

Construction of $SU(3)$ -Scheme and Pairing States and Results for Their Transformation Brackets

K.P. Drumev

Institute for Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia 1784, Bulgaria

Abstract. We calculate and present results for the construction of the pairing states and the rotational states of Elliott type in terms of single-particle Slater determinants as well as for their transformation brackets. The remaining cases for the model space of the sd shell are resolved so establishing a complete correspondence between the two sets of states allows us to develop a mixed-mode pairing-plus-quadrupole approach for nuclei where the two bases enter on an equal footing.

1 Introduction

It is well known that the pairing and the quadrupole-quadrupole interactions are the most important ingredients of the short- and the long-range part of the residual interaction in a nuclear system. This is a good reason to want to know the exact relation between their many-particle eigenstates. This knowledge could be an excellent starting point to build a model where the two modes participate on an equal footing.

One method to find this relation is by working in one of the bases while expressing the other interaction in terms of tensors typical for the first interaction and then to diagonalize the obtained energy matrix. The disadvantage of such an approach is that some of the obtained eigenstates (especially the ones which are obtained degenerate in energy) may turn out to be mixed. To avoid this it might be more advantageous to work with a method which does not lead to such mixture and generates each type of the states in terms of linear combinations in some common basis.

In this contribution, we review an old method [1] which generates the pairing and the rotational states in terms of Slater determinants built from single-particle states which are exactly related. With the goal of achieving some complete correspondence between the bases, we add some newly obtained results for the constructed states in the $SU(3)$ irreps for the $U(6)$ as well as for the transformation brackets not being the focus of an earlier paper [2].

2 Pairing-plus-Quadrupole Model

First introduced by Bohr and Mottelson [3] and Belyaev [4], the Pairing-plus-Quadrupole model has been widely used to reproduce both few-particle non-collective and many-particle collective features of nuclei [5, 6]. The most general form of a pairing-plus-quadrupole Hamiltonian that one could use includes isoscalar and isovector pairing parts where each of them has a strength of G_0 and G_1 , respectively. It is the following:

$$H = G_0 \sum_{\mu} D_{\mu}^{\dagger} D_{\mu} + G_1 \sum_{\mu} P_{\mu}^{\dagger} P_{\mu} - \frac{\chi}{2} Q \cdot Q \quad (1)$$

where

$$D^{\dagger} = \sum_l D^{\dagger}(l) = \sum_l \sqrt{\frac{2l+1}{2}} (a_{l\frac{1}{2}\frac{1}{2}} \times a_{l\frac{1}{2}\frac{1}{2}}^{\dagger})_{0,\mu,0}^{0,1,0}, \quad (2)$$

$$P^{\dagger} = \sum_l P^{\dagger}(l) = \sum_l \sqrt{\frac{2l+1}{2}} (a_{l\frac{1}{2}\frac{1}{2}} \times a_{l\frac{1}{2}\frac{1}{2}}^{\dagger})_{0,0,\mu}^{0,0,1}, \quad (3)$$

with $D(l) = (D^{\dagger}(l))^{\dagger}$ and $P(l) = (P^{\dagger}(l))^{\dagger}$. In the specific case of the supermultiplet Wigner model [7] the two pairing strengths are equal, i.e. $G_0 = G_1$.

The two chains of groups which can be used for the classification of the states of a many-particle nuclear system and which diagonalize the full pairing (with $G_0 = G_1$) or the quadrupole part of the Hamiltonian are

$$U(4\Omega) \supset U_{\{f\}}^{(\mu)}(\Omega) \otimes U_{\{\tilde{f}\}}^{(\sigma\tau)}(4) \supset [\mathbf{G} \supset SO_L(3)] \otimes [SU_S(2) \otimes SU_T(2)] \quad (4)$$

where $\mathbf{G} = SU_{(\lambda,\mu)}(3)$ or $SO_L(\Omega)$ are the relevant groups for the case of rotational and pairing states, respectively. The subscripts in these chains are the labels which describe the corresponding irreducible representations (irreps).

So, the states of N nucleons classified according to the above group-subgroup chains are denoted by

$$|\{f\}\alpha(\lambda_1\lambda_2\lambda_3)\kappa LM_L, \{\tilde{f}\}\beta SM_S TM_T\rangle \quad (5)$$

and

$$|\{f\}\alpha(\lambda, \mu)\kappa LM_L, \{\tilde{f}\}\beta SM_S TM_T\rangle \quad (6)$$

where α , κ and β are the multiplicity indices which account for the degeneracy in the corresponding reduction. In the rest of the text we will call these simply the pairing states of the Wigner supermultiplet scheme and the rotational states (or the Elliott $SU(3)$ scheme).

3 Method of Calculation

A method for the construction of the rotational and pairing eigenstates for the case with equal strengths $G_0 = G_1$ using Slater determinants built from the same set of single particle states of the harmonic oscillator has been developed by Moshinsky in the early sixties [1]. As a result, the relation between some states from these two sets of bases was found by calculating the dot product of the two sets resulting in finding the transformation brackets between these states.

Our goal is to achieve a completeness in the correspondence between the two bases which means to be able to express any state from one of the sets as a linear combination of the states from the other set. For the purpose, we need to complete the calculations for all the states from the two bases and to end up having all the values for the transformation brackets.

First, let us start with the explanation for the construction of the $SU(3)$ -scheme states (6). For a harmonic oscillator type of potential it is convenient to introduce the following creation and annihilation operators

$$a_q^\dagger = 1/\sqrt{2}(x_q - ip_q) \quad a^q = 1/\sqrt{2}(x_q + ip_q) \quad q = -1, 0, +1 \quad (7)$$

where $x_q, x^{q'}$ ($p_q, p^{q'}$) are respectively the covariant and contravariant spherical components of the position (momentum) vector.

As our system of nucleons obeys the Pauli principle we can introduce, as usual, the second-quantization picture by the correspondence $\psi \rightarrow b_{\mu s}^\dagger |0\rangle$, where $b_{\mu s}^\dagger$ is a fermion creation-operator and $b^{\mu s}$ is the corresponding annihilation operator, obeying the usual anticommutation rules; $|0\rangle$ is the vacuum state with the property $b^{\mu s} |0\rangle = 0$ for all μ, s . An N -particle state will then be $P |0\rangle \equiv b_{\mu_1 s_1}^\dagger b_{\mu_2 s_2}^\dagger \dots b_{\mu_N s_N}^\dagger |0\rangle$ where the letter P in this notation stands for a polynomial in the single-particle states.

For the case of the rotational states (6) we use the following single-particle states $\mu \equiv n_1 n_0 n_{-1}$, $s \equiv \sigma\tau$:

$$\begin{aligned} n_1 n_0 n_{-1} &\equiv 200, 110, 101, 020, 011, 002 \\ \sigma\tau &\equiv \frac{1}{2} \frac{1}{2}, \frac{1}{2} - \frac{1}{2}, -\frac{1}{2} \frac{1}{2}, -\frac{1}{2} - \frac{1}{2} \end{aligned} \quad (8)$$

where the sequences of non-negative integers $n_1 n_0 n_{-1}$ with $n_1 + n_0 + n_{-1} = 2$ describe the 6 possible single-particle states for the sd shell which is the model space of focus in this contribution. The weight of each of the single-particle states is known and is equal to $\{w_1, w_2, w_3\} = \{n_1, n_0, n_{-1}\}$. Knowing this, one can produce a table with the weights for any constructed N -particle state. Another possible choice for the single-particle states in a shell η of the harmonic oscillator can be characterized by the quantum numbers $\eta l m$ with $l = \eta, \eta - 2, \eta - 4, \dots, 1$ or 0 and $m = l, l - 1, \dots, -l$.

The method developed by Moshinsky for obtaining the transformation brackets between the two bases (5) and (6) follows the following two steps. First,

for each of the states (5) and (6), we use the single-particle states to construct the many-particle states and use the expansion of the $U(3)$ states of highest weight over the $U(\Omega)$ states. The HWS for a $U(3)$ irrep $(k_1 k_2)$ is labelled as $|\{f\}(k_1 k_2)L = M_L = k_1\rangle$. The highest L value is always unique and has the value $L = M_L = k_1$. As a second step, we use some lowering operators to find the rest of the states in that irrep.

The chains of groups that are used to classify the states of a nucleus start with the group $U(4\Omega)$ and since the state of the nucleus should be fully antisymmetric the only relevant representation for the case of N particles is $\{1^N\}$.

The $U(4\Omega)$ generators are

$$b_{\mu s}^\dagger b^{\mu' s'} \quad \mu, \mu' = 1, \dots, \Omega, \quad s, s' = 1, \dots, 4 \quad (9)$$

as their number is $(4\Omega)^2$ where Ω is the spatial degeneracy of the oscillator shell (which for the sd shell is $\Omega = 6$).

Related to the generators of $U(4\Omega)$ are the $U(\Omega)$ generators which raise ($\mu < \mu'$) the weight, give the weight ($\mu = \mu'$) and lower the weight ($\mu > \mu'$) $C_\mu^{\mu'} = \sum_s b_{\mu s}^\dagger b^{\mu' s}$ as well as their counterparts, the $U(4)$ generators on the spin-isospin side of the chains (4), which are $C_s^{s'} = \sum_\mu b_{\mu s}^\dagger b^{\mu s'}$.

Now, we can write the highest weight state (HWS) for an irrep of the $U(6) \times U(4)$ group. In a specific example, say for seven particles, the state can be written as

$$P|0\rangle \equiv b_{11}^\dagger b_{12}^\dagger b_{13}^\dagger b_{14}^\dagger b_{21}^\dagger b_{22}^\dagger b_{31}^\dagger |0\rangle = \frac{1}{4!2!1!} \Delta_{1111}^{1234} \Delta_{22}^{12} \Delta_3^1 |0\rangle \quad (10)$$

and it is the HWS of both the irrep $\{421\}$ of $U(6)$ and the conjugate irrep $\{3211\}$ of $U(4)$, as can be shown immediately if we apply the raising generators of $U(6)$ and $U(4)$ will give a zero as a result.

The weight of a $U(3)$ state can be determined as eigenvalues of some of the components of the $U(3)$ operator:

$$\mathcal{G}_1^1 P|0\rangle = f_1 P|0\rangle, \quad \mathcal{G}_0^0 P|0\rangle = f_2 P|0\rangle, \quad \mathcal{G}_{-1}^{-1} P|0\rangle = f_3 P|0\rangle, \quad (11)$$

where $f_1 f_2 f_3$ being the label of the irrep of $U(3)$ and $k_1 = f_1 - f_3, k_2 = f_2 - f_3$ being the corresponding label for $SU(3)$ which are related to the Elliott labels λ and μ by $k_1 = \lambda + \mu, k_2 = \mu$.

Moreover, the HWS for a $U(3)$ irrep should also satisfy the following equalities

$$\mathcal{G}_1^0 P|0\rangle = \mathcal{G}_1^{-1} P|0\rangle = \mathcal{G}_1^{-2} P|0\rangle = 0 \quad (12)$$

which means that the action of the $U(3)$ raising generators on such a state should result in zero.

It turns out that one can express other operators by using the operators $C_\mu^{\mu'}$ and $C_s^{s'}$. For example, the $U(3)$ raising, weight, and lowering operators $\mathcal{G}_q^{q'}$ can

be expressed in terms of the $U(\Omega)$ generators $\mathcal{C}_\mu^{\mu'}$:

$$\mathcal{G}_q^{q'} = \sum_{\mu, \mu'} \langle \mu | a_q^\dagger a^{q'} | \mu' \rangle \mathcal{C}_\mu^{\mu'} \quad q, q' = -1, 0, 1 \quad (13)$$

where $a_q^\dagger = 1/\sqrt{2}(x_q - ip_q)$ and $a^q = 1/\sqrt{2}(x_q + ip_q)$ with $q = -1, 0, +1$ are the creation and annihilation operators (7) for the harmonic oscillator Hamiltonian and the generators of a $U(3)$ group while the bra and the ket are harmonic oscillator wave functions (8) with $n_1 + n_0 + n_{-1} = \eta$. The expansion coefficients have been calculated in [1], Chapter 7.

Now, the key part in generating the HWS for a $U(3)$ irrep is the following. If we apply systematically products of raising operators $\mathcal{C}_\mu^{\mu'}$ (i.e. with $\mu > \mu'$) on the HWS in $U(6) \times U(4)$ (while maintaining the state of highest weight in $U(4)$) we get a linear combination of the same type of product of Slater determinants, so for our example in (10) the action will give

$$\mathcal{C}_{\mu''}^{\mu'''} \cdots \mathcal{C}_\mu^{\mu'} = \sum_{\{\mu\}} A_{\{\mu\}} \Delta_{\mu_1 \mu_2 \mu_3 \mu_4}^{1234} \Delta_{\mu'_1 \mu'_2}^{12} \Delta_{\mu'_1}^1 |0\rangle \quad (14)$$

as these are states which are not characterized by definite representations of the groups $U(3)$ and $SO(3)$. In order to become such a state we need to impose the conditions in (12) which determine the values for the expansion coefficients A_μ .

Similar expansions as in (13) over the operators $\mathcal{C}_\mu^{\mu'}$ can be found for the following three operators:

$$\mathcal{L}_{m=-1} = -\sqrt{2} \sum_{q, q'} (-)^{q'} \langle 1q; 1 - q' | 1m = -1 \rangle \mathcal{G}_q^{q'}, \quad (15)$$

$$\mathcal{Q}_\tau = - \sum_{q, q'} (-)^{q'} \langle 1q; 1 - q' | 2\tau \rangle \mathcal{G}_q^{q'}, \quad (16)$$

$$\mathcal{M}_{LL'}^{(2)} = \sum_{\tau=2}^{-2} (-)^\tau \sqrt{\frac{(2+\tau)!}{2^{2-\tau}(2-\tau)!}} \frac{(L-L'+2)!(L+L'+3)!}{(L-L'+\tau)!(L+L'+1+\tau)!} (\mathcal{L}_{-1})^{\tau+L-L'} \mathcal{Q}_\tau. \quad (17)$$

These are the lowering operator \mathcal{L}_{-1} in the orbital part of the group chain (4), the operator of the quadrupole momentum \mathcal{Q}_τ and the lowering operator $\mathcal{M}_{LL'}^{(2)}$ which from a state with $L, M_L = L$ produces a state of the same $U(3)$ irrep but with $L', M_{L'} = L'$. A further action of the operator \mathcal{L}_{-1} will then produce all the states with the lower values of the projection $M_{L'}$, e.g. $\mathcal{L}_{-1} |\{f\}(\lambda, \mu)L', M_{L'} = L'\rangle = |\{f\}(\lambda, \mu)L', M_{L'} = L' - 1\rangle$ and so on.

By construction, the operator $\mathcal{M}_{LL'}^{(2)}$ has the following property when applied to the rotational states:

$$\mathcal{M}_{LL'}^{(2)} |\{f\}\alpha(k_1 k_2)\kappa LL\rangle = \sum_{\kappa'} B_{\kappa'} |\{f\}\alpha(k_1 k_2)\kappa' L' L'\rangle \quad (18)$$

Construction of $SU(3)$ -scheme and Pairing States and Results for ...

so, the action produces a linear combination of states with a different pair of values $L' = M_{L'}$ over the possible values of the inner multiplicity index κ' .

In a similar fashion, on the $U(4)$ spin-isospin part of the chain (4), two operators are defined with a function similar to the operator $\mathcal{M}_{LL'}^{(2)}$ which from the $U(4)$ HWS which is the one with values for the spin and isospin of $S_{H.W.} = (\tilde{f}_1 + \tilde{f}_2 - \tilde{f}_3 - \tilde{f}_4)/2$, $T_{H.W.} = (\tilde{f}_1 - \tilde{f}_2 + \tilde{f}_3 - \tilde{f}_4)/2$ produces states with different spin but still of equal value for its projection, i.e. $S', M_{S'} = S'$ (and $T', M_{T'} = T'$). Then one can use the lowering operators

$$S_{-1} = (1/\sqrt{2})(C_3^1 + C_4^2) \quad (19)$$

$$T_{-1} = (1/\sqrt{2})(C_2^1 + C_4^3) \quad (20)$$

in order to obtain states with all other projections $M_{S'}$ and $M_{T'}$ of the spin S' and isospin T' , respectively.

Finally, to construct the pairing states (5), we choose the set of single-particle states to be $\mathbf{m} \equiv \eta l m$ which denotes the following six single-particle states:

$$\mathbf{m} \equiv \{2, 1, 0, -1, -2, 0'\} = \{222, 221, 220, 22 - 1, 22 - 2, 200\} \quad (21)$$

where the values for the transformation bracket between the single-particle states $|\eta l m\rangle$ and $|n_1 n_0 n_{-1}\rangle$ is well known [8]. The procedure of identifying the $SO(6)$ and $SO(5)$ generators is described in details in [9], Section 5. Here, we only mention that we use the following operator similar to $\mathcal{M}_{LL'}^{(2)}$:

$$\mathcal{M}_{LL'}^{(3)} = \sum_{\tau=3}^{-3} \sqrt{\frac{(3+\tau)!}{2^{3-\tau}(3-\tau)!}} \frac{(L-L'+3)!(L+L'+4)!}{(L-L'+\tau)!(L+L'+1+\tau)!} (\mathcal{L}_{-1})^{\tau+L-L'} \mathcal{Q}_\tau. \quad (22)$$

In order to construct this operator we recombine the generators of $SO(5)$ in the following way:

$$\mathcal{L}_q = \sqrt{\frac{5}{2}} \sum_{mm'} (-)^{m'} \langle 2m; 2-m' | 1q \rangle \dot{\Lambda}_m^{m'}, \quad (23)$$

$$\mathcal{T}_\tau = \sqrt{\frac{5}{2}} \sum_{mm'} (-)^{m'} \langle 2m; 2-m' | 3\tau \rangle \dot{\Lambda}_m^{m'} \quad (24)$$

where $\mathcal{L}_1, \mathcal{L}_0, \mathcal{L}_{-1}$ are the generators of the three-dimensional rotational group, while $\mathcal{T}_3, \dots, \mathcal{T}_{-3}$ are the components of a Racah tensor of rank three.

Since both types of states (5) and (6) can be written as expressions in terms of the same type of single-particle states we can take their dot product which gives us the transformation brackets between the two schemes.

4 Results

Moshinsky and his collaborators have published their results [2, 9] for a proton-neutron system of 2 and 4 particles in the sd shell. More specifically, the construction of all rotational states in the former case and from the $U(6)$ irreps $\{4\}$, $\{22\}$ (both for $S = 0$) and $\{31\}$ (only the result for $(\lambda, \mu), S = (71), 1$) expressed in terms of the Slater determinants for the latter. They also published the transformation brackets for most of the cases (all for the 2-particle system and those between the states with $U(6)$ irreps of $\{4\}$ and $\{22\}$ for the 4-particle system).

The cases we present here are for a proton-neutron system of 4 particles in the sd shell where we focus on the remaining three $U(6)$ irreps $\{31\}$, $\{211\}$, and $\{1^4\}$ as the complete list of states in chains (4) is displayed in Table 1 for the rotational states while for the pairing states only the $SO(6)$ irreps are shown. By adding these we achieving the goal of a one-to-one correspondence between the two bases with a future extension for other bases as well as other reduction chains.

Table 1. The possible $SU(3)$ - and $O(6)$ -scheme states which can be formed for the $U(6)$ irreps $\{31\}$, $\{211\}$ and $\{1^4\}$ in the sd shell. Superscripts in the label L denote the multiplicity of the states.

$U(6)$ $\{f\}$	Rotational states $SU(3) \supset SO(3)$ $(\lambda, \mu)L$	Pairing states $O(6)$ $[\lambda_1 \lambda_2 \lambda_3]$	$U(4) \supset SU(2) \times SU(2)$ $\{\tilde{f}\}(S, T)$
$\{31\}$	(61)1234567 (42)02 ² 34 ² 56 (23)123 ² 45 (31)1234 (12)123 ² 45 (20)02	[310] [200] [110]	$\{211\}(1, 0)$ (0,1) (1,1)
$\{211\}$	(50)135 (23)123 ² 45 (31)1234 (12)123 (01)1	[211] [21-1] [110]	$\{31\}(1, 0)$ (0,1) (2,1) (1,2) (1,1)
$\{1^4\}$	(12)123	(110)	$\{4\}(0, 0)$ (1,1) (2,2)

The highest-weight states for all the $U(3)$ irreps which belong to the three $U(6)$ irreps from Table 1 were built by selecting all possible and linearly independent products of Slater determinants composed from our knowledge of the $U(3)$ weights of the single-particle states (8).

Construction of $SU(3)$ -scheme and Pairing States and Results for ...

$$\begin{aligned}
 P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} & P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} \\
 P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} & P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} \\
 P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} & P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} \\
 P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} & P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} \\
 P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} & P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} \\
 P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12} & P|0\rangle &= \Delta_{12}^{12}\Delta_{15}^{12} - \sqrt{2}\Delta_{13}^{12}\Delta_{14}^{12} - \frac{\sqrt{2}}{2}\Delta_{12}^{12}\Delta_{23}^{12}
 \end{aligned}$$

Figure 1. Highest weight states for $U(3)$ irreps expressed in terms of (products of) Slater determinants $\Delta_{\{\mu_1\}}^{\{s_1\}} \dots \Delta_{\{\mu_j\}}^{\{s_j\}}$ built from single-particle states $|n_1 n_0 n_{-1}\rangle$ which are eigenstates of the harmonic oscillator.

The results expressed by using only the independent Slater determinants built from the harmonic oscillator single-particle eigenstates are given in Figure 1. Using the lowering operators, one can find expressions for the rest of the states in Table 1.

$\{f\}(\lambda, \mu)LS$	$\{f\}$ $\{\lambda_1 \lambda_2 \lambda_3\}$ LS	$\{f\}(\lambda, \mu)LS$	$\{f\}$ $\{\lambda_1 \lambda_2 \lambda_3\}$ LS
		$L = 1$	
		[310] [310]'	[310] [310]' [200] [110]
$L = 0$			
{31}(42)00	$\sqrt{\frac{2}{7}}$ $\sqrt{\frac{5}{7}}$	{31}(61)10	$\frac{2}{9}\sqrt{\frac{2}{7}}$ $-\frac{2}{3}\sqrt{\frac{2}{33}}$ $-\frac{1}{3}\sqrt{\frac{14}{3}}$ $-\frac{7}{45}\sqrt{\frac{14}{3}}$
{31}(20)00	$\sqrt{\frac{5}{7}}$ $-\sqrt{\frac{2}{7}}$	{31}(23)10	$\frac{1}{9}\sqrt{\frac{14}{3}}$ $\frac{2}{3}\sqrt{\frac{1}{3}}$ $-\frac{2}{15}\sqrt{3}$ $-\frac{1}{135}\sqrt{3}$
		{31}(31)10	0 0 $\frac{7}{\sqrt{130}}$ $-\frac{9}{\sqrt{130}}$
		{31}(12)10	$\frac{26}{9\sqrt{21}}$ $-\frac{13}{3}\sqrt{\frac{5}{273}}$ $2\sqrt{\frac{11}{273}}$ $\frac{14}{9}\sqrt{\frac{11}{273}}$
$\{f\}(\lambda, \mu)LS$	[310] [310]'		
$L = 1$		$\{f\}(\lambda, \mu)LS$	
(50)10		$L = 2$	
(23)10		(23)20	
(31)10		(31)20	
(12)10		(12)20	
(01)10			

Figure 2. Transformation brackets between wave functions in the $SU(3)$ rotational scheme and the $SO(6)$ pairing scheme for 4 particles in the sd shell.

Similar procedure has been performed for the pairing eigenstates (5). Finally, we compare the expressions obtained for the two bases as their dot product gives the transformation brackets between the two schemes. Some of the obtained results are presented in Figure 2.

5 Discussion

The approach described in this contribution has the advantage to be quite simple and universal. After we found the complete relation between all the states from the two sets of bases for a nucleus of choice, our next step is to apply it. The method may also be used to find some useful relations between different sets of bases, especially when a simple (or even a more complicated) analytical result is hard to achieve.

The obtained transformation brackets between the two bases allow us to avoid the knowledge of the expansion coefficients for the pairing operator in terms of $SU(3)$ tensors as well as the calculation and use of the $SU(3)$ Wigner coefficients - both of which are needed to obtain the matrix elements for the pairing operator and finally - to diagonalize the energy matrix, as is usually the procedure when working in a single-shell-model basis of Elliott type. Instead, this found relation gives us the opportunity to introduce and estimate the quality of a mixed-mode approach where the basis is composed of two sets - the Elliott rotational states and the pairing states [10].

Acknowledgements

Support by the Bulgarian National Science Fund under Contract No. KP-06-N48/1 is gratefully acknowledged.

References

- [1] M. Moshinsky, *Group Theory and The Many-Body Problem*, Gordon and Breach, New York (1966).
- [2] M. de Llano, P.A. Mello, E. Chacon, J. Flores, *Nucl. Phys.* **72** (1965) 379.
- [3] A. Bohr, B.R. Mottelson, D. Pines, *Phys. Rev.* **110** (1958) 936.
- [4] S.T. Belyaev, *Mat. Fys. Medd. Dan. Vid. Selsk.* **31** nr. 11 (1959) .
- [5] L.S. Kisslinger, R.A. Sorensen, *Mat. Fys. Medd. Dan. Vid. Selsk.* **32** nr. 9 (1960).
- [6] K. Kumar, M. Baranger, *Nucl. Phys.* **62** (1965) 113.
- [7] E.P. Wigner, *Phys. Rev.* **51** (1937) 106.
- [8] E. Chacon, M. de Llano, *Rev. Mex. Fis.* **12** (1963) 57.
- [9] J. Flores, E. Chacon, P.A. Mello, M. de Llano, *Nucl. Phys.* **72** (1965) 352.
- [10] K.P. Drumev, to be submitted.