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Multi-Step Neutron Emission Probabilities in Heaviest Nuclei

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Abstract. The probability of realization of xn channels and kinetic energy distribution of evaporated neutrons are calculated for superheavy nuclei with Monte Carlo method. The calculations are performed using the level densities obtained with a microscopic approach based on the superfluid formalism. This allows us to take into account pairing and shell effects in the calculation of energy dependent widths for fission and particle emissions. The kinetic energy distribution of neutrons in multi-step decay process is analyzed and applied to the calculation of survival probabilities.

1 Introduction

One of the major interests in nuclear physics is to investigate the properties of nuclei and nuclear reactions in superheavy mass region. The evaporation residue cross section depends on capture cross section, fusion probability and survival probability. Survival probability evaluates the deexcitation process via fission or particle emission.

Statistical models provide a straightforward approach to describe the fission process. However, survival probabilities calculated with statistical models are highly sensitive to the supplementary data *i.e.*fission barriers, neutron-separation energies and nuclear level densities (NLD). In this work, a microscopic approach based on the superfluid model [1, 2] is applied to calculate the ground-state NLDs of neutron, proton and α-particle emission residues as well as the NLDs of the fissioning nuclei at the saddle point. This method consistently accounts for pairing and shell effects in both evaporation and fission decay modes. We use the single-particle energies obtained with a microscopicmacroscopic model based on the deformed single-particle WoodsSaxon potential and Yukawa-plus exponential macroscopic energy [3–5]. The decay thresholds, *i.e.*fission barriers and separation energies are taken from [5]. In our previous publication, superfluid model together with the microscopic-macroscopic approach were shown to give a good agreement with the experimental fission probability for ^{236}U and ^{240}Pu [6].

The probability of realization of xn channel $P(xn, U)$, *i.e.*the probability that nucleus with initial energy U will emit exactly x neutrons, can be estimated as the difference between the probability that the residual nucleus will have an excitation energy greater than the binding energy of the last neutron from the probability that the original compound nucleus will emit at least x neutrons. Based on Maxwellian distribution of neutron's kinetic energy and assumption of a constant temperature T during whole neutron emission process these probabilities can be analytically given by the incomplete gamma function [7]. For large number of emitted neutrons, the decrease of nuclear temperature in evaporation process becomes important, and can be incorporated by an effective temperature $T_{\text{eff}} < T_{CN}$ where T_{CN} is the temperature of the initial compound nucleus [8,9].

In this work, we obtain $P(xn, U)$ without above mentioned assumptions in the straightforward Monte-Carlo (MC) method using the NLDs obtained with microscopic model. We then discuss the analytical expression of $P(xn, U)$ and the relation between T_{eff} and the average kinetic energy per emitted neutron obtained from MC calculation.

The survival probability under xn channel is factorized as the product of $P(xn, U)$ and a multiplicative factor describing the competition between neutron emission and other channels at each step j:

$$W(xn, U) = P(xn, U) \prod_{j=1}^{x} \frac{\Gamma_{nj}(U_j)}{\Gamma_{\text{tot}}(U_j)}. \tag{1}$$

Here, Γ_{nj} is the neutron decay width and the total width Γ_{tot} is the sum of the widths of fission and evaporation channels including neutron and charged particle decays. The excitation energy of the compound nucleus at j^{th} step

$$U_j = U - \sum_{i=1}^{j-1} (B_j + K_i) \tag{2}$$

depends on the neutron separation energies B_j and the average kinetic energies K_i taken away by the emitted neutrons throughout the evaporation chain. Therefore, the distribution of neutron kinetic energy is needed for the calculation of width ratios at U_j. The competition between various decay channels and its effective factors were analyzed in Ref. [6] for the superheavy nuclei (SHN) with $Z = 112 - 120$. In the present article, we focus on $P(xn, U)$ and the kinetic energy distribution. The details of NLD calculation with superfluid model are presented in Refs. [6, 10, 11].

2 Probability of Realization of xn Channels

In a multi-step routine, the MC sampling requires the probability distribution for neutron kinetic energy to check at each step whether emission of the next neutron will be energetically possible or not. We define the probability density to emit a particle with kinetic energy in a small interval around ε_i as

$$P_{\varepsilon_i}(U_i) = N \varepsilon_i \rho_{re}(U_i - B_i - \varepsilon_i), \tag{3}$$

where, $\rho_{re}(U_i - B_i - \varepsilon_i)$ is the level density of the residue and N is the normalization constant obtained as

$$N^{-1} = \int_0^{U_i - B_i} \varepsilon_i \rho_{re}(U_i - B_i - \varepsilon_i) d\varepsilon_i. \qquad (4)$$

We repeat the process of MC sampling for successive decays until the nucleus runs out of excitation energy needed for the next step. In the calculation process we count the kinetic energies of neutrons leading to xn decays and average them over the number of samples executing the considered channels.

As an example, the calculated $P(xn, U)$ for $^{299}119$ nucleus with $x = 1 - 5$ and the average kinetic energies corresponding to the emitted neutrons in $x =$

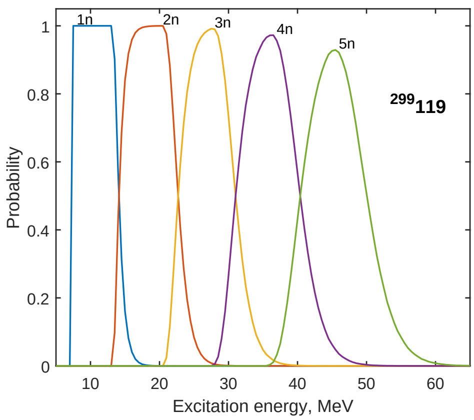

Figure 1. Probability of realization of $x = 1 - 5$ neutron decay channels calculated for $^{299}119$ with MC method using the microscopic level densities.

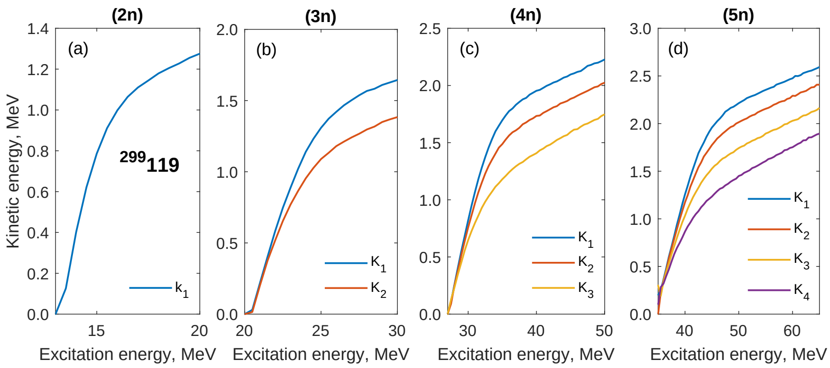

Figure 2. The average kinetic energies K_i taken away by the preceding neutrons in (xn) evaporation chains with $x = 2 - 5$ calculated with MC method for $^{299}119$.

$2-5$ decay chains are presented in Figures 1, and 2, respectively. As seen from the figures, the kinetic energy curves are growing steeply with the excitation energy up to the energies at which their corresponding $P(xn, U)$ curves reach maxima. Then after, they tend to have a form well described by the Maxwellian distribution of neutron kinetic energy ε

$$P_\varepsilon(T) = T^{-2}\varepsilon \exp\left[-\frac{\varepsilon}{T}\right], \tag{5}$$

which is in general appropriate for high excitation energies. From Eq. 5, the average kinetic energy is $K = 2T$. In the Fermi Gas model, $T = \sqrt{U/a}$ where a is the level density parameter. Energy and shell-correction dependencies of level density parameters and their relative values in various channels for SHN is discussed in details in Refs. [6, 12].

3 Results and Discussion

An alternative way to estimate K_i in evaporation chain is to calculate it with the probability distribution given in Eq. (3) as follows. For the i^{th} neutron in xn

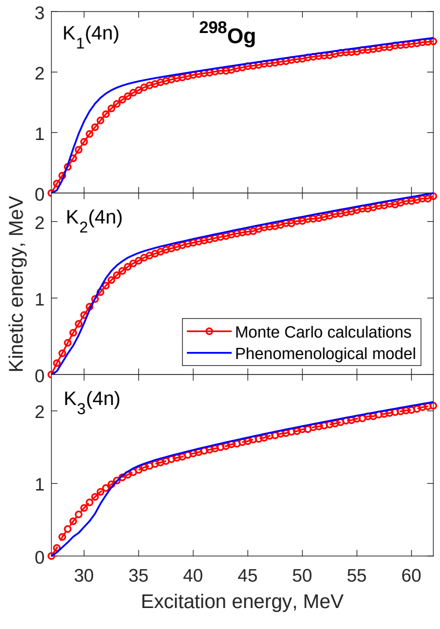

Figure 3. Average kinetic energies K_i taken away by $i = 1-3$ intermediate neutrons evaporated from ^{298}Og in 4n channel, obtained with MC method (red lines with circles) compared with those obtained with Eq. (6) (blue solid lines).

channel we have [8]

$$K_i(xn) = \int_0^{U-B_{xn}-K_1-\dots K_{i-1}} \varepsilon P_\varepsilon(U_i) d\varepsilon, \tag{6}$$

where, $B_{xn} = \sum_{i=1}^{x} B_i$. Energy dependence of $K_{1,2,3}$ in 4n channel obtained using Eq. 6 for ^{298}Og is displayed in Figure 3 together with the MC calculations. A good agreement between two approaches is seen from the figure.

It is clear that opening of new neutron emission channels and the absolute values of survival probabilities depend on the kinetic energies carried away by preceding neutrons. To study the effect of kinetic energy distribution on survival probabilities we present the ratios of the survival probabilities maxima, calculated using the neutron kinetic energies obtained with the MC method to those obtained with the assumption that the average kinetic energy is $K_i = 2T_i$. In our calculations the nuclear temperature values are taken from the microscopic model and the widths of various channels in Eq. (1) are calculated according to [7]. As shown in Figure 4 the discrepancy between two approaches starts to be seen form $3n$ channel and the ratio of $W(xn)$ gets at most 0.5 for $5n$ decays from ^{299}119. This means that the assumption of $K_i = 2T_i$ for the calculation of absolute values of $W(xn)$ is quite appropriate up to $5n$ channel.

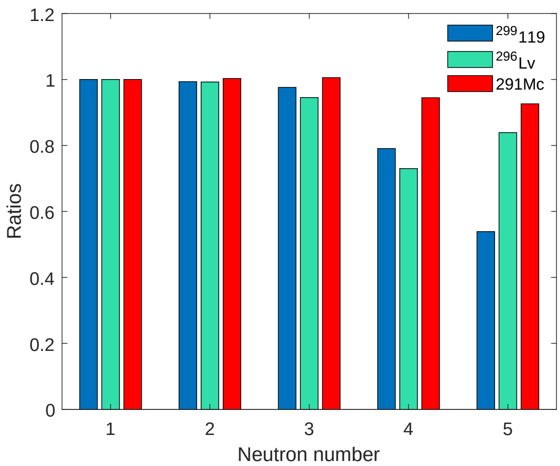

Figure 4. The ratios of the survival probabilities maxima, calculated using the neutron kinetic energies obtained with the MC method to those obtained with the assumption of $K_i = 2T_i$, where T_i is temperature of intermediate nuclei in evaporation chain.

Based on Maxwellian distribution of neutron energy spectrum (for more details see [8]) and assumption of a constant temperature T during whole neutron emission process $P(xn, U)$ is approximated by [7]

$$P(xn, U) = I(\Delta_x, 2x-3) - I(\Delta_{x+1}, 2x-1), \tag{7}$$

where

$$I(\Delta_x, 2x-3) = 1 - \exp[-\Delta_x]\left(1 + \sum_{i=1}^{2x-3} \frac{(\Delta_x)^i}{i!}\right) \tag{8}$$

is Pearson's incomplete gamma function and $\Delta_x = (U - B_{xn})/T$. For the final residual nucleus corresponding to the emission of x neutrons, with an excitation energy higher than the corresponding fission barrier but insufficient for

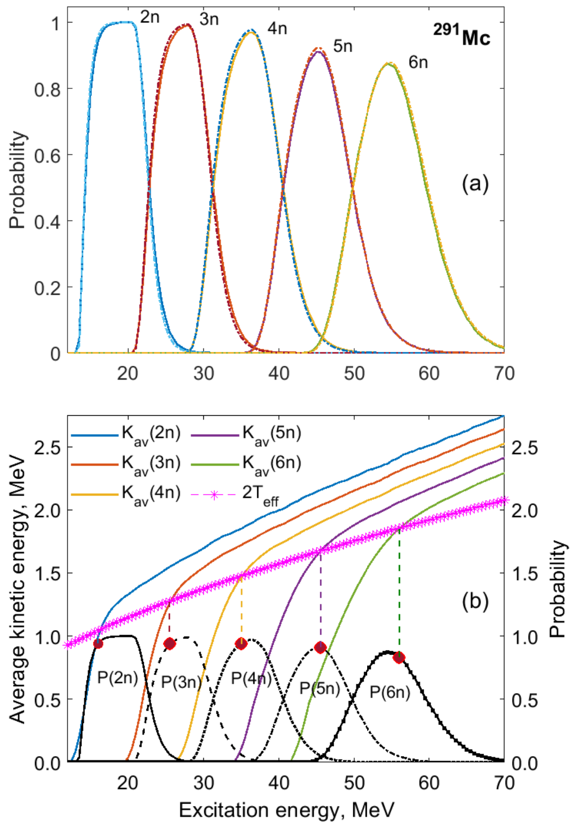

Figure 5. Panel (a): Comparison between the probability of realization of xn channel $P(xn)$ calculated with MC method (solid lines), and those obtained with Eq. (7) (dashed lines) versus the excitation energy of ^{291}Mc. Panel (b): The probabilities of $P(xn)$ (as indicated beside the curves), the average kinetic energy per emitted neutron in xn channel $K_{\rm av}(xn)$ (colored lines) as indicated in the legend, and the kinetic energy corresponding to the effective temperature $2T_{\rm eff}$ (dashed line with asterisks) versus the excitation energy of ^{291}Mc. The vertical dashed lines indicate the values of $P(xn)$ corresponding to equivalence of $K_{\rm av}(xn)$ and $2T_{\rm eff}$.

neutron evaporation, B_{x+1} in the calculation of Δ_{x+1} is replaced by fission barrier energy [7]. This approach is applicable for $x \geq 2$. Our analysis shows that using an effective temperature $T_{\text{eff}} = T_{CN}/\sqrt{2}$ for xn decays in the approximation $P(xn, U)$ by the Pearson's incomplete gamma function results in good agreement with the MC calculations. As example, in Figure 5(a) a very good agreement between the calculation of $P(xn)$ with MC method and those obtained with Eq. (7) is shown for ^{291}Mc.

It is interesting to see the relation between average kinetic energies and T_{eff}. We compare the average kinetic energy per neutron in xn decay channel calculated as

$$K_{\text{av}}(xn) = \sum_{i=1}^{x-1} \frac{K_i(xn)}{x-1}, \tag{9}$$

and $2T_{\text{eff}}$ in Figure 5 (b). As shown in the figure, the intersection points of $2T_{\text{eff}}$ and the $K_{\text{av}}(xn)$ correspond to the maxima of $P(xn)$. Therefore, one can describe the survival probability maxima by using Eq. (7) and simply taking kinetic energy as $2T_{\text{eff}}$.

4 Summary

The probabilities of realization of xn channels and the average kinetic energies carried away by emitted neutrons were obtained with MC method. The results are compared with analytical calculations based on assumption of Maxwellian distribution for neutron energy. Our study shows that at low excitation energies the probability for emission of neutrons with kinetic energies less than $2T$ becomes important for correct treatment of opening of the channels and survival probability maxima. The average kinetic energy can be estimated with Eq. 6 using the probability distribution defined with the NLDs obtained using the microscopic model. The relation between effective temperature in analytical expression of $P(xn, U)$ and the average kinetic energy per emitted neutron obtained with MC method is discussed. This study can be useful for the prediction of excitation functions of formation of superheavy nuclei in the same way as done in [13].

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