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Shapes Coexistence in the Frame of the Bohr Model

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Abstract. The Bohr Hamiltonian with a general sextic potential has been solved for the critical point of the shape phase transition from spherical vibrator to prolate rotor. The parameters defining the potential are constrained such that its form to manifest two degenerated minima, a spherical and a deformed one, separated by a maximum (a barrier). The eigenvalue problem is solved by diagonalizing the Hamiltonian in a basis of Bessel functions of first kind, which in turn are obtained by solving the same problem but for an infinite square well potential. By analyzing the density distribution probability for the states of the ground band and of the first β band, one can understand how the shape of the nucleus is changing as the barrier is introduced and increased step by step. Doing so, new interesting results have been found, as shape mixing and shape coexistence, properties which otherwise are absent where there is no separating barrier.

1 Introduction

The X(5) model [1] is an approximate solution proposed for the critical point of the nuclear shape phase transition from spherical vibrator [2, 3] to axially deformed rotor [4, 5], involving an infinite square well potential in β plus a harmonic oscillator potential for γ. On the other hand, it is well established [1, 6, 7] that a potential with two minima, a spherical and a deformed one separated by a small barrier, would be more adequate for this critical point. Since X(5) was proposed, many other solutions for the Bohr Hamiltonian [2, 8] have been analyzed [9, 10] by trying to get a better description of this critical point or to improve the agreement with the corresponding experimental data, but none of these solutions have considered the barrier separating the two minima. A first attempt [11, 12] was to use a quasi-exactly solvable sextic potential [13], which depending on its parameters can have indeed a single spherical minimum, a deformed one or simultaneously both of them. This potential proved to be useful in describing an evolution from a spherical shape to a deformed one within several isotope chains [11, 12, 15, 21–26], but due to the constraints imposed on its parameters such that to have analytical solutions, the barrier could not be appropriately introduced. Therefore, a more general solution of this potential has become necessary by relaxing the conditions for its free parameters. Thus, in [14,15], the Bohr Hamiltonian with a general sextic potential has been solved by numerical diagonalization using as a basis the solutions of the same Hamiltonian but for an infinite square well potential. Through this method, finally the barrier could be taken into account for this critical point and its effects carefully studied.

The goal of the this paper is to attract the attention on the present model in the context of its recent applications to experimental data [15, 16], to highlight its main achievements and to anticipate new possible developments and applications of the model for the topic of nuclear shape mixing and coexistence [17–20].

2 Model presentation and applications

In Refs. [14, 15], the equation of the Bohr-Mottelson Hamiltonian [2, 8]

$$\begin{aligned} H &= -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}\right] \\ &\quad +\frac{\hbar^2}{8B\beta^2}\sum_{k=1}^{3}\frac{Q_k^2}{\sin^2\left(\gamma-\frac{2\pi}{3}k\right)}+V(\beta,\gamma) \end{aligned} \tag{1}$$

is solved for the critical point of the phase transition from spherical vibrator to axially deformed rotor. In Eq. (1), β and γ are intrinsic deformation variables, with β describing the deviation from sphericity and γ from axiallity. B is the mass parameter, while Q_k are the angular momentum projections in the intrinsic reference frame. The novelty of the model consists in offering a numerical solution for the associated β equation,

$$\left[-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{L(L+1)}{3\beta^2}+v(\beta)\right]\Psi(\beta)=\varepsilon\Psi(\beta), \tag{2}$$

for a more general sextic oscillator potential,

$$v(\beta)=\beta^2+a\beta^4+b\beta^6, \tag{3}$$

and using as a diagonalization basis, the solutions of the X(5) model [1]:

$$\tilde{\Psi}_{\nu n}(\beta)=\frac{\sqrt{2}\beta^{-\frac{3}{2}}J_\nu(\alpha_n\beta/\beta_w)}{\beta_w J_{\nu+1}(\alpha_n)}. \tag{4}$$

In Eq. (4), J_ν are Bessel functions of the first kind of index

$$\nu=\sqrt{\frac{L(L+1)}{3}+\frac{9}{4}}, \tag{5}$$

while α_n is the n^{th} zero associated to the boundary conditions given by β_w. For a better evidence of some properties as shape mixing and coexistence, it is more adequate in the present study to bring Eq. (2) to a Schrödinger form by changing the function as $\Psi(\beta) = \beta^{-2}\psi(\beta)$:

$$\left[-\frac{\partial^2}{\partial\beta^2} + \frac{L(L+1)}{3\beta^2} + v_{eff}(\beta)\right]\psi(\beta) = \varepsilon\psi(\beta), \tag{6}$$

where,

$$v_{eff}(\beta) = \frac{2}{\beta^2} + \beta^2 + a\beta^4 + b\beta^6. \tag{7}$$

Here, a and b are free parameters, while the coefficient of the harmonic term β^2 is one due to a scaling property which is applied for energies [14]. The potential has two minima if and only if $a < 0$ and $b > 0$. Examples of potentials (3) and (7) can be seen in Figure 1 with parameters fitted to reproduce experimental data for certain nuclei. Graphical representations of the corresponding deformation probability distribution for the ground state and the first excited 0^+ state can be found in Refs. [16, 27]. A general description of various model cases made in Refs. [14, 15] as a function of the barrier height, revealed instances with shape coexistence, shape mixing, and shape fluctuation. Another relevant signature for these phenomena, as the monopole transition probability $\langle 0_1^+|\beta^2|0_2^+\rangle$ [14, 15], has been involved in the study.

The present model was first time applied for the experimental data of ^{238}Pu, ^{152}Nd and ^{170}Hf [14] trying to find some candidates for the critical point of the phase transition from spherical vibrator to axially deformed rotor, but also to point out the possibility of the model to describe shape mixing and shape coexistence. Because the results for a small barrier proved to be similar with those of X(5) where the barrier was neglected, further investigations were focused on increasing the height of the barrier and finding candidate nuclei for shape mixing and coexistence phenomena. Therefore, a first notable success of the model in this new direction was achieved in [15], where the energy spectra, the B(E2) transitions and the monopole transition between the first excited 0^+ and ground state of the ^{76}Kr nucleus, known for manifesting shape mixing and coexistence, were very well reproduced. The fitted parameters for ^{76}Kr, as well as the calculated value of the monopole transition $\langle 0_1^+|\beta^2|0_2^+\rangle$, indicated that this nucleus is a good candidate for the case of shape coexistence with mixing [15]. Recently, new candidates have been found, namely ^{72,74,76}Se [16], for the same cases of shape mixing and coexistence in the same state. Moreover, in [16], a shape evolution was observed in the ground band as a function of the total angular momentum. For example, analyzing the density distribution probabilities, one can see that the ground state prefers the less deformed minimum, while the 4^+ is already above the second more deformed minimum. Because the numerical results for these nuclei are already presented and largely discussed in [14–16], here we only have resumed ourselves to attract the attention on the main achievements of the model and of its possible future applications. On the other hand, in [16] the minima for the isotopes of Se are given in scaled values of the β deformation, while in [15] these values for ^{76}Kr have been omitted to be presented. The usually used quadrupole deformation values are especially in demand in the comunity of researchers studying shape coexistence phenomena with microscopic approaches. Therefore, in Figure 1, are plotted the potentials for ^{72,74,76}Se and ^{76}Kr and this time by indicating the quadrupole deformations for the two minima, which can be further compared with data coming from microscopic models.

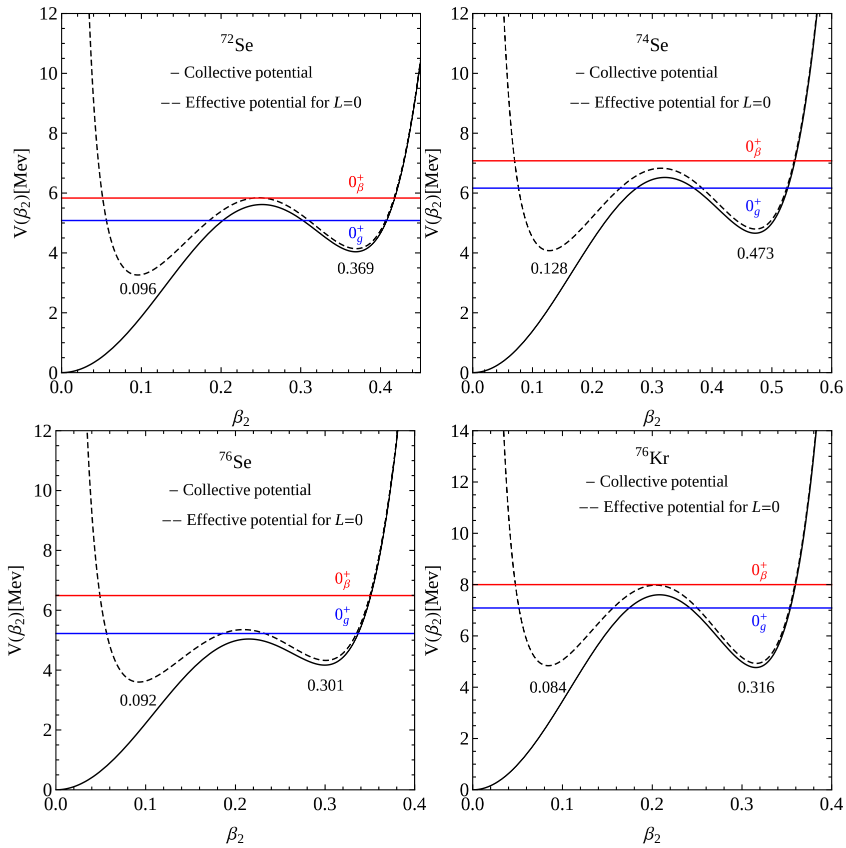

Figure 1. Potentials given by Eq. (3), dashed curve, and Eq. (7), continuous curve, are plotted as a function of the β quadrupole deformation for ^{72,74,76}Se and ^{76}Kr.

As can be seen from Figure 1, the present model [14, 15] was design to describe shape mixing and shape coexistence between spherical and deformed minima. The model can be further generalized to describe coexistence of two deformed shapes by considering high order terms as β^8 in the expression of the β potential (3). The diagonalization method developed in [14, 15] could be very easily applied also to such potentials.

3 Conclusion

The Bohr model was solved numerically for a general sextic potential [14, 15] using as a basis solutions of the X(5) model [1]. Taking the advantage of the barrier introduced to separate the spherical shape from the deformed one, new phenomena could be studied in the frame of the Bohr model as shape mixing and shape coexistence. Recent applications for ^{76}Kr [15] and ^{72,74,76}Se [16], suspected for manifesting such phenomena, revealed promising results and confirmed some of the model predictions. Also, new applications of the model for some Mo isotopes, namely ^{96,98,100}Mo, are to be published [27]. This recent study shown us that there are still many candidate nuclei for the present model which have to be tested in the future works. Moreover, the present model can be further improved by adding higher order terms in β to the potential. In this way shape mixing and shape coexistence between oblate and prolate deformations would be more accurately described, opening a new door for new applications. Another important aspect is that the model being a phenomenological one, needs support confirmation also from microscopical models. Thus, in the present paper, quadrupole β_2 deformation values are given for ^{76}Kr [15] and ^{72,74,76}Se, offering the possibility to make connections with microscopic models.

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