estimate and is close to results obtained from extensive DBHF calculations with a Bonn A potential, which produce the EoS stiff enough to describe neutron star properties (mass–radius constraint), especially the masses of "PSR J16142230" and "PSR J0348+0432", most massive ($\sim 2M_{\odot}$) known neutron stars [5].

1 EoS with an energy transfer

How an energy transfer between quarks and the repulsive (or attractive) nuclear medium, influence the Equation of State (EoS)? The complete answer will involve a complicated calculation. However, we have found a model [1] where we can compare two extreme scenarios **(A)** and **(B)**. I our model the pressure $p_H(\varrho)$ between the hadrons acts on the bag surface similarly to the bag "constant" $B(\varrho)$. In the scenario **(A)** the nucleon mass $M_N(\varrho) = M_N$ is a constant, independent of density. A mass M_N for finite $p_H(\varrho)$ can be obtained from [1]:

$$M_N(\varrho)=\frac{4}{3}\pi R^3[4(B+p_H)-p_H]=E^0_{\text{Bag}}\frac{R_0}{R}-p_H V_N. \tag{1}$$

The scaling factor R_0/R comes from a well-known model dependence ($E^0_{\text{Bag}} \propto 1/R_0$) in the spherical bag [6]. This simple radial dependence is now lost in (1) and responsible for that is the pressure dependent correction to the mass of

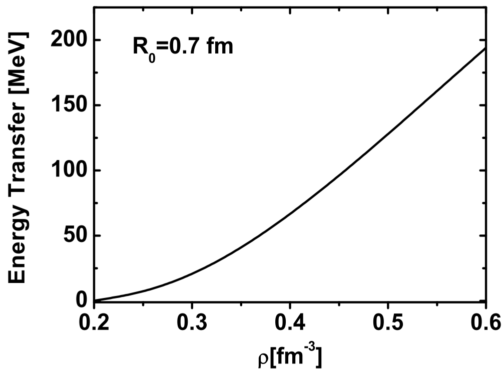

Figure 1. Energy transfer for constant nucleon mass and initial nucleon radius $R_0 = 0.7$ fm for increasing nuclear density.

a nucleon given by the energy transfer $\Delta E = p_H V_N(\varrho)$. This term is identical with the work $W_N = p_H V_N$ and disappear for the nucleon enthalpy

$$H_N(\varrho) = E^0_{\text{Bag}} \frac{R_0}{R(\varrho)} \propto 1/R(\varrho). \tag{2}$$

The nucleon radius $R(\varrho)$ reflects a scale of a confinement of partons. Generally, for increasing $R(\varrho)$, $H_N(\varrho)$ (2) decreasing, thus part of the nucleon rest energy is transferred from a confined region V_N to an remaining space between nucleons. For decreasing R, the H_N increasing; this allows the constant nucleon mass M_N (1). In such a case, the quarks inside the bag need the additional (1) energy transfer $\Delta E(\varrho)$ to keep the constant mass and the bag volume in the compressed medium. The accompanying energy transfer above the equilibrium density ϱ_0, shown in Figure. 1, will provide the volume energies W_N inside the bags.

Finally, the s.p. energy $\varepsilon^q_N(\varrho) = E_A/A$, the nucleon mass and the radius $R(\varrho)$ (1) can be written as [1]:

$$\varepsilon^q_N(\varrho) = \varepsilon^{\text{wal}}_N(\varrho) - \Delta E(\varrho) \tag{3}$$

Differentiating above equation one obtains the relation (4) which determines the new saturation density $\varrho_0 = 0.162\ \text{fm}^{-3}$ for the compressibility $K^{-1}(\varrho_0) = 250$ MeV. Please note, at the saturation point the derivative $d\varepsilon^q_N(\varrho)/d\varrho)\big|_{\varrho=\varrho_0} = 0$

[1] We extend in Egs.(3-6) the linear scalar-vector version of RMF [9] with ρ meson contributions to the symmetry energy [10, 11]. Parameters in [9] were fitted to obtain equilibrium density $\varrho_0 = 0.149$ fm and RMF energy $\varepsilon^{\text{wal}}_N(\varrho_0) = 15.75$ MeV at saturation point.

but derivative of Walecka part $\varepsilon_N^{\text{wal}}$ is ${\varepsilon_N^{\text{wal}}}'(\varrho_0) > 0$.

$$K^{-1}(\varrho_0) = 9\frac{(1-\varrho_0 V_N(\varrho_0))}{V_N}{\varepsilon_N^{\text{wal}}}'(\varrho)\bigg|_{\varrho=\varrho_0} \tag{4}$$

$$R_0/R(\varrho) = 1 + \Delta E(\varrho)/M_N(\varrho) \tag{5}$$

$$\text{where} \;\; \Delta E(\varrho) = p_H V_N = \frac{\varrho^2 {\varepsilon_N^q}'(\varrho) V_N(\varrho)}{(1-\varrho V_N(\varrho))} \tag{6}$$

For negative pressure p_H the nucleon bag increases its radius (1), so the energy is transfer in opposite direction - from bags to the meson field. Summarizing, our model in scenario (**A**) consists of four self-consistent equations (3-6) including usual equation for nucleon effective mass M^* in a linear scalar field σ [3].

In order to show a thermodynamical consistency let us express the chemical potential μ_N^q by the uniform pressure $p(\varrho) = \varrho^2(\varepsilon_N^q)'(\varrho)$(Hugenholz-van Hove Theorem):

$$\mu_N^q = \varepsilon_N - p_H V_N + p_H V_-/A = \varepsilon_N^q + (pV)/A\,. \tag{7}$$

In the uniform system of NM the grand canonical potential $\Omega = -pV$ with $d\Omega = -pdV - Ad\mu_N^q$ given by a relation (7), satisfies the following thermodynamical relation for the average number of particles A,

$$A = -\left[\frac{\partial\Omega}{\partial\mu_N^q}\right]_V = \left[\frac{\partial(pV)}{\partial\mu_N^q}\right]_V \tag{8}$$

which proofs the thermodynamical consistency of our model.

The energy transfer ΔE, shown in Figure 1 was not taken into account in scenario **(A)** in our previous findings [1]. Let us compare these new results with energy transfer obtained in scenario **(A)** with scenario **(B)** obtained in [12] where $\Delta E(\varrho) = 0$. In **(B)** the nucleon mass M_{pr} decreases with density (1) by the volume work: $M_{pr}(\varrho) = M_N - \Delta E$ at the expense of maintaining the volume of the bag. In contrast to the discussion in [1], the values of s.p. energies $\varepsilon_N^q(\varrho)$ and the Fermi energies E_F^q are similar in both scenarios because the mentioned decrease of mass in scenario **(B)** is close to the decrease of s.p. energy by the energy transfer (3,7) in **(A)**. Therefore, both Fermi energies in **(A)** and **(B)** are smaller than the Fermi energy $E_F = \mu_N$ calculated for point-like nucleons (7), by a volume energy $p_H V_N(\varrho)$ which weakens "effectively" the repulsion between nucleons.

2 Results for EoS

The EoS present in scenario (A) is shown in Figure 2 versus scenario (B). Walecka [3] and DBHF calculations [8] with a Bonn A interaction are shown for references. Results for pressure in both scenarios are similar, however critical densities are very different [12]. This difference illustrates Figure 4 in [12],

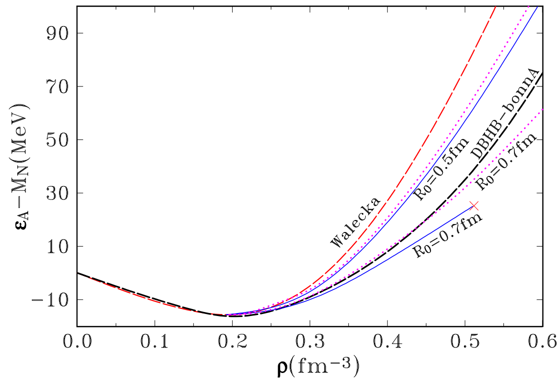

Figure 2. Energy of NM above the equilibrium density for different models. Walecka [3] and Dirac-Bruckner-Hartree-Fock (DBHF) [8] calculations with the Bonn A interaction are shown as long dashes. Results for const nucleon mass (for $R = 0.5$ fm, 0.7 fm) are denoted by dotted red lines and for const. nucleon radii (**B**) by solid blue lines.

where the nuclear energy density $\varrho\varepsilon_N^q(\varrho)$ grows with density while the nucleon energy density $M_{pr}(\varrho)/V_N(\varrho)$ in scenario (**B**) declines and finally both energy densities for $\varrho \sim 0.5$ fm^{-3} are equal. For that density, nucleon bags with constant $R_0 \sim 0.7$ fm starts to overlap in case (**B**) and multi-quark bags would be possibly formed. The alignment density depends strongly on the nucleon radius, in turn the points where $B(\varrho)$=0 depend mainly from the starting value $B(\varrho_0)$. For example, for $R_0 = 0.75$ fm the alignment density $\varrho_{al} = 0.44$ fm^{-3} almost coincides the vanishing bag constant $B(\varrho_0) = 100$ MeV fm^{-3} (see Figure 3 in [12]). Therefore, scenario (**B**) with constant nucleon radius and the gradual alignment of the energy densities inside and outside the bag suggests the crossover transition below $\varrho = 0.45$ fm^{-3}. However, such a transition is not observed in heavy ion experiments. Also in neutron stars [13], for that density of star core we would expected for the quark core to decrease the radius of the star, but such a decrease is not expected in comparison to lighter stars with a standard neutron core. The scenario (**A**) with constant nucleon mass [12] is more realistic then scenario (**B**) [1] without energy transfer. For constant nucleon mass in scenario (**A**), a nucleon volume decreases with ϱ, therefore nucleon bags do not overlap for large density and the energy density of the nucleon increases due to the energy transfer into nucleon bags.

3 Symmetry Energy in Dense Nuclear Matter

The symmetry energy E_s is defined as the coefficient of the quadratic term in the expansion of the energy per nucleon $\varepsilon_N^q(\varrho)$ in neutron excess $t = (\varrho_n - \varrho_p)/\varrho$:

$$E_s = \frac{\partial^2 \varepsilon_N^q(\varrho)}{2\partial t^2} = E_s^{\text{wal}} - \frac{L}{3}\frac{V_N(\varrho)}{(1-\varrho V_N(\varrho))}\bigg|_{\varrho=\varrho_0} \qquad \frac{L}{\varrho_0} \doteq 3\varrho \frac{dE_s}{d\varrho}\bigg|_{\varrho=\varrho_0} \tag{9}$$

$$\text{where } E_s^{\text{wal}} = \frac{\partial^2 \varepsilon_N^{\text{wal}}}{\partial t^2} = \frac{g_\rho^2}{8m_\rho^2}\varrho + \frac{P_F^2}{6\sqrt{P_F^2 + M_N^{*2}}} \tag{10}$$

Calculations of the symmetry energy [10] E_s^{wal} without nonlinear contribution proportional to L given by the equation (10) give the value $E_s^{\text{wal}} = 24.8$ MeV for $R_0 = 0.7$ fm, which is a few MeV too low from phenomenological extrapolation $E_s{}^{\text{exp}} = 30.5 \pm 3$ MeV [14]. Differentiating equation (9) we get a following expression for the second derivative of symmetry energy K_{sym}.

$$K_{\text{sym}} = \varrho \frac{\partial E_s^{\text{wal}}}{\partial \varrho}\frac{(1-\varrho V_N)}{\varrho V_N} - \frac{\varrho}{1-\varrho V_N}\frac{\partial E_s}{\partial \varrho}(1 + \varrho^2 \frac{\partial V_N(\varrho)}{\partial \varrho})\bigg|_{\varrho=\varrho_0} \tag{11}$$

The symmetry slope (9) parameter $L = 88$ MeV is higher then the phenomenologically extrapolated value $L^{\text{exp}} = 52.5 \pm 20$ MeV [14].

It is straightforward to include the additional coupling g_ρ to the ρ meson [10], which contribute only to the E_s of NM (9) and correct the energy of asymmetric neutron matter. In our model the inclusion of meson ρ contributions with $((g_\rho/m_\rho)^2 = 1.38\ \text{fm}^2)$ together with the pressure correction present

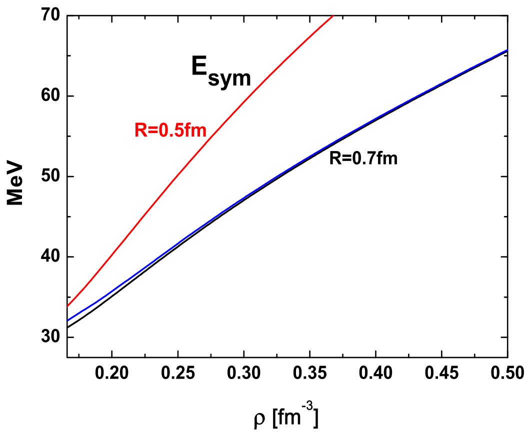

Figure 3. Symmetry energy of NM above the equilibrium density for different nucleon radii $R = 0.5$ fm, 0.7 fm

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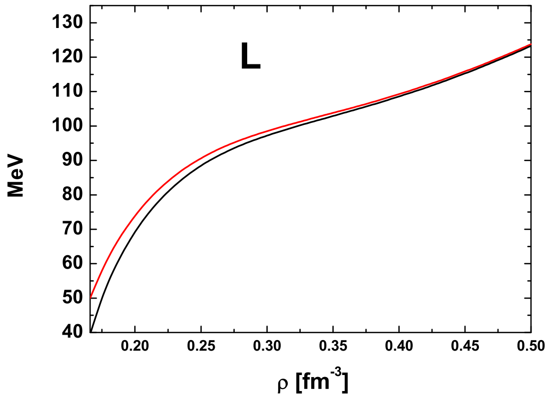

Figure 4. The symmetry energy derivative L density as a function of the nuclear density for $R = 0.7$ fm and $R = 0.5$ fm

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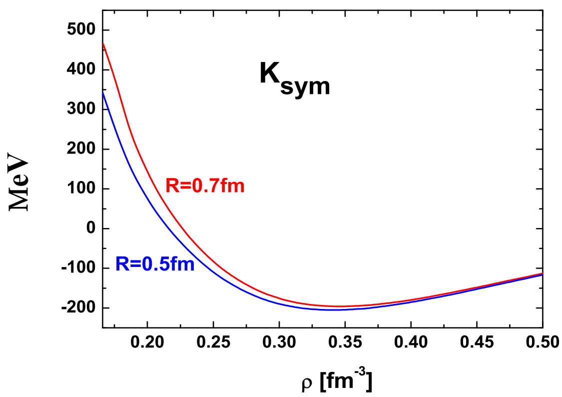

Figure 5. The second derivative of symmetry energy K_{sym} as a function of the nuclear density for nucleon radii $R = 0.7$ fm and $R = 0.5$ fm

in equation (9) gives $E_s = 31$ MeV for the slope $L = 55$ MeV – in very good agreement with the phenomenological estimate ${E_s}^{\text{exp}} = 30.5 \pm 3$ and $L^{\text{exp}} = 52.5 \pm 20$ MeV [14]. Also their density dependence shown in Figures (3,4) agree with terrestrial and astrophysical constrains [4, 14]. We also present the plot of the second derivative of symmetry energy K_{sym} given by equation (11) in Figure 5.

4 Conclusions

We have shown, how nucleon volumes in compressed NM affect the nuclear compressibility at equilibrium, keeping the constant nucleon mass thanks to the decrease of the nucleon volume in nuclear medium. It effectively corresponds to pressure dependent modifications of a nuclear scalar potential. In our model the nonlinear term originated from the finite sizes of nucleons with quarks degrees of freedom. It allows to connect the nuclear compressibility with the saturation density and determine their values from equation (4). Also the value of the symmetry slope L is well fitted with the help of differential term in equation (9). Therefore the presented model of compressed nucleons in dense NM is suitable for studying Equation of State of nuclear matter and properties of neutron stars. Not accidentally, in the widely used standard [2, 11] RMF model with point-like nucleons the good compressibility is fit by nonlinear modifications of a scalar mean field with the help of two additional parameters. Thus, our results suggests to reconsider these mean field parameters.

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