NUCLEAR THEORY, Vol. 39 (2022)
eds. M. Gaidarov, N. Minkov, Heron Press, Sofia

Alternative Facts in Microscopic Nuclear Structure

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Abstract. I review *ab initio* predictions for light and intermediate-mass nuclei as well as nuclear matter. The problems we are facing in this area are outlined and their relationship to the two- and many-nucleon forces currently in use are discussed. A recent claim that those problems have been solved is critically investigated.

One of the most fundamental aims in theoretical nuclear physics is to understand nuclear structure and reactions in terms of the basic forces between nucleons. As discussed in numerous review papers [1–5], the nuclear physics community presently perceives chiral effective field theory (EFT) as the authoritative paradigm for the derivation of those forces. This perception is based upon a clearly defined relationship between the fundamental theory of strong interactions, QCD, and chiral EFT via symmetries.

Since a while, it is well established that predictive nuclear structure must include three-nucleon forces (3NFs), besides the usual two-nucleon force (2NF) contribution. The advantage of chiral EFT is that it generates 2NFs and multi-nucleon forces simultaneously and on an equal footing. In the Δ-less theory [1], 3NFs occur for the first time at next-to-next-to-leading order (NNLO) and continue to have additional contributions in higher orders. Four-nucleon forces (4NFs) start at next-to-next-to-next-to-leading order (N^{3}LO), but are difficult to implement, which is why they are left out in most present-day calculations. If an explicit Δ-isobar is included in chiral EFT (Δ-full theory [6–9]), then 3NF contributions start already at next-to-leading order (NLO), which leads to a smoother convergence when advancing from leading order (LO) to NNLO. However, summing up all contributions up to NNLO leads to very similar results for both versions of the theory [9]. The convergence of both theories beyond NNLO is expected to be very similar.

In the initial phase, the 3NFs were typically adjusted in $A = 3$ and/or the $A = 4$ systems and the *ab initio* calculations were driven up to the oxygen region [10]. It turned out that for $A \lesssim 16$ the ground-state energies and radii are predicted about right, no matter what type of chiral or phenomenological potentials were applied (local, nonlocal, soft, hard, etc.) and what the details of the 3NF adjustments to few-body systems were [10–13]. It may be suggestive to perceive the α substruture of ^{16}O to be part of the explanation.

The picture changed, when the many-body practitioners were able to move up to medium-mass nuclei (e. g., the calcium or even the tin regions). Large variations of the predictions now occurred depending on what forces were used, and cases of severe underbinding [14] as well as of substantial overbinding [15] were observed. Ever since the nuclear structure community understands that the *ab initio* explanation of intermediate and heavy nuclei is a severe, still unsolved, problem.

A seemingly successfull interaction for the intermediate mass region appears to be the force that is commonly denoted by "1.8/2.0(EM)" (sometimes dubbed "the Magic force") [16, 17], which is a similarity renormalization group (SRG) evolved version of the N^{3}LO 2NF of Ref. [18] complemented by a NNLO 3NF adjusted to the triton binding energy and the point charge radius of ^{4}He. With this force, the ground-state energies all the way up to the tin isotopes are reproduced perfectly—but with charge radii being on the smaller side [19, 20]. Nuclear matter saturation is also reproduced reasonably well, with a slightly too high saturation density [16]. However, these calculations are not consistently *ab initio*, because the 2NF of "1.8/2.0(EM)" is SRG evolved, while the 3NF is not. Moreover, the SRG evolved 2NF is used like an original force with the induced 3NFs omitted. Still, this force is providing clues for how to get the intermediate and heavy mass region right.

Thus, in the follow-up, there have been attempts to get the medium-mass nuclei under control by means of more consistent *ab initio* calculations [21]. Of the various efforts, we will now single out three, which demonstrate in more detail what the problems are.

In Ref. [22], recently developed soft chiral 2NFs [23] at NNLO and N^{3}LO were picked up and complemented with 3NFs at NNLO and N^{3}LO, respectively, to fit the triton binding energy and nuclear matter saturation. These forces were then applied in in-medium similarity renormalization group (IM-SRG [24]) calculations of finite nuclei up to ^{68}Ni predicting underbinding and slightly too large radii [25].

In a separate study [26], the same 2NFs used in Refs. [22,25] were employed, but with the 3NFs now adjusted to the triton and ^{16}O ground-state energies. The interactions so obtained reproduce accurately experimental energies and point-proton radii of nuclei up to ^{78}Ni [26]. However, when the 2NF plus 3NF combinations of Ref. [26] are utilized in nuclear matter, then dramatic overbinding and no saturation at reasonable densities is obtained [27].

Obviously, there is a problem with achieving simultaneously reasonable results for nuclear matter and medium mass nuclei: In Refs. [22, 25], nuclear matter is saturated right, but nuclei are underbound; while in Ref. [26], nuclei are bound accurately, but nuclear matter is overbound.

On this background, it came across like a bombshell when, recently, the Gőteborg-Oak Ridge (GO) group [28,29] claimed that they had solved that problem by way of an NNLO model which includes Δ-isobars. With this model, the authors obtain "accurate binding energies and radii for a range of nuclei

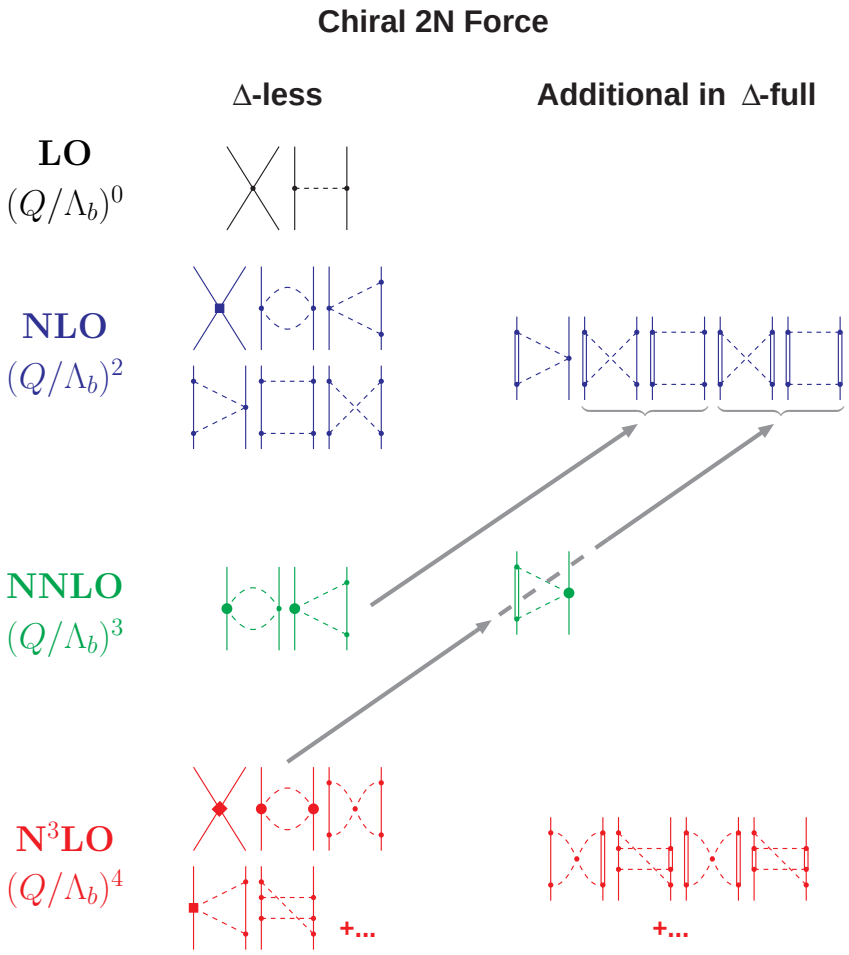

Figure 1. Chiral 2NF without and with Δ-isobar degrees of freedom. Arrows indicate the shift of strength when explicit Δ's are added to the theory. Note that the Δ-full theory consists of the diagrams involving Δ's *plus* the Δ-less ones. Solid lines represent nucleons, double lines Δ-isobars, and dashed lines pions. Small dots, large solid dots, solid squares, and diamonds denote vertices of index $\delta_i = 0$, 1, 2, and 4, respectively. Λ_b denotes the breakdown scale.

from $A = 16$ to $A = 132$, and provide accurate equations of state for nuclear matter" [29]. However, the accuracy of the NN interactions applied was not checked against NN data. Another aspect of interest (not investigated in Refs. [28, 29]) is if the inclusion of Δ-degrees of freedom leads to a higher degree of softness. Note that the successful "Magic" 1.8/2.0(EM) potential is very soft since it is SRG evolved. Moreover, a recent study [30], which investigated

the essential elements of nuclear binding using nuclear lattice simulations, has come to the conclusion that proper nuclear matter saturation requires a considerable amount of non-locality in the NN interaction implying a high degree of softness.

Thus, there is a need for a deeper understanding of the elements in the recent model by the GO group [28, 29], and how they come together to produce the reported favorable nuclear structure predictions. To gain this deeper insight, the following issues need to be addressed:

1. What are the precision and accuracy of the Δ-full NN potentials developed in Ref. [29]? In the context of chiral EFT, this amounts to asking whether the precision of the Δ-full potentials is consistent with the uncertainty of the chiral order at which they have been derived. And, is the accuracy sufficient for meaningful *ab initio* predictions? If there are problems with precison and/or accuracy, how does that impact the predictions for nuclear many-body systems?

2. Does the inclusion of Δ-isobars increase the smoothness of the interaction and, if so, how does the Δ degree of freedom accomplish that?

We have investigated the above-raised questions in Ref. [31]. Following the notation introduced in Ref. [29], potentials at NNLO of the Δ-full theory will be denoted by "ΔNNLO." The diagrams to consider are displayed in Figure 1. For illustrative purposes, the figure includes also the graphs that occur at N^{3}LO. The powers that are associated with the various orders are calculated as follows. For a connected diagram of NN scattering, the power is given by [1]

$$\nu = 2L + \sum_i \delta_i \,, \tag{1}$$

with vertex index

$$\delta_i \equiv d_i + \frac{f_i}{2} - 2 \,, \tag{2}$$

where L denotes the number of loops. Moreover, for each vertex i, d_i is the number of derivatives or pion-mass insertions and f_i the number of fermion fields. The sum runs over all vertices i contained in the diagram under consideration.

In Ref. [29], the GO group presented two ΔNNLO models, which — following the GO notation — are marked by ΔNNLO(450)$_{\mathbf{GO}}$ and ΔNNLO(394)$_{\mathbf{GO}}$, where the parenthetical number denotes the value for the cutoff Λ in units of MeV used in the regulator function. In Figure 2, we display the phase parameters for neutron-proton scattering as predicted by the GO models [solid red line ΔNNLO(450)$_{\mathbf{GO}}$, dashed red ΔNNLO(394)$_{\mathbf{GO}}$] and compare them with the Nijmegen [32] and the Granada [33] phase-shift analyses. It is clearly seen that the predictions deviate substantially from the analyses. This is also reflected in the χ^2/datum for the reproduction of the NN data shown in Table 1.

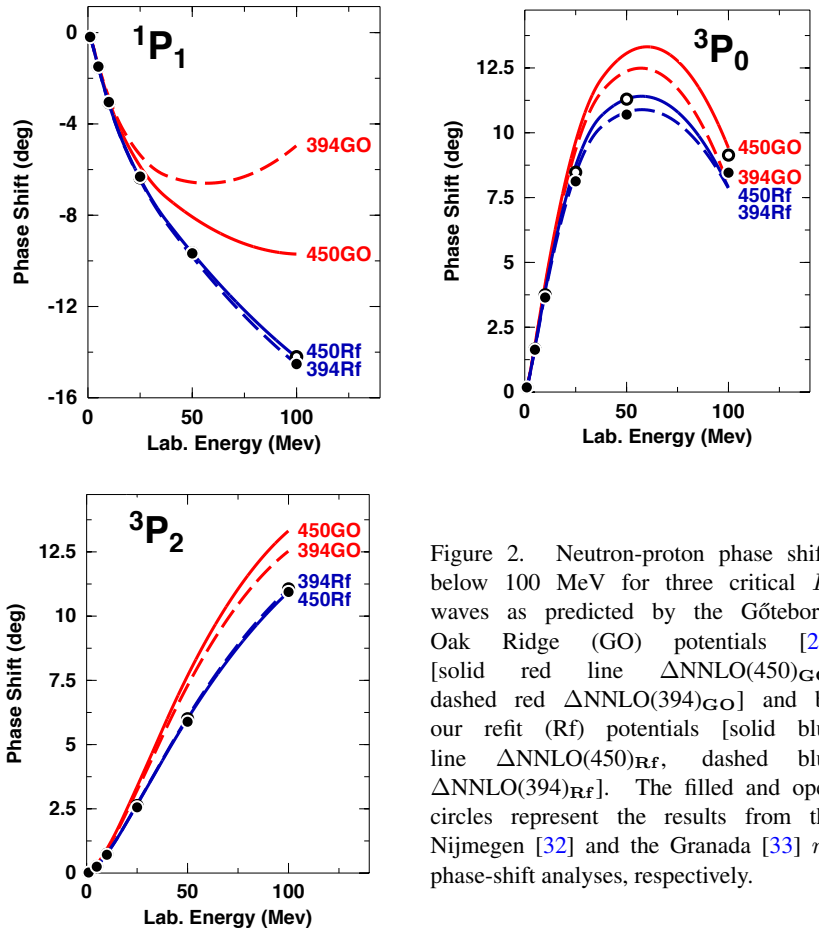

Figure 2. Neutron-proton phase shifts below 100 MeV for three critical P-waves as predicted by the Gőteborg-Oak Ridge (GO) potentials [29] [solid red line ΔNNLO(450)$_{\mathbf{GO}}$, dashed red ΔNNLO(394)$_{\mathbf{GO}}$] and by our refit (Rf) potentials [solid blue line ΔNNLO(450)$_{\mathbf{Rf}}$, dashed blue ΔNNLO(394)$_{\mathbf{Rf}}$]. The filled and open circles represent the results from the Nijmegen [32] and the Granada [33] np phase-shift analyses, respectively.

We have then constructed ΔNNLO models with improved fits—for the purpose of explicitly checking out whether, within the Δ-full theory, we can achieve χ^2 that are consistent with the χ^2 obtained in Ref. [23] for the Δ-less theory. We have dubbed our refits ΔNNLO(450)$_{\mathbf{Rf}}$ and ΔNNLO(394)$_{\mathbf{Rf}}$ (where "Rf" stands for Refit) and, indeed, their χ^2 are very similar to the ones of Ref. [23] of the Δ-less theory. The phase shifts of the refits are displayed in Figure 2 by the blue solid and blue dashed lines. The conclusion is that, within the Δ-full theory, fits can be achieved that are of the same quality as in the Δ-less theory.

Our investigation [31] has come to the following conclusions:

1. The Δ-full NN potentials at NNLO constructed by the Gőteborg-Oak Ridge (GO) group [29] are up to 40 times outside the theoretical error of chiral EFT at NNLO and are, therefore, inconsistent with the EFT that the

Table 1. χ^2/datum for the reproduction of the NN data by the Gőteborg-Oak Ridge (GO) potentials. The Hamada-Johnston potential [34] is included for comparison

Bin (MeV)	Hamada-Johnston potential of 1962 [34]	ΔNNLO(450)$_{\mathbf{GO}}$	ΔNNLO(394)$_{\mathbf{GO}}$
		proton-proton	
0–100	19.6	60.7	34.3
0–200	13.8	46.3	39.7
		neutron-proton	
0–100		5.87	8.58
0–200		14.2	26.2
		pp plus ***np***	
0–100		28.8	19.3
0–200		29.6	32.6

potentials are intended to be based upon (cf. the phase-shifts of some crucial P-waves shown in Figure 2). Moreover, these potentials reproduce the NN data with a very large χ^2/datum (cf. Table 1). This is unacceptable based on contemporary precision standards.

2. The predictions by the GO NN potentials for the energy per nucleon in nuclear matter are very attractive, similar to the predictions by the 1.8/2.0(EM) NN potential of Ref. [16], also known as 'Magic' (cf. Figure 3). The extremely attractive nature of both the GO and the Magic potentials is the reason for the favorable reproduction of the energies (and radii) of intermediate-mass nuclei, which have proven to be a problem in *ab initio* nuclear structure physics. However, the extra attraction in the GO potentials which brings them to the level of Magic can be traced to incorrect P-wave and ϵ_1 mixing parameters.

3. When all phase parameters, including the P-wave and the ϵ_1-mixing parameters, are fitted within the NNLO truncation error, then the extra attraction disappears and the nuclear matter predictions become very similar to the ones by NN potentials constructed within the Δ-less theory (cf. Figure 3). Thus, we find claims that Δ-full potentials lead to more attraction in nuclear many-body systems to be incorrect.

4. The extraordinarily attractive nature of Magic is due to its high degree of nonlocality which, in turn, is due to its SRG construction. This degree of nonlocality is not achieved by chiral NN potentials, no matter if Δs are included or excluded, because all two-pion exchange (2PE) contributions in both version of the theory are local (at least up to NNLO) and nonlocality is generated only by the regulator function, which adds only moderate nonlocality.

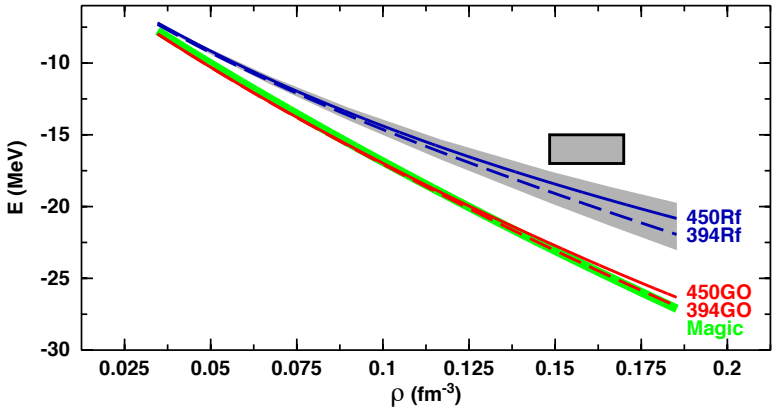

Figure 3. Energy per nucleon in symmetric nuclear matter, E, as a function of density, ρ, as generated by some two-body forces. Notation for the ΔNNLO potentials as in Figure 2. Magic (solid green line) refers to the 1.8/2.0(EM) potential of Ref. [16]. The shaded band includes the theoretical uncertainties associated with the predictions by the Rf potentials (blue lines). Note that this shaded band also covers the predictions by the Δ-less NNLO and N^{3}LO potentials of Ref. [23] applied in Ref. [25] to intermediate-mass nuclei. The grey box outlines the area where nuclear saturation is expected to occur.

5. The problem with a microscopic description of intermediate mass nuclei with realistic chiral nuclear forces remains, unfortunately, unsolved.

Acknowledgement

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-03ER41270.

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