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An Alternative Approach within the CDFM for Studies of Nuclear Symmetry Energy Components and their Ratio

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Abstract. A new alternative approach to calculate the ratio of the surface to volume components of the nuclear symmetry energy is proposed in the framework of the coherent density fluctuation model (CDFM). A new expression (scheme II) for the ratio is derived consistently within the model. This expression appears in a form more direct and physically motivated than the expression (scheme I) that was used in our previous works within the CDFM and avoids preliminary assumptions and mathematical ambiguities in scheme I. The calculations are based on the Skyrme and Brueckner energy-density functionals for nuclear matter and on nonrelativistic Brueckner-Hartree-Fock method with realistic Bonn B and Bonn CD nucleon-nucleon potentials. The approach is applied to isotopic chains of Ni, Sn, and Pb nuclei using nuclear densities obtained in self-consistent Hartree-Fock+BCS calculations with SLy4 Skyrme effective interaction. The applicability of both schemes within the CDFM is demonstrated by a comparison of the results with the available empirical data and with results of other theoretical studies of the considered quantities. Although in some instances the results obtained for the studied ratio and the symmetry energy components are rather close in both schemes, the new scheme II leads to more realistic values that agree better with the empirical data and exhibits conceptual and operational advantages.

1 Introduction

Measurements of nuclear structure characteristics including masses, densities, and collective excitations have resolved some of the basic features of the equation of state (EOS) of nuclear matter. The EOS allows one to constrain the bulk

and surface properties of the nuclear energy-density functionals (EDFs) quite effectively via the symmetry energy and related properties. The latter are significant ingredients of the EOS and their study in both asymmetric nuclear matter and finite nuclei are of particular importance.

While there is enough collected information for the key EOS parameters, e.g., for the nuclear symmetry energy (NSE), the neutron pressure, and the asymmetric compressibility, the volume and surface symmetry energies have been poorly investigated till now. This concerns mostly the surface contribution to the NSE and comes from the fact that many nucleons are present at around the nuclear surface. For instance, the combined experiment at GANIL, where the VAMOS spectrometer was coupled with the 4π INDRA detector to study the isotopic distributions produced in ^{40,48}Ca+^{40,48}Ca collisions at 35 MeV/nucleon, allowed one to estimate the relative contribution of surface and volume terms to the symmetry energy in the nuclear EOS [1]. The knowledge of this contribution and, especially, the relevance of the surface term are important to explore to what extent one can learn about the density dependence of the symmetry energy in infinite nuclear matter (NM) from multifragmentation of finite nuclei and from nuclear reaction dynamics.

The volume and surface contributions to the NSE and their ratio at zero temperature were calculated in Ref. [2] within the CDFM (see, e.g., Refs. [3, 4]) using two EDFs, namely, the Brueckner and Skyrme ones. The obtained results in the cases of Ni, Sn, and Pb isotopic chains were compared with results of other theoretical methods and with those from approaches which used experimental data on binding energies, excitation energies to isobaric analog states (IAS), and neutron-skin thicknesses. An investigation of the thermal evolution of the NSE components and their ratio for isotopes belonging to the same chains around the double-magic nuclei performed in Ref. [5] has extended our previous analysis of these nuclei for temperatures different from zero.

In the present paper (see also Ref. [6]) we suggest a new alternative approach to the one proposed in Refs. [2, 7] for calculation of the ratio between the volume and surface components of the NSE within the CDFM in a more direct and physically motivated way, namely to avoid the preliminary assumptions and mathematical ambiguities in our previous scheme I. To achieve this goal, in the new scheme II, we apply the general relation based on the Droplet Model between the symmetry energy and its components to the building units ("fluctons") of the CDFM model, and we construct from them the ratio between the NSE components for finite nuclei following the standard CDFM procedure. We also search for the dependence of the results on several sets of nuclear potentials. In the new approach we perform calculations for the symmetry energy components $S^V(A)$ and $S^S(A)$ and their ratio for the same isotopes in Ni (A=74-84), Sn (A=124-156), and Pb (A=202-214) chains considered before and compare the obtained results with the previous ones (including $S^V(A)$, $S^S(A)$, and their ratio κ) obtained by the procedure in Refs. [2, 7]. The applicability of our both schemes within the CDFM is also demonstrated by a comparison of the results

with the available empirical data and with results of other theoretical studies for the considered quantities.

2 Theoretical Scheme

The symmetry energy $S(A)$ is expressed by the volume $S^V(A)$ and modified surface component $S^S(A)$ in the droplet model (see Ref. [8], where it is defined as S_s^*):

$$S(A) = \frac{S^V(A)}{1 + \dfrac{S^S(A)}{S^V(A)} A^{-1/3}} = \frac{S^V(A)}{1 + q(A) A^{-1/3}}, \tag{1}$$

where

$$q(A) \equiv \frac{S^S(A)}{S^V(A)}. \tag{2}$$

We note that in the present work we use Eq. (1) as a basic relation between the symmetry energy $S(A)$ and its volume $S^V(A)$ and surface $S^S(A)$ components. The reason to use Eq. (1) in contrast to the relation in another approach used in, e.g., Refs. [9–12], and also in our work [2], was discussed in detail in our previous work [7]. It is motivated by the necessity to have a correct behavior of the denominator in Eq. (1) in the infinite nuclear matter limit. More precisely, in the limit $A \to \infty$ the ratio in Eq. (1) $S^S/S^V \to 0$, so that $[S^S/S^V]A^{-1/3} \to 0$ and the symmetry energy in Eq. (1) has the correct limit $S \to S^V$. Contrary to this, in the approach of Refs. [9–12] in the limit $A \to \infty$ the term $[S^V(A)/S^S(A)]A^{-1/3}$ is not well determined. The use of the latter approach needs a condition to be imposed, namely the surface coefficient $S^S(A)$ to go to zero more slowly than $A^{-1/3}$ as $A \to \infty$. This is the reason to use in our work Eq. (1) instead of the relation in the approach in e.g., Refs. [9–12].

At very large A we may write the symmetry energy in the known form (see Ref. [13]):

$$S(A) \simeq S^V(A) - \frac{S^S(A)}{A^{1/3}}, \tag{3}$$

which follows from Eq. (1) for large A.

The relations of $S^V(A)$ and $S^S(A)$ with $S(A)$ in terms of $q(A)$ can be found from Eqs. (1) and (2):

$$S^V(A) = S(A)\left[1 + \frac{q(A)}{A^{1/3}}\right], \tag{4}$$

$$S^S(A) = q(A)S(A)\left[1 + \frac{q(A)}{A^{1/3}}\right]. \tag{5}$$

The following expression for the ratio of the volume to the surface symmetry energy coefficients was given by Danielewicz [9] (see also Ref. [14]):

$$\kappa(A) = \frac{S^V(A)}{S^S(A)} = \frac{3}{r_0}\int dr \frac{\rho(r)}{\rho_0}\left\{\frac{S^{NM}(\rho_0)}{S^{NM}[\rho(r)]} - 1\right\}, \tag{6}$$

where $S^{NM}[\rho(r)]$ is the nuclear matter symmetry energy, $\rho(r)$ is the half-infinite nuclear matter density, ρ_0 is the nuclear matter equilibrium density, and r_0 is the radius of the nuclear volume per nucleon. The latter two quantities are related by

$$\frac{4\pi r_0^3}{3} = \frac{1}{\rho_0}. \tag{7}$$

In the present work we calculate the EOS parameters in finite nuclei, such as the nuclear symmetry energy and its surface and volume components using the CDFM. The model is based on the δ-function limit of the generator coordinate method [4, 15], it is a natural extension of the Fermi-gas model and includes nucleon-nucleon correlations of collective type. An important feature of the CDFM is that it allows us to make the transition from nuclear matter quantities to the corresponding ones in finite nuclei.

In our first scheme to calculate the ratio $\kappa(A)$ we started from the expression of Eq. (6) (see, e.g., [9, 14]) making in it a preliminary assumption replacing the density $\rho(r)$ for the half-infinite nuclear matter in the integrand by the density distribution of a finite nucleus, namely, by the expression in the CDFM:

$$\rho(\mathbf{r}) = \int_0^\infty dx |F(x)|^2 \rho_0(x) \Theta(x - |\mathbf{r}|), \tag{8}$$

where

$$\rho_0(x) = \frac{3A}{4\pi x^3} \tag{9}$$

and $|F(x)|^2$ is the weight function. In the case of monotonically decreasing local density ($d\rho/dr \leq 0$) the weight function $|F(x)|^2$ can be obtained from a known density (obtained theoretically or experimentally):

$$|F(x)|^2 = -\frac{1}{\rho_0(x)} \left. \frac{d\rho(r)}{dr} \right|_{r=x}. \tag{10}$$

The following expression for the nuclear symmetry energy in finite nuclei $S(A)$ can be obtained within the CDFM on the base of the infinite matter one $S^{NM}(\rho)$ by weighting it with $|F(x)|^2$:

$$S(A) = \int_0^\infty dx |F(x)|^2 S^{NM}[\rho(x)]. \tag{11}$$

Following the procedure whose details are given in our work [7], we obtain the formula for $\kappa(A)$ in the form:

$$\kappa(A) = \frac{3}{r_0 \rho_0} \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_0(x) \int_0^x dr \left\{ \frac{S^{NM}(\rho_0)}{S^{NM}[\rho_0(x)]} - 1 \right\} \tag{12}$$

that leads finally to

$$\kappa(A) = \frac{3}{r_0 \rho_0} \int_0^\infty dx |\mathcal{F}(x)|^2 x \rho_0(x) \left\{ \frac{S^{NM}(\rho_0)}{S^{NM}[\rho_0(x)]} - 1 \right\}. \tag{13}$$

The right-hand side of Eq. (13) is an one-dimensional integral over x, the latter being the radius of the "flucton" that is perpendicular to the nuclear surface. We refer to the expression in Eq. (13) as scheme I, because this was the first equation that we used for the numerical calculations of the results presented in [2, 7].

Here we would like to underline the main differences in the construction of scheme II in comparison with the previous scheme I: i) we do not use the method in Refs. [9, 14], and ii) we avoid the assumption concerning the replacement of the density $\rho(r)$ for the half-infinite nuclear matter by the density distribution of a finite nucleus. A third and important reason to choose a new scheme is that the integrand in Eq. (13) for κ in scheme I presents singularities for some of the potentials (e.g., for the Brueckner one). Thus, the results for κ become extremely sensitive to the choice of the integration interval, mainly to the value of the lower limit of integration in Eq. (13). In the new scheme II we start from the general relationship [Eq. (1)] between the NSE S and its components S^V and S^S. The procedure of the derivation of $q(A)$ for finite nuclei is as follows: i) we determine the ratio $q(x) = S^S(x)/S^V(x)$ for the "fluctons" of the CDFM from the basic Eqs. (1) and (3), and ii) we construct $q(A)$ within the CDFM rules weighting $q(x)$ by the weight function $|F(x)|^2$. First, to construct $q(x) = S^S(x)/S^V(x)$ in the x-flucton we recall that the x-flucton is a sphere of nuclear matter of radius x with density $\rho_0(x)$. This implies that inside each flucton we may apply Eq. (3) in the form $S^S/S^V \simeq (1 - S/S^V)A^{1/3}$, with A, the number of nucleons in the flucton, given by $(x/r_0)^3[\rho_0(x)/\rho_0]$ [see Eqs. (7) and (9)], and S the nuclear matter symmetry energy in the flucton $[S^{NM}(\rho_0(x))]$ with volume component $S^V \simeq S^{NM}(\rho_0)$. This results in the following expression for $q(x)$:

$$q(x) = \frac{S^S(x)}{S^V(x)} = \frac{x}{r_0}\left[\frac{\rho_0(x)}{\rho_0}\right]^{1/3}\left[1 - \frac{S^{NM}[\rho_0(x)]}{S^{NM}(\rho_0)}\right]. \tag{14}$$

Weighting $q(x)$ by the function $|F(x)|^2$ leads to the following relationship for the ratio (2):

$$\begin{aligned} q(A) &= \int_0^\infty dx|F(x)|^2 q(x) \\ &= \int_0^\infty dx|F(x)|^2 \frac{x}{r_0}\left[\frac{\rho_0(x)}{\rho_0}\right]^{1/3}\left[1 - \frac{S^{NM}[\rho_0(x)]}{S^{NM}(\rho_0)}\right]. \end{aligned} \tag{15}$$

We refer to the expression in Eq. (15) as scheme II. Here we would like to note the following: i) the expression Eq. (14) for a flucton is obtained in a direct and natural way starting from the known formula Eq. (3) that follows from the general relationship Eq. (1) at large A; ii) Eq. (15) is obtained without preliminary assumptions that were imposed to obtain Eq. (13) in scheme I and is free from singularities; iii) as a result of i) and ii) the calculated quantity $1/q = S^V/S^S$ that follows from Eq. (15) is not equal to the previously calculated quantity κ following Eq. (13). We note that both quantities are obtained within different

schemes, though both are within the framework of the CDFM. Of course, the values of the results for $1/q(A)$ coming from Eq. (15) and $\kappa(A)$ [Eq. (13)] can be compared and this is done in the next section.

3 Results and Discussion

We show in Figure 1 the results for the ratio $1/q = S^V/S^S$ as a function of the mass number A for the isotopic chains of Ni, Sn, and Pb with SLy4 force. In Table 1 the values of this ratio obtained within the new scheme are compared with the values of κ [Eq. (13)] calculated from our previous scheme within the CDFM [2, 7]. We would like to emphasize that this comparison is between quantities obtained in two different CDFM schemes and it can serve basically to show the influence and the importance of the preliminary assumptions and shortcomings made of scheme I and the advantage of the new scheme that is free from them.

In general, the values of $1/q$ within the new CDFM scheme calculated using the Skyrme EDF for the isotopic chains of Ni, Sn, and Pb are between 1.70 and 2.40. This range of values is similar to the estimations for $\kappa(A)$ [Eq. (13)] of Danielewicz *et al.* obtained from a wide range of available data on the binding energies [11], of Steiner *et al.* [8], and from a fit to other nuclear properties, such as the excitation energies to IAS and skins [10] $2.6 \leq \kappa \leq 3.0$ and from masses and skins [10] $2.0 \leq \kappa \leq 2.8$. The values of $1/q$ obtained using the Brueckner EDF for the Ni isotopic chain with SLy4 force are in agreement partly with that obtained in Ref. [14] by Dieperink and Van Isacker from the analyses

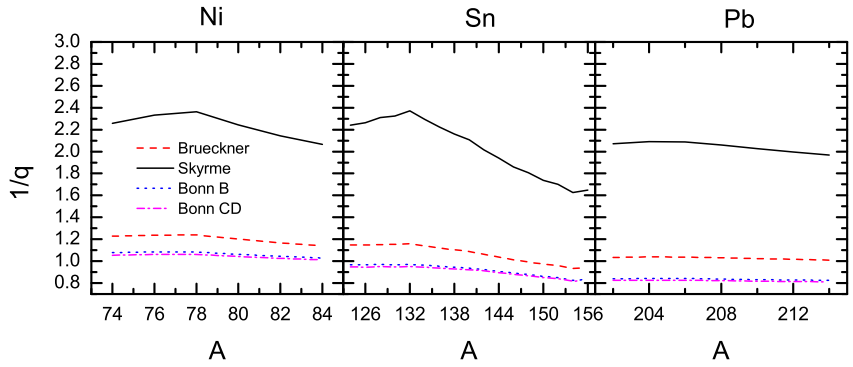

Figure 1. The quantity $1/q = S^V/S^S$ [following from Eq. (15)] as a function of A for the isotopic chains of Ni, Sn, and Pb obtained using Brueckner EDF (dashed line), Skyrme EDF (solid line) and BHF method with Bonn B (dotted line) and Bonn CD (dash-dotted line) potentials from Refs. [16, 17]. The weight function $|F(x)|^2$ [Eq. (10)] used in the calculations is obtained by means of the densities derived within a self-consistent Skyrme-Hartree-Fock plus BCS method with SLy4 force.

Table 1. The ranges of changes of $1/q$ (scheme II) and κ (scheme I) [2, 7] with Skyrme and Brueckner EDFs and BHF method with Bonn B and Bonn CD potentials for the Ni, Sn, and Pb isotopic chains.

	Ni		Sn		Pb	
	$1/q$	κ	$1/q$	κ	$1/q$	κ
Skyrme	2.07–2.36	1.53–1.70	1.63–2.37	1.58–2.02	1.97–2.09	1.67–1.71
Brueckner	1.14–1.24	2.22–2.44	0.94–1.16	2.40–2.90	1.01–1.04	2.62–2.64
Bonn B	1.03–1.08	1.80–1.90	0.83–0.97	2.00–2.48	0.84–0.88	2.54–2.80
Bonn CD	1.01–1.06	1.80–2.00	0.82–0.95	2.00–2.48	0.81–0.83	2.54–2.80

of masses and skins $1.6 \leq \kappa \leq 2.0$. The obtained values of $1/q$ for Sn and Pb isotopes using the Brueckner EDF together with the ones when using both Bonn potentials are close to the value of 1.14 given by Bethe in Ref. [13] and to the estimated value of 1.1838 by Myers and Swiatecki [18]. Generally, we can note that the results of the new scheme for $1/q$, in particular using Skyrme and Brueckner EDFs, cover reasonably the estimated values of κ (between 1.14 and 2.80) in a better way than in the previous scheme.

Here we note the observed peaks in the ratio $1/q$ at $A = 78$ and $A = 132$ for Ni and Sn isotopes, respectively. They are more pronounced for the choice of the Skyrme EDF, less pronounced for Brueckner EDF, and are somewhat smoothed out for Bonn B and Bonn CD potentials. We attribute these peaks to the sharp nuclear density transition when passing double-magic nuclei, such as ^{78}Ni and ^{132}Sn, in an isotopic chain. The peculiarities of $\rho(r)$ (and consequently the derivative of $\rho(r)$ which determines the weight function $|F(x)|^2$) for the closed shells lead to the existence of "kinks" that had been found and discussed in our previous works (e.g., in [2, 5, 7]). In the case of Pb isotopic chain (see Figure 1) such kink does not exist at $A = 208$ and this reflects the smooth behavior without kinks of $S(A)$ [Eq. (11)] and related quantities for the Pb isotopic chain. Similar peaks in the ratio κ as a function of the mass number have been observed in our previous studies [2, 7].

The values of the symmetry energy S [Eq. (11)] and its volume S^V [Eq. (4)] and surface S^S [Eq. (5)] components as functions of A deduced within the new scheme for the same isotopic chains are presented in Figure 2. The calculated symmetry energy for the three isotopic chains and all considered potentials turns out to be between 24 and 31 MeV (see Figure 2). In practice, predictions for the symmetry energy vary substantially (28–38 MeV), e.g., an empirical value of the symmetry energy 30 ± 4 MeV is given in Refs. [19, 20]. The values of the volume contribution S^V to the NSE obtained within the new scheme in the case of Brueckner and Skyrme EDFs are smaller than the ones derived from the previous CDFM scheme I (presented in Tables I and III of Ref. [2]). We would like to emphasize that the results for S^V in the scheme II (between 29 and 34 MeV) are more realistic than the ones previously obtained within our scheme

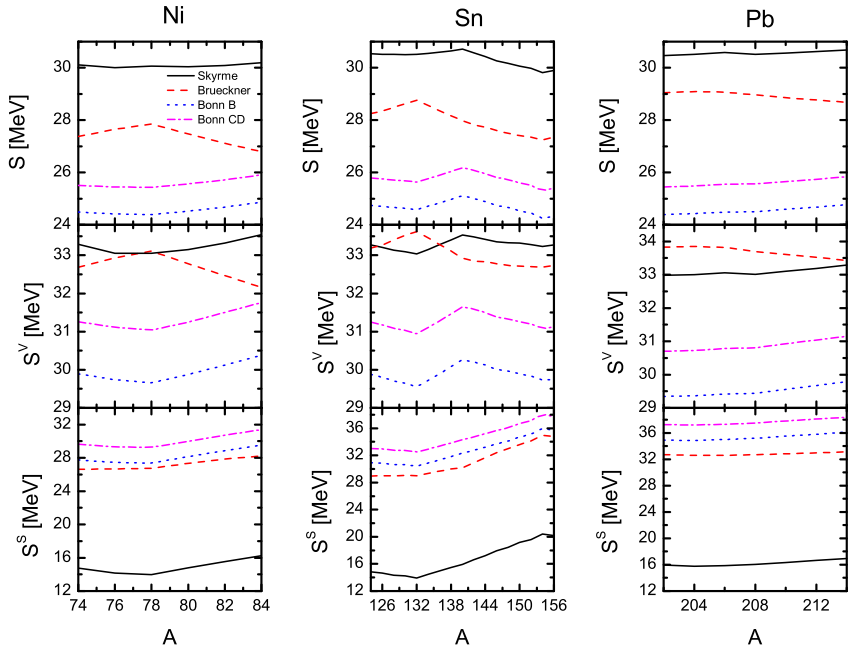

Figure 2. The symmetry energy S [Eq. (11)] and its volume S^V [Eq. (4)] and surface S^S [Eq. (5)] components for the isotopic chains of Ni, Sn, and Pb obtained using Brueckner EDF (dashed line), Skyrme EDF (solid line) and BHF method with Bonn B (dotted line) and Bonn CD (dash-dotted line) potentials from Refs. [16, 17]. The weight function $|F(x)|^2$ [Eq. (10)] used in the calculations is obtained by means of the densities derived within a self-consistent Skyrme-Hartree-Fock plus BCS method with SLy4 force.

I, for instance, using Brueckner EDF (between 41.5 and 43 MeV). The new results with scheme II are in good agreement with the available phenomenological estimations, as follows: Ref. [10]: $30.0 \leq S^V \leq 32.5$ MeV, Ref. [12]: $31.5 \leq S^V \leq 33.5$ MeV. In the case of Ni isotopic chain our previous calculations [7] with SLy4 force provided values of the volume symmetry energy within 27.6 and 28.1 MeV for Bonn B potential and within 28.4–29.1 MeV for Bonn CD potential. In the new approach for the same potentials the corresponding values of S^V are larger by 2 MeV and are better compared with the results presented in Refs. [10,12]. Concerning the surface component of the NSE S^S, it is known that this component is poorly constrained by empirical data. Figure 2 shows that the range of the values obtained for S^S and for Ni, Sn, and Pb isotopes in the case of Skyrme EDF is 14–18 MeV. These results come closer to the limits on the surface symmetry parameter 11 MeV$\leq \beta \leq$14 MeV established in Ref. [11]. The new CDFM scheme gives larger values for the surface component in the case of the three other potentials (Brueckner, Bonn B, and Bonn CD).

We would like to note that the same peculiarities (as for the ratio $1/q = S^V/S^S$ presented in Figure 1), namely "kinks" appear in the cases of S, S^V, and S^S as functions of the mass number A at the double-magic ^{78}Ni and ^{132}Sn isotopes. They are stronger or weaker and depending on the use of a given nuclear potential. In Figure 2 a kink appears for $S(A)$ and $S^V(A)$ not only for the double-magic ^{132}Sn but also for the semimagic ^{140}Sn nucleus. As was discussed in Ref. [2], the latter is related to the closed $2f_{7/2}$ subshell for neutrons. Kinks of the A dependence of the symmetry energy and its components in the Pb isotopic chain are not observed.

4 Summary and Conclusions

The main results of the present work can be summarized as follows:

i) We provide an alternative approach (scheme II) to calculate the ratio $q(A) = S^S(A)/S^V(A)$ of the surface to volume components of the NSE within the framework of the CDFM in a more direct and simple way and having stronger physical grounds than the former one (scheme I) that had been used in our previous works [2, 7]. In the new approach we firstly determine the ratio $q(x)$ for a flucton in the CDFM model from the basic Droplet Model mass formula and then we use the convolution of $q(x)$ with $|F(x)|^2$ to construct $q(A)$ for finite nuclei following the standard CDFM procedure. In this way the new scheme avoids some conceptual and mathematical shortcomings that were met in the previous scheme.

ii) We would like to note the dependence of the results for the ratio of S^S to S^V on the effective nuclear potentials used in the calculations. In this respect, the results of our calculations using Skyrme EDF turn out to be close to the different estimations obtained from a fit to nuclear properties, such as the excitation energies to IAS and neutron-skin thickness [10], masses, and others. The values of $1/q$ obtained using the Brueckner EDF for the Ni isotopic chain are in agreement with those obtained in Ref. [14] from the analyses of masses and skins. In the case of Bonn B and Bonn CD two-body potentials the results for the ratio $1/q$ approach the estimated values from the works of Bethe [13] and Myers and Swiatecki [18]. Overall, the results of the new scheme for $1/q$ cover reasonably the whole region of estimated values for κ (between 1.14 and 2.80) and in some cases are somewhat better than the values obtained in the previous scheme.

iii) The values of the symmetry energy S for the three isotopic chains and all considered potentials are between 24 and 31 MeV that is in accordance with the region of its empirical values 30 ± 4 MeV given in Refs. [19, 20]. The results for the volume component $S^V(A)$ of NSE in scheme II (between 29 and 34 MeV) are in good agreement with those of Refs. [10, 12] (between 30 and 33.5 MeV). The values of the surface contribution $S^S(A)$ in scheme II in the case of Skyrme EDF (14-18 MeV) come closer to the region of 11-14 MeV established in Ref. [11].

iv) Analyzing the isotopic sensitivity of $S^V(A)$, $S^S(A)$, and their ratio $1/q(A)$ we observe peculiarities ("kinks") of these quantities as functions of the mass number A in the cases of the double-magic ^{78}Ni and ^{132}Sn isotopes, as well as a "kink" of $S^V(A)$ for ^{140}Sn. No pronounced peak at the double-magic nucleus with $A = 208$ in the Pb chain is found.

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References

[1] Q. Fable *et al.*, *Phys. Rev. C* **106** (2022) 024605.

[2] A.N. Antonov, M.K. Gaidarov, P. Sarriguren, and E. Moya de Guerra, *Phys. Rev. C* **94** (2016) 014319.

[3] A.N. Antonov, V.A. Nikolaev, and I.Zh. Petkov, *Bulg. J. Phys*. **6** (1979) 151; *Z. Phys. A* **297** (1980) 257; *ibid* **304** (1982) 239; *Nuovo Cimento A* **86** (1985) 23; A.N. Antonov et al., *ibid* **102** (1989) 1701; A.N. Antonov, D.N. Kadrev, and P.E. Hodgson, *Phys. Rev. C* **50** (1994) 164.

[4] A.N. Antonov, P.E. Hodgson, and I.Zh. Petkov, *Nucleon Momentum and Density Distributions in Nuclei* (Clarendon Press, Oxford, 1988); *Nucleon Correlations in Nuclei* (Springer-Verlag, Berlin-Heidelberg-New York, 1993).

[5] A.N. Antonov, D.N. Kadrev, M.K. Gaidarov, P. Sarriguren, and E. Moya de Guerra, *Phys. Rev. C* **98** (2018) 054315.

[6] M.K. Gaidarov, E. Moya de Guerra, A.N. Antonov, I.C. Danchev, P. Sarriguren, and D.N. Kadrev, *Phys. Rev. C* **104** (2021) 044312.

[7] I.C. Danchev, A.N. Antonov, D.N. Kadrev, M.K. Gaidarov, P. Sarriguren, and E. Moya de Guerra, *Phys. Rev. C* **101** (2020) 064315.

[8] A.W. Steiner, M. Prakash, J.M. Lattimer, and P.J. Ellis, *Phys. Rep.* **411** (2005) 325.

[9] P. Danielewicz, arXiv: 0607030 [nucl-th].

[10] P. Danielewicz, arXiv: 0411115 [nucl-th].

[11] P. Danielewicz, *Nucl. Phys. A* **727** (2003) 233.

[12] P. Danielewicz and J. Lee, *Nucl. Phys. A* **818** (2009) 36.

[13] H.A. Bethe, *Theory of Nuclear Matter*, Annual Review of Nuclear Science, vol. 21, (Palo Alto, California, USA, 1971), p.93.

[14] A.E.L. Dieperink and P. Van Isacker, *Eur. Phys. J. A* **32** (2007) 11.

[15] J.J. Griffin and J.A. Wheeler, *Phys. Rev.* **108** (1957) 311.

[16] R. Machleidt, *Adv. Nucl. Phys.* **19** (1989) 189.

[17] R. Machleidt, *Phys. Rev. C* **63** (2001) 024001.

[18] W.D. Myers and W.J. Swiatecki, *Nucl. Phys. A* **81** (1966) 1.

[19] P.E. Haustein, *At. Data Nucl. Data Tables* **39** (1988) 185.

[20] N. Nikolov, N. Schunk, W. Nazarewich, M. Bender, and J. Pei, *Phys. Rev. C* **83** (2011) 034305.