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The Bizarreness of Symmetry Energy Behavior in Peculiar Cases of Meson Interactions

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Abstract. Symmetry energy of nuclear matter is one of the most essential parameters in the field of nuclear astrophysics. There are common types of meson interactions that have been profoundly investigated within RMF framework over recent years (ρ-σ or ρ-ω [1,2]). In this article, yet another cross-scalar terms of the kind σ-δ have been introduced. Such cross-couplings may greatly modify the symmetry energy and its slope. An interesting point is that one may obtain desired values of a slope, and thus the values consistent with observations and experimental data, by playing coupling constants of δ meson field and a newly adopted σ-δ meson interaction. A broad research has been performed of wide coupling constant range where most extreme cases may be seen. The results indicate a unique bizarreness of symmetry energy under such distinctive conditions.

1 Introduction

The density dependence of symmetry energy at supra-saturation densities is still very uncertain due to lack of experimental measurements at terrestrial laboratories that would reveal the high-energy matter properties. Theoretical models, on the other hand, show rather divergent results. Indeed, the recent work of Li *et al.* [3] shows the variety of symmetry energy behavior at densities higher than saturation density $n_0 = 0.16\,\mathrm{fm}^2$ for numerous models. The variety comes from, inter alia, poor knowledge of the isospin dependence of strong interactions. The observation of neutron stars merger event GW170817 by LIGO/Virgo collaboration [4] in 2017, supplied the research on internal neutron star structure with some constraints. The measurements based on GW170817 indicate rather stiff equation of state (EOS), therefore higher maximum mass of the star family. In fact, the neutron stars with the mass above $2M_\odot$ have been already observed (PSRJ0348+0432 [5], PSRJ2215+5135 [6]). Another constraint that the neutron star merger provided is the upper and lower limit of the star radius. It was shown by Fattoyev *et al.* [7] that the upper limit for the radius of $1.4M_\odot$ neutron star is $R = 13.76$km. On the other hand, Bauswein *et al.* [8] show the lower limit for $1.6M_\odot$ neutron star for which the radius of the star should be not less than 10.64 km. In their work, it was also indicated that the maximum radius $R^{\max}$, *i.e.* the radius at which maximum mass M^{max} of the star family is reached, should be greater than about 9.6 km. The radius constraints are vital for the nuclear matter parameters. Indeed, it was shown that the neutron star radii has clear connection with symmetry energy slope L [9]. The authors obtained the results of the slope to be between 43 and 52 MeV for a radius of $1.4M_{\odot}$ neutron star being in the range 11–13 km. The slope is defined as derivation of symmetry energy S_2 in regards to density given at n_0. Therefore the symmetry energy behavior in saturation vicinity rules out the value of the slope. Notwithstanding, the density dependence of symmetry energy at higher densities remain disputable despite observational and experimental data which makes it one the major theoretical challenge for nuclear astrophysics branch. In this work we show the results of symmetry energy behavior at supra-saturation densities when the meson crossing of the scalar-scalar kind is included. We show that such crossed terms can greatly modify symmetry energy.

2 Model and Parametrization

In our calculations, we use the Relativistic Mean Field (RMF) theory that provides an elegant framework for nuclear matter description. We start with defining Lagrangian from which the equations of motions for mesons and nucleons can be derived. The model consists of nucleons and four mesons. Here we show the interaction part of the Lagrangian $\mathcal{L}_{\text{int}}$ only. The model description and total Lagrangian can be found in [10].

$$\mathcal{L}_{\text{int}} = g_\sigma \sigma \bar{\psi}\psi - g_\omega \omega_\mu \bar{\psi}\gamma^\mu \psi - \frac{1}{2} g_\rho \vec{\rho}_\mu \bar{\psi}\gamma^\mu \vec{\tau}\psi + g_\delta \vec{\delta}\bar{\psi}\vec{\tau}\psi - U(\sigma) + \tilde{g}_\alpha \sigma^\alpha \vec{\delta}^2. \quad (1)$$

The interaction Lagrangian includes the coupling constants g_σ, g_ω, g_ρ, g_δ for σ, ω, ρ and δ meson fields respectively, $U(\sigma) = \frac{1}{3} b\ m (g_\sigma \sigma)^3 + \frac{1}{4} c\ (g_\sigma \sigma)^4$ represents σ meson self-interaction, and g_α is σ-δ meson coupling constant. The equations of motions (not shown in this paper, but their full forms can be found in [10]), allows expressing energy density in the following form

$$\begin{aligned}\epsilon = \frac{2}{(2\pi)^3}\Big(\int_0^{k_p} d^3k\sqrt{k^2 + m_p^{*2}} + \int_0^{k_n} d^3k\sqrt{k^2 + m_n^{*2}}\Big) \\ + \frac{1}{2}\frac{1}{C_\sigma^2}(m - \frac{(m_p^* + m_n^*)}{2})^2 + \frac{1}{2}C_\omega^2 n^2 + \frac{1}{8}C_\rho^2(2x-1)^2 n^2 \\ + \frac{1}{8}\frac{1}{C_\delta^2}(m_n^* - m_p^*)^2 + g_\alpha (m_n^* - m_p^*)^2 (m - \frac{(m_p^* + m_n^*)}{2})^\alpha \\ + U(m - \frac{(m_p^* + m_n^*)}{2}), \quad (2)\end{aligned}$$

The σ-δ meson interaction sits in the term with g_α that is connected with $\tilde{g}_\alpha$ by $g_\alpha = \frac{\tilde{g}_\alpha}{4 g_\sigma^\alpha g_\delta^2}$. $C_i^2 = \frac{g_i^2}{m_i^2}$ for $i = \sigma, \omega, \rho$, and δ due to density-independent coupling constants. The m_n^* and m_p^* are effective masses for neutrons and protons respectively. Both are depended on σ and δ fields

$$m_p^* = m - g_\sigma \bar{\sigma} - g_\delta \bar{\delta}_3, \tag{3}$$

$$m_n^* = m - g_\sigma \bar{\sigma} + g_\delta \bar{\delta}_3. \tag{4}$$

Now, the symmetry energy is defined as the second derivative of energy density per particle in respect to proton fraction x

$$S_2(n) = \frac{1}{8n} \frac{\partial^2 \epsilon\,(n,x)\,/n}{\partial x^2}\bigg|_{x=\frac{1}{2}}. \tag{5}$$

The analytical calculation leads to the form of symmetry energy that for considered model is depended on C_ρ^2, C_δ^2 and g_α. Therefore, these three constants from isevector channel becomes an essential set that should be properly selected in order to obtain the constraints given by experimental and observational data. Complete formalism has been derived in [10]. The constrains of symmetry energy at the vicinity of saturation density n_0 are well determined based on heavy-ion experiments and nuclear structure observables [11]. The binding energy of symmetric matter $B = \epsilon(n, 1/2)/n\Big|_{n_0} = -16$ MeV, the compressibility $K = 230$ MeV and symmetry energy at saturation $S_2 = 30$ MeV allows us to find the coupling constants of the considered model. Nevertheless, the model consist of more coupling constants than empirical data. One of them is C_σ^2 which is chosen to obtain stiff equation of state and consequently higher maximum mass of the star family [12]. Three isovector couplings are adjustable in the way that C_ρ^2 and C_δ^2 are connected to keep $S_2 = 30$ MeV, where C_δ^2 together with third coupling of σ-δ meson interactions g_α can be modified to receive the desired slope of symmetry energy L. Recent studies shows the slope to be in the range 40–60 MeV [13]. In this work we show that the additional meson-meson interactions of the σ-δ kind can greatly modify symmetry energy and thus we can obtain the slope in a wide range of its value.

3 Results

The manipulation of coupling constants C_δ^2 and g_α allows obtaining the desired value of the slope of symmetry energy. In the work [10] such investigation on these couplings has been already done for specific g_α which was defined and kept constant for two cases, linear $\alpha = 1$ and quadratic $\alpha = 2$ interactions. Namely $g_1 = -0.009$ fm^{-1} and $g_2 = -0.004$. In this paper, we investigate behavior of symmetry energy and its slope for a broader range of C_δ^2 and g_α. Particularly, we mostly focus on the role of g_α. The specific correlation was found for these two couplings. We noticed that it is possible to find the same value of the slope L for different values of g_α and C_δ^2. Indeed, the same value of L is obtained for high C_δ^2 and g_α close to zero and for smaller C_δ^2 and g_α having lower values. In order to obtain identical L, manipulation has to be very precise. Another feature is that when considering only negative values of g_α, the higher these two parameters, the slope becomes lower, whereas for negative g_α such combination gives high values of L. Positive g_α always give high slope (95 MeV and higher), no matter of how much is C_δ^2. The slope $L = 95$ MeV can be obtained also for negative g_α. In this case, with increasing C_δ^2, the value of L drops which makes it a perfect candidate to attain the slope range 40–60 MeV.

In Figure 1 we show the density dependence of symmetry energy for 22 various RMF models, all being based on the same Lagrangian and the same isoscalar sector parameters, but with different values of the parameters from isovector sector. The figure is presented for illustration purposes. Due to the fact that only negative values of g_α lead to smaller L, in the further considerations we consider only $g_\alpha < 0$. In Figure 2, the density dependence of symmetry energy is presented for constant C_δ^2 and various g_α. Both diagrams are calculated within quadratic interaction $\alpha = 2$. It can be seen that, for high values of $C_\delta^2 = 10$ fm^2 (upper diagram), small changes of g_α (from -0.0015 to -0.0009) are needed to obtain a wide range of the slope. For lower $C_\delta^2 = 5$ fm^2 (lower diagram), lower values of g_α are required (from -0.0033 to -0.001) to receive similar range of the slope as in the upper diagram. The choice of the parameters has its explanation. Indeed, taking even lower g_α, the slope becomes negative. For example, for the $C_\delta^2 = 10$ fm^2, already $g_\alpha = -0.0016$ gives slope $L = -3.8$ MeV. The upper limit, on the other hand, is justified on the basis of conclusions regarding too high values of the slope for negative values of g_α. Taking $g_\alpha = -.001$ when $C_\delta^2 = 10$ fm^2, the slope becomes greater than 100 MeV.

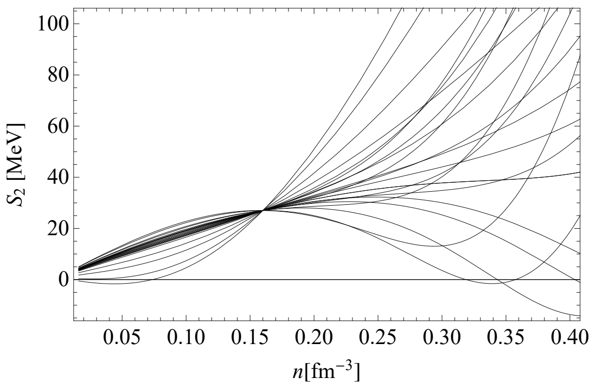

Figure 1. Symmetry energy as a function of density for various C_δ^2 and g_α.

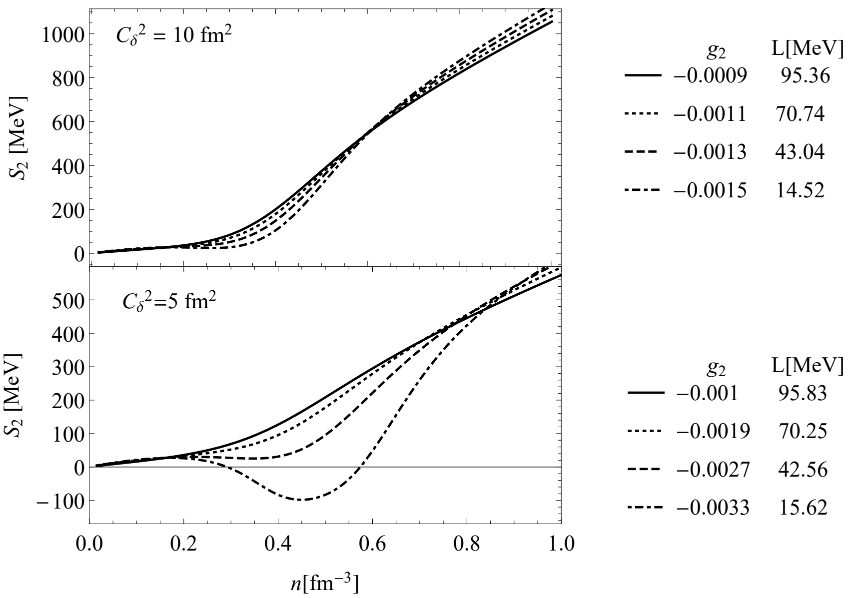

Figure 2. Symmetry energy as a function of density for various g_α and $C_\delta^2 = 10$ fm^2 (upper diagram) and $C_\delta^2 = 5$ fm^2 (lower diagram).

In Figure 3 we present visually the influence of g_α on the slope. The figure shows the relation between these two quantities for $C_\delta^2 = 10$, 8, 5 and 3 fm^2. It reveals the strength of coupling constant of the additional meson-meson inter-

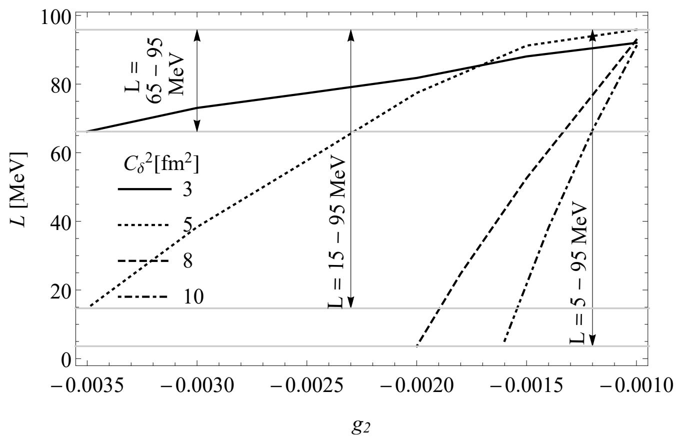

Figure 3. Relation between slope L and the coupling constant of scalar-scalar meson interaction g_α for four different C_δ^2 =const.

action to the standard RMF [1]. Taking high coupling C_δ^2, small range and small changes of g_α are required to acquire quite wide range values of the slope. For $C_\delta^2 = 10$ fm^2, small g_α range gives the slope L = 5-95 MeV whereas for $C_\delta^2 = 3$ fm^2, the manipulation of g_α has to be much wider. One may see that three times bigger range of g_α is necessary than for the case with the highest C_δ^2. Still, the slope is only L = 65–95 MeV. Even more precise choice of C_δ^2 and g_α, despite the fact that g_α changes are on the fourth decimal place, to attain the exact values of the slope. The more precise the coupling constants, the more exact L can be obtained.

4 Conclusion

In this paper, we have shown the effect of scalar-isoscalar σ meson with scalar-isovector δ meson interaction on the symmetry energy. In general, in models based on RMF approach, the symmetry energy tends to increase with increasing density. The introduction of meson interaction of σ-δ kind allows getting a broad spectrum of S_2 behavior, from hard to super-soft, by manipulation of isovector coupling constants. The models for which values of the slope of symmetry energy are in the range 40-60 MeV were investigated towards neutron star properties. The maximum masses of the star family are above $2M_\odot$ for all models, and maximum radii are no less than 11 km which is consistent with the 9.6 km constrain.

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[1] Standard RMF consist of four meson fields σ, ω, ρ, and δ that couple to nucleons, with no additional meson-meson interactions.