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Microscopic Study of Nuclear Monopole Excitations

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Abstract. A systematic study of the isoscalar giant monopole resonance (ISGMR) in spherical and deformed nuclei from various isotopic chains is performed within the microscopic self-consistent Skyrme HF+BCS method and coherent density fluctuation model. The calculations for the incompressibility in finite nuclei are based on the Brueckner energy-density functional for nuclear matter. The good agreement achieved between the calculated centroid energies of the ISGMR and their recent experimental values for various nuclei demonstrates the relevance of the proposed theoretical approach. The latter can be applied to analyses of neutron stars properties, such as incompressibility, symmetry energy, slope parameter, and other astrophysical quantities.

1 Introduction

The detailed information from measurements and theoretical studies of the isoscalar giant monopole resonance (ISGMR) plays an important role in constraining the nuclear equation of state (EOS) [1–6]. An important result is that the energy of this resonance is closely related to the nuclear incompressibility. The latter can be connected to the incompressibility of the infinite nuclear matter, which represents an important ingredient of the nuclear matter EOS. To make the EOS isospin asymmetry term more precise, recent experimental measurements of isoscalar monopole modes are being extended in isotopic chains from the nuclei on the valley of stability towards exotic nuclei with larger proton–neutron asymmetry. For instance, different measurements have been conducted on Ni isotopes far from stability, namely ^{56}Ni [7, 8] and ^{68}Ni [9, 10]. In particular, the ^{68}Ni experiment is the first measurement of the isoscalar monopole response in a short-lived neutron-rich nucleus using inelastic alpha-particle scattering. The peak of the ISGMR was found to be fragmented, indicating a possibility for a soft monopole resonance.

In the present work (as well as in Ref. [11]), the incompressibility and the centroid energy of ISGMR are investigated for Ni, Sn, and Pb isotopic chains on the basis of the Brueckner energy-density functional (EDF) for nuclear matter [12, 13] and using the coherent density fluctuation model (CDFM) (e.g., Refs. [14, 15]). Our main purpose is to validate the CDFM for studies of collective vibrational modes by using as a main theoretical ground the self-consistent

Hartree–Fock (HF)+BCS method with Skyrme interactions. The mentioned above model gives a link between nuclear matter and finite nuclei in studying of their properties, such as binding energies and rms radii of light, medium, and heavy nuclei. We present and discuss the values of the centroid energies in Sn isotopic chain (A=112-124) studying its isotopic sensitivity. The main reason to select these chains of nuclei is partly supported by their recent intensive ISGMR measurements so that we focus too on the comparison with the available experimental data for Ni [16], Sn [17], and Pb [18, 19] isotopes. In addition, new results for the excitation energies of ISGMR for Ca, Fe, Zn, and Zr nuclei are reported, as well as for deformed Mo and Cd isotopes inspired by the new experimental data for them and the fully self-consistent quasiparticle random-phase-approximation (QRPA) calculations (e.g., in [20]).

2 Theoretical Formalism

The centroid energy of ISGMR E_{ISGMR} is generally related to a finite nucleus incompressibility $\Delta K(N,Z)$ for a nucleus with Z protons and N neutrons ($A = Z + N$ is the mass number). Among the various definitions of E_{ISGMR} we will mention the one from, e.g., Ref. [21]:

$$E_{\text{ISGMR}} = \frac{\hbar}{r_0 A^{1/3}} \sqrt{\frac{\Delta K(N,Z)}{m}}\,, \tag{1}$$

where r_0 is deduced from the equilibrium density and m is the nucleon mass. In the present work, describing the monopole vibrations in terms of harmonic oscillations of the nuclear size and assuming an $A^{1/3}$ law for it, we calculate E_{ISGMR} by using Eq. (1). In it values of the parameter r_0 between 1.07 and 1.2 fm are adopted, which are determined from experiments on particle scattering off nuclei.

The symmetry energy $S(\rho)$ is defined by the energy per particle for nuclear matter (NM) $E(\rho,\delta)$ in terms of the isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$

$$S(\rho) = \frac{1}{2} \left.\frac{\partial^2 E(\rho,\delta)}{\partial \delta^2}\right|_{\delta=0}, \tag{2}$$

where

$$E(\rho,\delta) = E(\rho,0) + S(\rho)\delta^2 + O(\delta^4) + \cdots \tag{3}$$

and $\rho = \rho_n + \rho_p$ is the baryon density with ρ_n and ρ_p denoting the neutron and proton densities, respectively (see, e.g., [22–24]). The incompressibility (the curvature) of the symmetry energy ΔK^{NM} is given by

$$\Delta K^{NM} = 9\rho_0^2 \left.\frac{\partial^2 S}{\partial \rho^2}\right|_{\rho=\rho_0}, \tag{4}$$

where ρ_0 is the density at equilibrium.

The CDFM was suggested and developed in Refs. [14, 15] (see also our recent papers [22, 25]). Within the model the one-body density matrix (OBDM) of the nucleus $\rho(\mathbf{r},\mathbf{r}')$

$$\rho(\mathbf{r},\mathbf{r}') = \int_0^\infty dx|F(x)|^2\rho_x(\mathbf{r},\mathbf{r}') \tag{5}$$

is expressed by OBDM's of spherical "pieces" of nuclear matter ("fluctons") with radius x of all A nucleons uniformly distributed in it:

$$\rho_x(\mathbf{r},\mathbf{r}') = 3\rho_0(x)\frac{j_1(k_F(x)|\mathbf{r}-\mathbf{r}'|)}{(k_F(x)|\mathbf{r}-\mathbf{r}'|)}\Theta\left(x - \frac{|\mathbf{r}+\mathbf{r}'|}{2}\right). \tag{6}$$

In Eq. (6) j_1 is the first-order spherical Bessel function and $k_F(x)$ is the Fermi momentum. It can be seen from Eq. (5) that the density distribution in the CDFM is:

$$\rho(\mathbf{r}) = \int_0^\infty dx|F(x)|^2\rho_0(x)\Theta(x-|\mathbf{r}|) \tag{7}$$

with

$$\rho_0(x) = \frac{3A}{4\pi x^3}. \tag{8}$$

It follows from Eq. (7) that the weight function $|F(x)|^2$ of CDFM can be obtained in the case of monotonically decreasing local densities (*i.e.*, for $d\rho(r)/dr \leq 0$) by

$$|F(x)|^2 = -\frac{1}{\rho_0(x)}\left.\frac{d\rho(r)}{dr}\right|_{r=x} \tag{9}$$

being normalized as

$$\int_0^\infty dx|F(x)|^2 = 1. \tag{10}$$

In the case of the Brueckner method for nuclear matter energy [12, 13] the asymmetric incompressibility has the form [26, 27]:

$$\Delta K^{NM}(x) = -83.4\rho_0^{2/3}(x) + 4b_5\rho_0^{4/3}(x) + 10b_6\rho_0^{5/3}(x) \tag{11}$$

and contains the following values of the parameters: $b_5 = 372.84$ and $b_6 = -769.57$. According to the CDFM scheme, the curvature for finite nuclei can be expressed in the following form:

$$\Delta K = \int_0^\infty dx|F(x)|^2\Delta K^{NM}(x). \tag{12}$$

In our calculations we apply self-consistent deformed Hartree–Fock method with density-dependent Skyrme interactions [28] with pairing correlations. We use the Skyrme SLy4 [29], Sk3 [30] and SGII [31] parametrizations. In addition, we probe the SkM parameter set [32], which led to an appropriate description of bulk nuclear properties. All necessary expressions for the single-particle functions and densities in the HF+BCS method can be found, e.g., in Ref. [26].

It is known that the value of the nuclear matter incompressibility ΔK^{NM} plays a key role in determining the location of the ISGMR centroid energy [17]. The different Skyrme parameter sets used in the present calculations are chosen since they are characterized by different values of the nuclear incompressibility, ΔK^{NM} = 230, 217, 215, and 355 MeV for SLy4, SkM, SGII, and Sk3, respectively [33].

3 Results and Discussion

First, in Figure 1 we draw, as examples, the density distributions of ^{56}Ni and ^{208}Pb and the corresponding CDFM weight function $|F(x)|^2$ as a function of x. To fully specify the role of both quantities $\Delta K^{NM}[\rho_0(x)]$ and $|F(x)|^2$ in the expression (12) for the finite nuclei incompressibility ΔK and to locate the relevant region of densities in finite nucleus calculations, we apply the same physical criterion related to the weight function $|F(x)|^2$, as in [22]. The criterion is related to the width Γ of the weight function $|F(x)|^2$ at its half maximum, which is a good and acceptable choice. More specifically, we define the lower limit of integration as the lower value of the radius x, x_{min}, corresponding to the left point of the half-width Γ (for more details see the discussion in Refs. [22, 25]). One can see also in Figure 1 the part of the density distribution $\rho(r)$ (at $r \geq x_{min}$) that is involved in the calculations.

The obtained centroid positions of the monopole mode calculated using Brueckner EDF in the procedure [Eqs. (1), (11), and (12)] are compared with available experimental data in Tables 1–3. It can be seen from Table 1 that a very good agreement with the experimental data for ^{56,58,60}Ni is obtained, while the results with both Skyrme interactions underestimate the experimental energy of the soft monopole vibrations of ^{68}Ni. The excitation energy of this ISGMR in

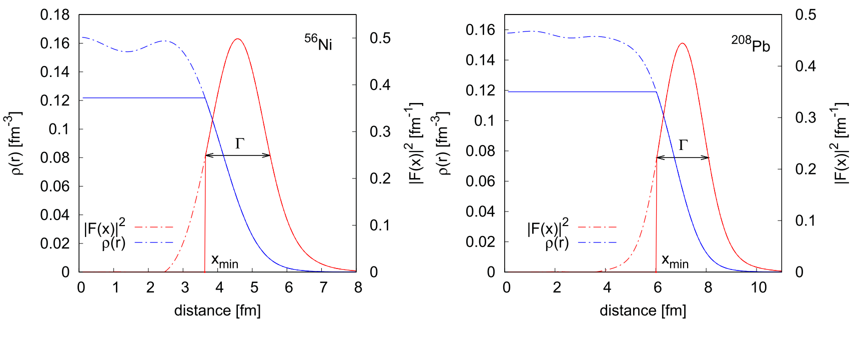

Figure 1. The densities $\rho(r)$ (in fm^{-3}) of ^{56}Ni and ^{208}Pb calculated in the Skyrme HF + BCS method with SLy4 force (normalized to $A = 56$ and $A = 208$, respectively) and the weight function $|F(x)|^2$ (in fm^{-1}) normalized to unity [Eq. (10)].

Table 1. The values of the centroid energies $E_{\rm ISGMR}$ (in MeV) of Ni isotopes obtained from HF+CDFM calculations in this work using SLy4 and SkM Skyrme forces compared with the experimental data found in the literature.

Nucleus	SLy4	SkM	Exp.
^{56}Ni	19.41	19.57	19.1 $\pm$ 0.5 [8]
			19.3 $\pm$ 0.5 [7]
^{58}Ni	18.95	19.18	18.43 $\pm$ 0.15 [16]
^{60}Ni	18.62	18.79	18.10(29) [16]
^{68}Ni	17.46	17.70	21.1 $\pm$ 1.9 [9, 10]

Table 2. The values of the centroid energies $E_{\rm ISGMR}$ (in MeV) of Sn isotopes (A=112-124) obtained from HF+CDFM calculations in this work using SLy4, SGII, and Sk3 Skyrme forces. The experimental data are taken from Table III of Ref. [17].

Nucleus	SLy4	SGII	Sk3	Exp.
^{112}Sn	15.04	15.30	14.89	16.2 $\pm$ 0.1
^{114}Sn	15.03	15.20	14.70	16.1 $\pm$ 0.1
^{116}Sn	14.94	15.08	14.56	15.8 $\pm$ 0.1
^{118}Sn	14.82	15.13	14.48	15.8 $\pm$ 0.1
^{120}Sn	14.69	15.08	14.58	15.7 $\pm$ 0.1
^{122}Sn	14.68	15.00	14.61	15.4 $\pm$ 0.1
^{124}Sn	14.68	14.96	14.51	15.3 $\pm$ 0.1

Table 3. The values of the centroid energies $E_{\rm ISGMR}$ (in MeV) of Pb isotopes obtained from HF+CDFM calculations in this work using SLy4 and SkM Skyrme forces compared with the experimental data found in the literature.

Nucleus	SLy4	SkM	Exp.	Theory
^{204}Pb	12.16	12.29	13.98 [18]	
^{206}Pb	12.12	12.23	13.94 [18]	
^{208}Pb	12.10	12.15	13.96 $\pm$ 0.2 [19]	14.453 [34]

^{68}Ni is located unexpectedly at higher energy (21.1 MeV) for the Ni isotopic chain, having at the same time large error bars. The reason is due to the large fragmentation of the isoscalar monopole strength in the unstable neutron-rich ^{68}Ni nucleus, much more than in stable nuclei [9, 10]. The obtained values of $E_{\rm ISGMR}$ for Sn isotopes (A = 112–124) exhibit small difference regarding the Skyrme parametrization (see Table 2). The theoretical results for the centroid energies for the same Sn isotopes obtained in Ref. [17] by using the SkP (between 14.87 and 15.60 MeV), SkM* (between 15.57 and 16.23 MeV), and SLy5

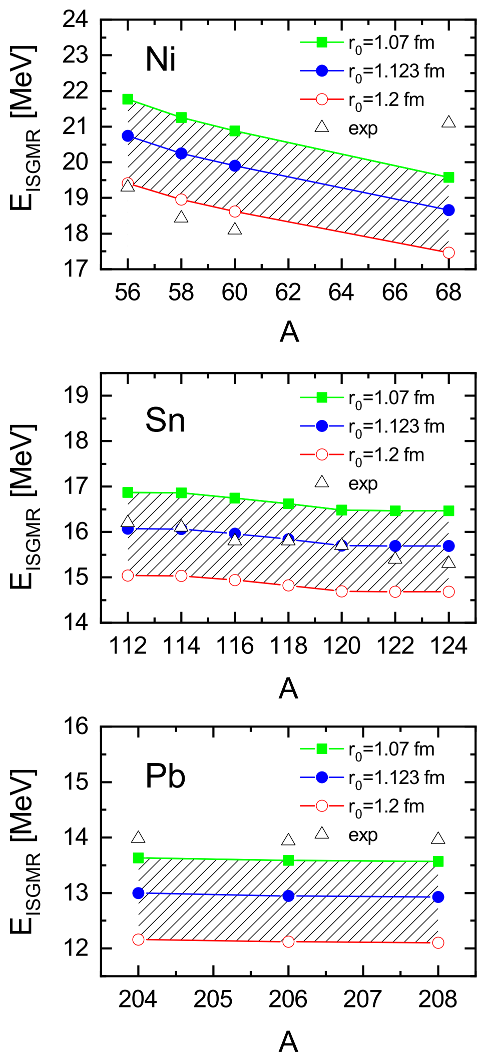

Figure 2. The centroid energies E_{ISGMR} as a function of the mass number A for Ni, Sn, and Pb isotopes in the case of SLy4 force obtained with three different values of the parameter $r_0 = 1.07, 1.123, 1.2$ fm [Eq. (1)] compared with the experimental data (see Refs. in Tables 1-3).

(between 15.95 and 16.61 MeV) parameter sets are in good agreement with our results. Almost no dependence on the Skyrme forces used in the calculations of the centroid energies is found for Ni and Pb isotopes being slightly larger in the case of SkM interaction than when using the SLy4 one.

The collective (bulk) character of the giant resonances and nuclear incompressibility presumes a quite smooth variation of the properties of the ISGMR with mass, thus not expecting very strong variations related to the internal nuclear structure. The isotopic evolution of the centroid energies E_{ISGMR} for the Ni, Sn, and Pb isotopes is presented in Figure 2. As a test of the role of the half-density radius parameter r_0 on the centroid energy [Eq. (1)], the results in the case of SLy4 force with $r_0 = 1.2$ fm (e.g., in Refs. [35, 36]), $r_0 = 1.07$ fm (for instance, in Ref. [37]), and $r_0 = 1.123$ fm [38] are presented. It is seen that with the increase of r_0 the agreement with the experimental data becomes better for lighter isotopes. Particularly, the value of $r_0 = 1.123$ fm leads to fair agreement of the ISGMR energies for Sn isotopes, while for Ni isotopes the experimental data are reproduced better with $r_0 = 1.2$ fm and for Pb isotopes with $r_0 = 1.07$ fm. Here we would like to note that the specific choice of the r_0 parameter values adopted to calculate the values of the centroid energies by using (1) is often used in the literature. The values of the measured nuclear radii are deduced from processes with strongly interacting particles or electron (muon) scattering. It is well known that the A-dependence of r_0 exhibits a smooth decrease with A being 1.07 fm for nuclei with $A > 16$ and increasing to 1.2 fm for heavy nuclei. The results for the calculated values of E_{ISGMR} and the corresponding ranges of change in respect to r_0 are illustrated in Figure 2 by hatched areas. Thus, we find a sensitivity of the results for centroid energies of ISGMR to the radial parame-

Table 4. The values of the centroid energies E_{ISGMR} (in MeV) of Ca, Fe, Zn, Zr, and Mo isotopes obtained from HF+CDFM calculations in this work using SLy4 and SkM Skyrme forces and $r_0 = 1.2$ fm compared with the experimental data found in the literature.

Nucleus	SLy4	SkM	Exp.
^{40}Ca	20.03	19.99	19.18 ± 0.37 [39]
^{42}Ca	19.83	19.98	19.7 ± 0.1 [40]
^{44}Ca	19.71	19.95	19.49 ± 0.34 [41]
^{46}Ca	19.69	19.91	
^{48}Ca	19.71	19.89	19.88 ± 0.16 [39]
^{54}Fe	19.45	19.62	19.66 ± 0.37 [42]
^{64}Zn	17.82	17.94	18.88 ± 0.79 [42]
^{68}Zn	17.24	17.42	16.60 ± 0.17 [42]
^{90}Zr	16.05	16.17	16.9 ± 0.1 [43]
^{92}Zr	15.82	15.94	16.5 ± 0.1 [43]
^{92}Mo	15.99	16.12	16.6 ± 0.1 [43]
^{94}Mo	15.78	15.90	16.4 ± 0.2 [43]
^{96}Mo	15.52	15.62	16.3 ± 0.2 [43]

Table 5. The values of the centroid energies $E_{\rm ISGMR}$ (in MeV) of Cd isotopes obtained from HF+CDFM calculations in this work using SLy4 and SkM Skyrme forces and $r_0 = 1.123$ fm. The experimental data are taken from Ref. [44].

Nucleus	SLy4	SkM	Exp.
^{106}Cd	16.15	16.27	16.27 ± 0.09
^{110}Cd	15.88	15.98	15.94 ± 0.07
^{112}Cd	15.73	15.89	15.80 ± 0.05
^{114}Cd	15.64	15.75	15.61 ± 0.08
^{116}Cd	15.49	15.67	15.44 ± 0.06

ter r_0 and this fact has to be taken into account when considering resonances in light, medium, and heavy nuclei.

The calculated values of $E_{\rm ISGMR}$ with SLy4 and SkM forces for Ca, Fe, Zn, Zr, Mo and Cd isotopes are given in Tables 4 and 5, respectively. An excellent agreement with the available experimental data is achieved for Ca isotopic chain and also for Cd chain. For the latter case our results fit very well the theoretical predictions from QRPA calculations for the ISGMR peaks obtained with the SV-bas Skyrme force [20].

4 Summary and Concluding Remarks

The main results of the present work can be summarized as follows:

i) A very good agreement is achieved between the calculated centroid energies of the ISGMR and corresponding experimental values for Ni isotopes when $r_0 = 1.2$ fm. Especially this concerns the exotic double-magic ^{56}Ni nucleus, for which the obtained (with SLy4 Skyrme force) value is 19.41 MeV, in consistency with the centroid position of the ISGMR found at 19.1 ± 0.5 MeV.

ii) The comparative analysis of the centroid energies in the case of Sn and Pb isotopes shows less agreement with $r_0 = 1.2$ fm, but still in acceptable limits.

iii) The agreement with the experimental values of $E_{\rm ISGMR}$ can be improved also by varying the parameter r_0 in strong connection with the mass dependence of this parameter and its effect for the considered isotopes.

iv) In general, the obtained results demonstrate the relevance of our theoretical approach to probe the excitation energy of the ISGMR in various nuclei. The future goals are to extend this theoretical study by employing more realistic energy-density functionals for nuclear matter and to expand the nuclear spectrum to lighter and medium mass nuclei including isotopes with large proton-neutron asymmetry.

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