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Shape Phase Transition in Nuclei within a Correlation between Two Quantum Concepts

S. Ait Elkorchi[1], **M. Chabab**[1], **A. El Batoul**[1], **A. Lahbas**[1,2], **M. Oulne**[1]

[1]High Energy Physics and Astrophysics Laboratory, Faculty of Sciences Semlalia, Cadi Ayyad University, P. O. B. 2390, Marrakesh 40000, Morocco
[2]ESMaR, Department of Physics, Faculty of Sciences, Mohammed V University, P. O. B. 1014, Rabat, Morocco.

Abstract. In this work, we present a correlation that we have revealed between the well-known quantum concepts: the Minimal Length (ML) and the Deformation Dependent Mass (DDM) in the study of transitional nuclei near the critical point symmetries X(3) and Z(4).

1 Introduction

The theoretical study of the Quantum phase transitions (QPT) in nuclear structure have attracted a lot of attention [1] to understand low-energy quantum rovibrational modes of nuclei in the shape phase transitional region [2]. Therefore, different approaches have been developed in this context particularly in the framework of the Bohr-Mottelson model [3] and Interacting Boson Model (IBM) [4]. Moreover, the interest devoted to such a thematic has increased even more with the occurrence of Critical Point Symmetries (CPSs). Among these symmetries, one can cite for example E(5) [5] and X(5) [6] corresponding to the shape phase transitions U(5)↔O(6) and U(5)↔ SU(3) respectively. Later, a γ-rigid (with $\gamma = 0$) version of X(5), called X(3), has been introduced [7]. In the same way, other CPS have been developed like for example Z(5) and its γ-rigid version Z(4) (with $\gamma = \pi/6$) corresponding to shape phase transitions from prolate to triaxial symmetry [8]. In this context, considerable attempts have been done for several potentials to achieve analytical solutions of Bohr Hamiltonian, either in the usual case where the mass parameter is assumed to be a constant [9–11], within Deformation Dependent Mass (DDM) formalism [12–15], or by introducing the the Minimal Length (ML) concept in nuclear structure [16–18]. The DDM concept [19], which is widely used in quantum physics, is equivalent to a deformation of the canonical commutation relations:

$$[x_i, x_j] = 0, \quad [x_i, p_j] = i\hbar\delta_{i,j}, \quad [p_i, p_j] = 0, \tag{1}$$

where $i = 1, 2, 3$. By replacing the momentum components $p_i = -\mathrm{i}\hbar\nabla_i = -\mathrm{i}\hbar\partial/\partial x_i$ by some deformed hermitian operators:

$$\pi_i = \sqrt{f(x)}\, p_i \sqrt{f(x)}, \tag{2}$$

where the positive real deforming function $f(x)$ depends on the coordinates $x = (x_1, x_2, x_3)$, both last commutators in Eq.(1) transform into:

$$[x_i, \pi_j] = \mathrm{i}\hbar f(x)\delta_{i,j}, \quad [\pi_i, \pi_j] = \mathrm{i}\hbar[f_j(x)\pi_i - f_i(x)\pi_j]. \tag{3}$$

On the other hand, several studies in string theory [20] and quantum gravity [21] in the view of Heisenberg algebra propose a small correction to the Heisenberg uncertainty relation of the form

$$\Delta X \Delta P \geq \frac{\hbar}{2}(1 + \alpha(\Delta P)^2), \tag{4}$$

Therefore, this correction results in the modification of the canonical commutation relation between the position operator and momentum operator.

In the present work the attention is focused on the study of correlation between the two quantum concepts: the Minimal Length (ML) and the Deformation Dependent Mass (DDM), through solutions of Bohr-hamiltonian for transitional nuclei in the limits of CPS X(3) and Z(4) with ISW and Davidson potentials.

2 First Concept: Deformation Dependent Mass

The DDM approach consists in a generalized Bohr Hamiltonian by allowing the nuclear mass to depend on the deformation coordinate β, such as $B(\beta) = B_0/f^2(\beta)$, where B_0 is a constant and $f(\beta)$ is the deformation function depending only on the radial coordinate β. This formalism has been firstly achieved by applying Davidson [12] and Kratzer [14] potentials to huge number of γ-unstable and axially symmetric prolate deformed nuclei with a good prediction of the corresponding experimental data in comparison with the constant mass models.

The Bohr Hamiltonian, in DDM formalism, is equivalent to a modified Bohr hamiltonian with different metric and different effective potential. The resulting equation reads as,

$$\Bigg[-\frac{1}{2}\frac{\sqrt{f}}{\beta^4}\frac{\partial}{\partial\beta}\beta^4 f \frac{\partial}{\partial\beta}\sqrt{f} - \frac{f^2}{2\beta^2 \sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma \frac{\partial}{\partial\gamma} + \frac{f^2}{8\beta^2}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)} + v_{\mathrm{eff}}\Bigg]\Psi = \epsilon\Psi, \tag{5}$$

with,

$$v_{\text{eff}} = v(\beta,\gamma) + \frac{1}{4}(1-\delta-\lambda)f\,\nabla^2 f + \frac{1}{2}\left(\frac{1}{2}-\delta\right)\left(\frac{1}{2}-\lambda\right)(\nabla f)^2, \quad (6)$$

where δ and λ are free parameters originated from the construction procedure of the kinetic energy term within the DDM formalism. The reduced energies and potentials are defined as $\epsilon = \frac{B_0}{\hbar^2}E$, $v(\beta,\gamma) = \frac{B_0}{\hbar^2}V(\beta,\gamma)$, respectively. Note that the deformation function $f(\beta)$ depends on the potential shape, for example:

$$\begin{cases} f_D(\beta) = 1 + a\beta^2 & \text{in the case of Davidson potential} \quad v_D(\beta) = \beta^2 + \frac{\beta_0^2}{\beta^2} \\ f_K(\beta) = 1 + a\beta & \text{in the case of Kratzer potential} \quad v_K(\beta) = \frac{-1}{\beta} + \frac{1}{\beta^2} \end{cases}$$

It is clear that in the case of the Bohr Hamiltonian with DDM formalism as seen from Eq. (5), the moment of inertia is defined by $(\beta^2/f^2(\beta))\sin^2(\gamma - 2\pi k/3)$. The effect of the function $\beta^2/f^2(\beta)$ on the moment of inertia is shown in Figure (1) for different values of the deformation parameter a. It is apparent that the increase of the moment of inertia is slowed down by the function of deformation $f(\beta)$.

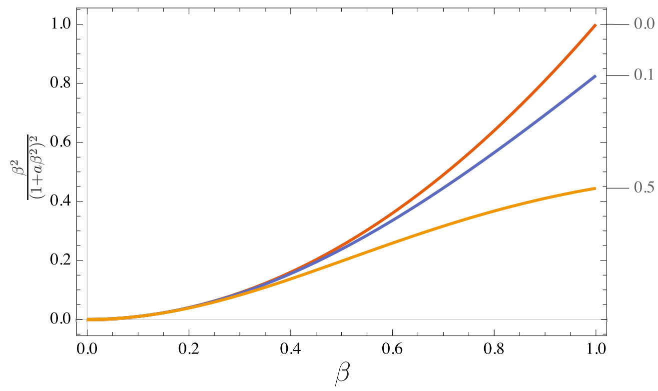

Figure 1. The function $\beta^2/B_\beta(1+a\beta^2)^2$, to which moments of inertia are proportional, plotted as a function of the nuclear deformation β, for different values of the parameter a.

3 Second Concept: Bohr Hamiltonian in the Presence of a Minimal Length

This formalism has introduced the minimal length in nuclear structure [16]. Such an approach, which is inspired from Heisenberg algebra, modifies the momentum operator according to some requirements of the Generalized Uncertainty Principle (GUP). In the framework of this formalism, the generalization of the deformed canonical commutation relation is given by [22]

$$\left[\hat{X}_i, \hat{P}_j\right] = i\hbar\left(\delta_{ij} + \alpha\hat{P}^2\delta_{ij} + \alpha'\hat{P}_i\hat{P}_j\right), \tag{7}$$

where α' is an additional parameter which is of the order of α. In this case, the components of the momentum operator commute to one another

$$\left[\hat{P}_i, \hat{P}_j\right] = 0. \tag{8}$$

However, the commutator between two position operators is in general different from zero

$$\left[\hat{X}_i, \hat{X}_j\right] = i\hbar\frac{(2\alpha - \alpha') + (2\alpha + \alpha')\alpha\hat{P}^2}{1 + \alpha\hat{P}^2}\left(\hat{P}_i\hat{X}_j - \hat{P}_j\hat{X}_i\right). \tag{9}$$

The operators $\hat{X}_i$ and $\hat{P}_i$ up to the first order in α are given by:

$$\hat{X}_i = \hat{x}_i,\ \hat{P}_i = \left(1 + \alpha\hat{p}^2\right)\hat{p}_i. \tag{10}$$

The collective motion of the γ-rigid Z(4), in the presence of ML, is achieved by considering the Hamiltonian [16]

$$\hat{H} = -\frac{\hbar^2}{2B_m}\Delta + \frac{\alpha\hbar^4}{B_m}\Delta^2 + V(\beta), \tag{11}$$

with

$$\Delta = \left[\frac{1}{\beta^3}\frac{\partial}{\partial\beta}\beta^3\frac{\partial}{\partial\beta} - \frac{1}{\beta^2}\left(\hat{Q}^2 - \frac{3}{4}\hat{Q}_1^2\right)\right]. \tag{12}$$

In order to determine the eigenfunctions and eigenvalues of the operator $\hat{H}$, we put

$$\Psi(\beta, \Omega) = \left[1 + 2\alpha\hbar^2\Delta\right] F_{n_\beta}(\beta)\, Y^L_{\mu,\omega}(\Omega),\ \Omega = (\theta_1, \theta_2, \theta_3). \tag{13}$$

Thus, we obtain the equation in the variable β:

$$\begin{aligned}\Big[\frac{1}{\beta^3}\frac{\partial}{\partial\beta}\beta^3\frac{\partial}{\partial\beta} &- \frac{W_{L,\omega}}{4\beta^2}\\ &+ \frac{2B_m}{\hbar^2}\Big(\frac{E - V(\beta)}{(1 + 4B_m\alpha(E - V(\beta)))}\Big)\Big]F_{n_\beta}(\beta) = 0,\end{aligned} \tag{14}$$

where n_β is radial quantum number and $W_{L,\omega} = \left(4\hat{Q}^2 - 3\hat{Q}_1{}^2\right)Y_{L,\omega}(\Omega) = 4L(L+1) - 3\omega^2$ is found by using the following symmetrized wave function,

$$Y^L_{\mu,\omega}(\Omega) = \sqrt{\frac{2L+1}{16\pi^2(1 + \delta_{\omega,0})}}\left[\mathcal{D}^{(L)}_{\mu,\omega}(\theta_i) + (-1)^L\mathcal{D}^{(L)}_{\mu,-\omega}(\theta_i)\right], \tag{15}$$

with $\mathcal{D}(\theta_i)$ represent the Wigner functions of the Euler angles.

4 Correlation between DDM and ML

After presenting the theoretical background of two quantum concepts, namely: the Minimal Length (ML) and the Deformation Dependent Mass (DDM), we have now to reveal the correlation between them, through solutions of Bohr-hamiltonian for transitional nuclei in the limits of CPS X(3) and Z(4).

4.1 The case of X(3)-model

Here, we consider the above mentioned equation in the previous section, with an Infinite Square-Well ISW potential defined by

$$V(\beta) = \begin{cases} 0, & \text{if } \beta \leq \beta_\omega \\ \infty, & \text{if } \beta > \beta_\omega \end{cases}, \tag{16}$$

where β_w indicates the width of the well. The peculiarity of this potential resides in the fact that it admits an infinite number of minimums and moreover, it obviously cannot have unbound states, all possible energies will therefore be quantified. The deforming function coming from DDM formalism, is chosen in the case of an ISW with a null depth in the following form

$$f(\beta) = \beta^{-a},\ a \in [0, 1[\,, \tag{17}$$

where a is deformation mass parameter. An important consequence of this chose is the possibility to derive the energy spectrum, as a function of zeros of the Bessel functions, employing the same method in Ref. [7]. Thus, the energies of X(3) model, characterized by the principal quantum number s together with total angular momentum L are given in this frame by:

$$E_{s,L} = \frac{\hbar^2}{2B_m}\bar{k}^2_{s,\eta}, \quad \bar{k}_{s,\eta} = \frac{\chi_{s,\eta}}{\beta_\omega^{a+1}}, \tag{18}$$

where $\chi_{s,\eta}$ is the s-th zero of the Bessel function and β_ω is the potential's width. η is a parameter given by

$$\eta = \frac{\sqrt{a(a+1) + \frac{L(L+1)}{3} + \frac{1}{4}}}{2(a+1)}. \tag{19}$$

In the ML concept, the eigenenergies formula reads [16]

$$E_{s,L} = \frac{\hbar^2}{2B_m} \times \frac{\bar{k}^2_{s,\eta}}{1 - 2\hbar^2\alpha\bar{k}^2_{s,\eta}}, \quad \bar{k}_{s,\eta} = \frac{\chi_{s,\eta}}{\beta_\omega} \tag{20}$$

where the parameter η, is given by

$$\eta = \left(\frac{L(L+1)}{3} + \frac{1}{4}\right)^{\frac{1}{2}}. \tag{21}$$

By using the energy spectrum of equations (18),(20), we have calculated for each quantum concept the energy ratios $R_{L_g/2_g}$ and $R_{L_\beta/2_g}$ of different levels L_g and L_β of the ground state (g.s) and β bands, respectively, normalized to the first excited level of the g.s band, for 36 even-even nuclei having the ratio $R_{4g/2_g}$ not far from the value 2.44, which is a reference point for the X(3) model, such as the following isotope chains: ^{104}Ru, ^{106}Cd, ^{112}Pd, $^{106,116-120}$Cd, $^{116-134}$Xe, ^{132,138}Ce, $^{132-136,142}$Ba, $^{140-144}$Gd, ^{152}Gd, ^{154}Dy, ^{172}Hf, ^{172,176}Os, ^{190}Os, $^{186-190}$Pt, $^{194-196}$Pt, ^{140,148}Nd. Moreover, the parameters a (DDM) and α

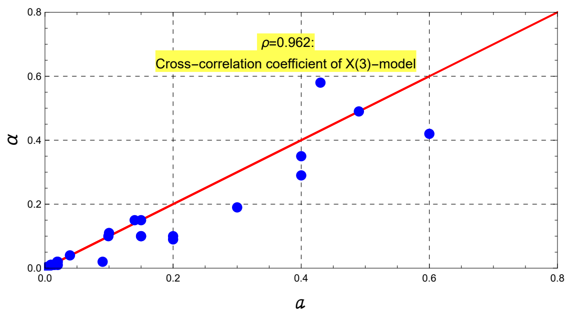

Figure 2. The Correlation between the parameters a and α in X(3).

(ML) for each nucleus are obtained by fitting their available experimental data and are depicted in Figure (2). In fact, this figure shows a strong correlation between ML and DDM formalisms where the cross-correlation coefficient is equal to 0.96. The nuclei situated on the bisectrix have been proved to be the best candidates for X(3) symmetry [7,23] namely: ^{120,126}Xe, ^{148}Nd, ^{172}Os and ^{186}Pt, including two new ones: ^{124}Xe and ^{190}Pt.

4.2 The case of Z(4)-model

As to the CPS Z(4) within DDM concept, by using the deformation function Eq. (17) and through asymptotic iteration method, the eigenvalues are determined by the following formula:

$$E_{s,L} = \frac{\hbar^2}{2B_m}\bar{k}^2_{s,\eta}, \quad \bar{k}_{s,\eta} = \frac{\chi_{s,\eta}}{\beta_\omega^{a+1}} \ , \tag{22}$$

with,

$$\eta = \frac{\sqrt{L(L+4) + 3n_\omega\left(2L - n_\omega\right) + 2a(3a-4) + 4}}{2(a+1)}. \tag{23}$$

where n_w is the wobbling quantum number, while in the ML concept, the equations above are defined respectively by [16]:

$$E_{s,L} = \frac{\hbar^2}{2B_m} \times \frac{\bar{k}^2_{s,\eta}}{1 - 2\hbar^2\alpha\bar{k}^2_{s,\eta}}, \quad \bar{k}_{s,\eta} = \frac{\chi_{s,\eta}}{\beta_\omega} \tag{24}$$

and,

$$\eta = \frac{\sqrt{L(L+4) + 3n_\omega (2L - n_\omega) + 4}}{2}. \tag{25}$$

The calculations of the energy ratios $R_{L_g/2_g}$, $R_{L_\beta/2_g}$ and $R_{L_\gamma/2_g}$, by the above equations (22),(24) have been carried out for several isotopes for which the ratio $R_{4_g/2_g}$ is nearby to 2.23, namely: $^{98-104}$Ru, $^{102-116}$Pd, $^{106-120}$Cd, $^{118-134}$Xe, $^{130-136}$Ba, $^{134-138}$Ce, ^{142}Ba, $^{142-144}$Gd, ^{152}Gd, $^{186-200}$Pt. The obtained parameters a (DDM) and α (ML), by fitting Eq. (22) and Eq. (24) on all available experimental levels, are plotted in Figure 3. One can observe a strong correla-

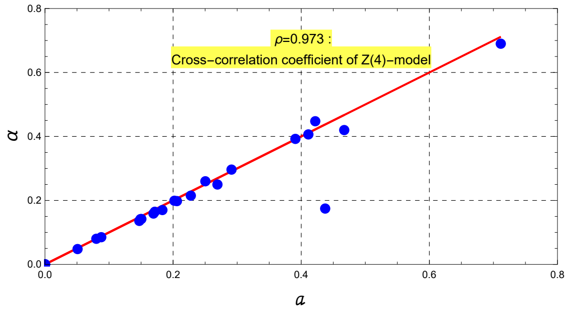

Figure 3. The correlation between the parameters a and α in Z(4).

tion between both concepts (DDM and ML). The cross correlation coefficient is equal to 0.97. The best candidate nuclei for this CPS are set on the bisectrix, which are: $^{128-132}$Xe and $^{192-196}$Pt. These isotopes have been already proved to be the best candidates for Z(4) model [24, 25] including the new one ^{114}Pd.

4.3 The case of X(3)-model with Davidson potential

In order to see further whether the above found correlation is or not adversely impacted by the form or type of the used potential, we apply the above concepts to Davidson potential,

$$U(\beta) = c\beta^2 + \frac{b}{\beta^2}, \quad \beta_0 = \left(\frac{b}{c}\right)^{1/4} \tag{26}$$

where b and c are two free scaling parameters, and β_0 represents the position of the minimum of the potential. In the CPS X(3) within DDM formalism, through AIM we obtained the energy eigenvalues

$$E_{n_\beta,L} = \frac{\hbar^2}{2B_m}\Big(k_0 + \frac{1}{2}(3 + 2\sigma_2 + 2\sigma_{-2} + \sigma_{-2}\sigma_2) \\ + 2an_\beta(2 + \sigma_{-2} + \sigma_2) + 4an_\beta^2\Big) \tag{27}$$

where

$$\begin{aligned}
\sigma_2 =& \sqrt{1+8\frac{k_2}{a^2}}, \qquad \sigma_{-2} = \sqrt{1+8k_{-2}},\\
k_0 =& a\Big(\frac{2L(L+1)}{3} + 3(1-\delta-\delta)+1\Big), k_{-2} = b + \frac{L(L+1)}{6},\\
k_2 =& c + a^2\Big[\frac{3}{2}(1-\delta-\lambda) + 2\Big(\frac{1}{2}-\delta\Big)\Big(\frac{1}{2}-\lambda\Big) + \frac{L(L+1)}{6}\Big].
\end{aligned} \tag{28}$$

For the γ-rigid nuclei within minimal length X(3)-ML by considering a scaled Davidson potential (26), the energy spectrum has been obtained by quantum perturbation method in Ref [17] of the form

$$\begin{aligned}
E_{n_\beta,L} = E^{(0)}_{n_\beta,L} + 4B_m\alpha\Big[\Big(E^{(0)}_{n_\beta,L}\Big)^2 + 2cb\\
- 2E^{(0)}_{n_\beta,L}\Big(c\bar{\beta}^2 + b\bar{\beta}^{-2}\Big) + \Big(c^2\bar{\beta}^4 + b^2\bar{\beta}^{-4}\Big)\Big]
\end{aligned} \tag{29}$$

The fit of the formulas (27)-(29) for the energy ratios, in the model X(3) on the available experimental data [26] for all above used isotope chains has lead to the parameters values of a (DDM) and α (ML), which are plotted in Figure (4). It appears clearly that a very strong correlation exists between the two quantum concepts. In fact, the cross-correlation coefficient is 0.94 in the X(3) case.

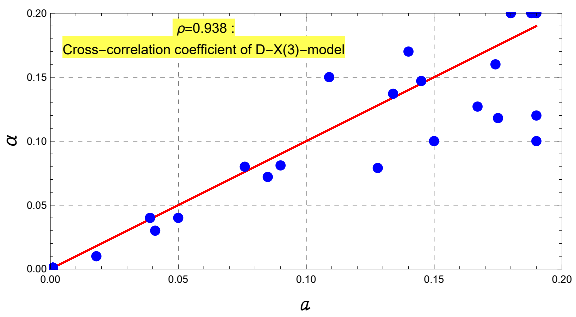

Figure 4. The correlation between the parameters a and α in D-X(3).

4.4 The case of Z(4)-model with Davidson potential

For Davidson potential, given in Eq. (26). The energy eigenvalues of Z(4)-model within DDM concept through AIM are given by

$$E = 2a\Big(2n_\beta+K_1+1\Big)K_2+2a\Big(2n_\beta^2+K_1(2n_\beta+1)+2n_\beta+3+\Lambda+4C_1\Big), \tag{30}$$

where

$$\begin{aligned} \Lambda =& L(L+1) - \frac{3}{4}\alpha^2 \\ C_1 =& \frac{1}{2}\Big(1-\delta-\lambda\Big), \qquad C_2 = \Big(\frac{1}{2}-\delta\Big)\Big(\frac{1}{2}-\lambda\Big), \\ K_1 =& \sqrt{1+\Lambda+b}, \qquad K_2 = \sqrt{\lambda+12C_1+4C_2+4+\frac{c}{a^2}}. \end{aligned} \tag{31}$$

Concerning the energy eigenvalues of Z(4)-model within ML concept for Davidson potential Eq. (26), they are obtained in the form

$$\begin{aligned} E_{n_\beta,L} = E^{(0)}_{n_\beta,L} + 4B_m\alpha\Big[\Big(E^{(0)}_{n_\beta,L}\Big)^2 + 2cb \\ - 2E^{(0)}_{n_\beta,L}\Big(c\bar{\beta}^2 + b\bar{\beta}^{-2}\Big) + \Big(c^2\bar{\beta}^4 + b^2\bar{\beta}^{-4}\Big)\Big] \end{aligned} \tag{32}$$

The fit of the energy ratios (30)-(32) normalized to the first excited level $R_{L_g/2_g}$, $R_{L_\beta/2_g}$ and $R_{L_\gamma/2_g}$, in the model Z(4) on the available experimental data [26] for all above used isotope chains has lead to the parameters values of a (DDM) and α (ML), which are depicted in Figure (5). It is obvious that a very strong correlation exists between the two quantum concepts. In fact, the cross-correlation coefficient is 0.99 in the X(4) case.

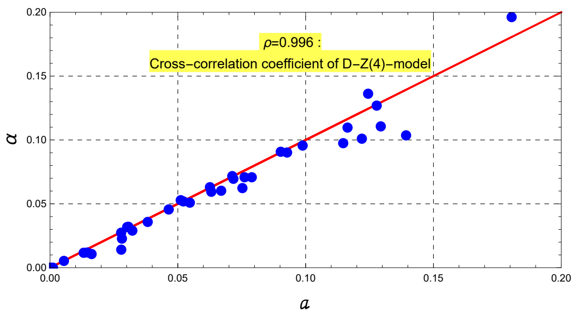

Figure 5. The correlation between the parameters a and α in D-Z(4).

5 Conclusion

We have studied the correlation between both quantum concepts, namely: the Minimal Length (ML) and the Deformation Dependent Mass (DDM) in transitional nuclei near the critical points symmetries (CPS) X(3) and Z(4) with ISW and Davidson potentials. The ML and DDM are well and truly strongly correlated. The uncovered correlation has been used as a new signature for some nuclear CPS allowing us to predict new candidate nuclei to these symmetries.

The present revelation will pave the way for further investigations of such correlation in other shape phase transitions in nuclei at other CPS.

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References

[1] R. Casten, N. Zamfir, *Phys. Rev. Lett*. **85** (2000) 3584.
[2] R.F. Casten, *Nat. Phys.* **2** (2006) 811820.
[3] A. Bohr, B.R. Mottelson, *Mat. Fys. Medd. Dan. Vid. Selsk*. **27** (1953) 16.
[4] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, 1987).
[5] F. Iachello, *Phys. Rev. Lett.* **85** (2000) 3580.
[6] F. Iachello, *Phys. Rev. Lett*. **87** (2001) 052502.
[7] D. Bonatsos, D. Lenis, D. Petrellis, P.A. Terziev, I. Yigitoglu, *Phys. Lett. B* **632** (2006), 238.
[8] D. Bonatsos, D. Lenis, D. Petrellis, P.A. Terziev, I. Yigitoglu, *Phys. Lett. B* **621** (2005) 102.
[9] M. Chabab, A. Lahbas, M. Oulne, *Eur. Phys. J. A* **51** (2015) 131.
[10] M. Chabab, A. Lahbas, M. Oulne, Internat. *J. Modern Phys. E* **24** (2015) 1550089.
[11] M. Chabab, A. El Batoul, M. Hamzavi, A. Lahbas and M. Oulne, *Eur. Phys. J. A* **53** (2017) 157.
[12] D. Bonatsos, P.E. Georgoudis, D. Lenis, N. Minkov, C. Quesne, *Phys. Rev. C* **83** (2011) 044321.
[13] M. Chabab, A. Lahbas, M. Oulne, *Phys. Rev. C* **91** (2015) 064307.
[14] D. Bonatsos, P.E. Georgoudis, N. Minkov, D. Petrellis, C. Quesne, *Phys. Rev. C* **88** (2013) 034316.
[15] M. Chabab, A. El Batoul, A. Lahbas and M. Oulne, *J. Phys. G: Nucl. Part. Phys.* **43** (2016) 125107.
[16] M. Chabab, A. El Batoul, A. Lahbas, M. Oulne, *Phys. Lett. B* **758** (2016) 212.
[17] M. Chabab, A. El Batoul, A. Lahbas, M. Oulne, *Ann. Phys.* **392** (2018) 142.
[18] M. Chabab, A. El batoul, M. Hamzavi, A. Lahbas, I. Moumene, M. Oulne, *Ann. Phys. (N. Y.)* **403** (2019) 47.
[19] C. Quesne and V.M. Tkachuk, *J. Phys. A: Math. Gen.* **37** (2004) 4267.
[20] D.J. Gross and P.F. Mende, *Nucl. Phys. B* **303** (1988) 407.
[21] C. Rovelli and L. Smolin, *Nucl. Phys. B* **442** (1995) 593.
[22] L. Chang, D. Minic, N. Okamura, T. Takeuchi, *Phys. Rev. D* **65** (2002) 125027.
[23] P. Buganu and R. Budaca, *J. Phys. G: Nucl. Part. Phys*. **42** (2015) 105106.
[24] R. Budaca, P. Buganu, M. Chabab, A. Lahbas, M. Oulne, *Ann. Phys. (N. Y.)* **375** (2016) 65.
[25] P. Buganu, M. Chabab, A. El Batoul, A. Lahbas, M. Oulne, *Nucl. Phys. A* **970** (2018) 272.
[26] Nuclear Data Sheets. http://nndc.bnl.gov/.